Public enforcement, political competition, and the timing of sanctions*

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Abstract

This paper analyzes the determinants of the timing of sanctions when citizens and public authorities disagree on the assessment of harm associated with offenses. We consider two kinds of institutions that may influence the decisions of the public law enforcer: on the one hand, the public enforcer may behave as a benevolent planner; on the other hand, its decisions may be influenced by citizens’ pressure through electoral competition. A central result of the paper is that the utilitarian contract approach and the political market view yield opposite recommendations. We show that act-based sanctions are preferred to (dominated by) harm-based sanctions under the utilitarian (political competition) approach when the citizens’ expected harm is larger than the enforcer’s one. Moreover, we show that as compared to the utilitarian approach, political competition promotes harsher (less severe) punishments and more (less) deterrence when citizens’ expectation of harm is large (small) enough.

Keywords: timing of sanction, law enforcement, deterrence, monetary sanctions, political competition, majority rule, democracy.

JEL classification codes: D72, D73, H1, K14, K23, K32, K4, Q52, Q53, Q58.

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1 Introduction

In the classical analysis of public law enforcement, existing differences between the enforcer and citizens regarding information relative to the dangerousness of acts play a central role in order to explain the timing of enforcement measures (see for example Bowles, Faure and Garoupa, 2008; Shavell, 2005; Van der Bergh and Visscher, 2008). The usual justification relies on the failure or the lack of effectiveness of public campaigns informing citizens of the danger associated with some acts and promoting relevant behaviors in specific circumstances, which results from the costs of information transmission. The main prediction regarding the timing of enforcement measures is that the higher the quality of information held by the state relative to the knowledge of citizens themselves, the more attractive will be legal intervention focused at the early stages of private decisions and mostly, illegal behaviors. Following this perspective, the penal code and the structure of criminal punishments are generally viewed as harm-based, while regulatory laws and administrative sanctions (fines) are mainly seen as act-based.

It is obvious that this taxonomy of laws is not so ironclad, since criminal punishments for example are both act and harm-based in practice (see also Garoupa and Obidzinski, 2011). Thus, the issue of the structure of law and punishments is still in debate. The point deserves more focused analysis, since important factors may have been missed up to now or their influence have not been captured in a satisfying manner. The purpose of our paper is to provide a formal analysis of the timing of punishment, where the driving force is that citizens and enforcers have a disagreement regarding the assessment of the external harm associated with illegal behaviors. This disagreement may be explained by pure differences in the assessment of harms, a lack of citizens’ trust in public authorities, or a weakness of enforcer’s credibility (see our conclusion for a discussion). We believe that this is an important issue, which proves to be relevant and powerful both from the analytical and practical points of view.

Casual observations regarding the recent history in Europe, suggest a complex relationship between citizens’ expectations in terms of public policies and the response of public authorities. On the one hand, the issue of criminality is a main concern in electoral campaigns for more than two decades in most of the European countries; moreover, one can observe different attitudes (and the existence of a ongoing debate) among European countries about the criminalization/legalization of some offenses (drug consumption, prostitution, except in link with international traffics and criminal networks). On the other hand, the demand for new public regulations has increased among European citizens, regarding for example the spread of new technologies (GMOs, electromagnetic waves). After several nutritional and health crises with which Europe has been particularly confronted, European citizens have also exerted pressures in order to criminalize specific behaviors regarding the diffusion and use of many

\footnote{Confer to the so-called contaminated blood affair 1991, the episode of spongiform encephalitis 1986, Creutzfeldt-Jakob disease 1996, the avian 'flu crisis and the H5N1 virus, later the H1N1 virus (influenza A) 2009.}
products and substances in the area of environmental or health harms (dioxins, pesticides, medications). In the case of USA, Zimring and Johnson (2006) insist that it is the distrust into the law and politics that leads to the sharp expansion in repression of crimes experienced in UE after 1970. In their attempt to explain why policies against crimes became more and more severe after 1970, these authors argue that the (general) hostile attitude of the population about criminals and the (largely spread) belief that sanctions for criminals were too lenient can hardly be seen as powerful explanations: in a sense both are necessary conditions for the development of such repressive policies; but, to the extent that these feelings have been (and are) always present among the American population, they are not enough. For Zimring and Johnson (2006), the "growth in the salience of crime as a citizens concern and the increasing public distrust of government and legitimacy", mainly explain that these two ever-present feelings have been exacerbated and have fed a long run movement of fight against criminality with harsh punishments.

To our knowledge, the literature about public law enforcement has not yet addressed in a systematic way neither the issue of citizens’ distrust vis-à-vis public enforcers, nor the issue of the influence of democratic institutions and/or political pressures. The economic literature more focused on incentives and the design of regulation assumes a benevolent public authority and relies on informational asymmetries to explain the divergence in beliefs between the regulatory agency and the regulated party (see Laффont and Martimort, 2002; Laффont and Tirole, 1993). Jeleva and Rossignol (2009), Salanié and Treich (2009) discuss the consequences of a paternalistic (or populist) attitude on the part of public authorities, in a general context of the optimal regulation of a risky activity; their respective findings show that there is no general argument explaining that a utilitarian government produce levels of regulation which are higher than in other regimes (i.e. populist, paternalistic, or Leviathan). In the Law & Economics literature, Becker (1968) and Polinsky and Shavell (1979) considered the effects of risk aversion, but not the disagreement in risk assessment. Garoupa and Obidzinski (2010) assume that the public enforcer and the potential offenders do not assess the same value for the expected external cost of crime, and find that act and harm based sanctions have equivalent effects in terms of deterrence and social welfare. Finally, Garoupa and Klerman (2002) and Dittman (2005) have analyzed the issue of law enforcement with a rent seeking authority, but assumed that (the timing of) sanctions are (is) exogenously set (at the maximum level). Recent papers by Friche and Tabbach (2013), Leshem and Tabbach (2009) and Van Wijck (2011) analyze the rational for preventive enforcement measures by a benevolent policy maker, that is expenditures in the control of illegal activities that allow pre-crime interventions.

On the empirical side, Levitt (1997) analyses the influence of the electoral cycle on the orientation of public policies against criminality, and specifically, documents the increase in enforcement expenditures (mainly, number of police

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2Harel and Segal (1999) provide a general discussion about risk perception and crime deterrence.

3In a sense, their paper can be understood as considering the case of a paternalistic enforcer.
officers) before electoral terms. Attempts have been also realized in order to verify empirically whether differences exist in criminal law enforcement policies according to the age and quality of democracies. But both the economic literature (Dušek, 2005; Lin, 2007) as well as political sciences (Fernandez and Kuenzi, 2007; Karstedt and LaFree, 2006; LaFree and Tseloni, 2006; Zimring and Johnson, 2006) have been more interested in the performances of democracy regarding the level of criminality experienced both in the short and long run, than in the impact of democracy on the penal law content per se. Lin (2007) more specifically uses an index of political liberty from the comparative freedom survey to distinguish "low democracies" from "high democracies"; he has shown that countries characterized by a higher level of democracy tend to punish major crimes more severely as compared to countries with a low level of democracy, the reverse being true for minor crimes. He finds that the deterrence of homicides is quite strong\(^4\) and the homicide rate lower in high\(^5\) democracy by comparison with low democracy. On the contrary, it seems that democracy has a negative impact on less serious crimes such as burglary, robbery, car theft.

Thus, a more careful analysis of the impact citizens’ opinion on the design of public policies in the area of law enforcement, and specifically, on the timing of sanctions is required. Yet, little has been done also to understand how democratic institutions deal with differences in opinions in the area of law enforcement. Furthermore, it is not easy to understand how it may work, beyond a basic and too simple distinction between what is seen as legal and illegal behaviors.

In this paper, we focus on two alternative interpretations of democracy. First, we assume that democracy, as a political regime where every citizen is free (and more specifically, free to participate to the public debate and collective choices), is sustained by an agreement between citizens to commit to a social contract. We follow this way the standard utilitarian approach, according to which this social contract is defined with the respect to the collective preferences (as the solution to the maximization of the social welfare). Second, we analyze the effects that imperfections of the political market may have on the design of public policies in the area of crime deterrence. In this sense, collective preferences do not reflect the preferences of all citizens, but only those of the majority of voters. We introduce a basic model of electoral competition, where citizens vote between two candidates, and where we consider the simple majority rule. The main result of the paper is that these two conceptions of democracy yield opposite recommendations. We show that act-based sanctions are preferred to harm-based sanctions under the utilitarian approach when the citizens’ expected harm is larger than the enforcer’s one. In contrast, harm-based sanctions arise as an equilibrium with electoral competition under the same condition that the citizens’ expected harm is larger than the enforcer’s one. Moreover, we show that political competition promotes harsher (less severe) punishments and more (less) deterrence than the optimal (i.e. utilitarian)

\(^4\)According to multiple criteria: average prison length, average clearance rates.
\(^5\)The index of political liberty from the comparative freedom survey is used to distinguish "low democracies" from "high democracies".
ones, when citizens’ expectation of harm is large (small) enough.

The paper is organized as follows. Section 2 focuses on the case of a benevolent enforcer, and analyses the choice of the socially optimal timing of sanction (act-based versus harm-based sanction) when a disagreement exists between citizens and the enforcer regarding the probability of harm associated with illegal acts. Section 3 analyzes the timing of sanction that emerges at equilibrium when the political process is imperfect, using a simple model of downian electoral competition. Section 4 concludes.

2 Deterrence, sanction, and the utilitarian contract

We introduce here our basic framework, which elaborates on the model of law enforcement à la Becker. We depart from the usual approach only in the sense that we assume a disagreement exists between the population and the public enforcer regarding the assessment of (the value of) the expected external cost associated with offenses.

In this interpretation, democracy is seen as the result of an agreement between citizens to commit to a social contract. In the utilitarian tradition, this contract is based on the welfare of the society, defined as an aggregate of individual preferences. In our set up, this implies that punishment is the result of the decision of a utilitarian government.

2.1 general assumptions

Let us consider the case where the illegal activity allows the (risk neutral) criminal to obtain a benefit equal to $b$, while an honest citizen earns an income normalized to 0; $b$ will be called the type of the criminal. Public authorities do not observe the type $b$, and only know that $b$ follows a uniform distribution function on $[0, B]$. The (external) loss/harm to the rest of the society is $h$ in case of crime, whatever the private benefit for the criminal.

The citizens and the public authority do not agree on the probability of occurrence of $h$; for the former, the likelihood of $h$ is denoted as $\sigma \in [0, 1]$, while $\pi \in [0, 1]$ is for the government. We require the following assumption to hold in the text:

Assumption 1: $B > \max\{\pi, \sigma\} \times h$.

This assumption affords the advantage that any quantity $\max\{\pi, \sigma\} \times \frac{h}{B}$ has the dimension of a probability throughout the paper. One of the main consequence of this assumption is also that it does not matter (see Kaplow (1992) for this discussion) whether acts are definitively undesirable ($h > B$) or not ($h < B$).
Monitoring the criminal activity entails a cost for public authorities, equal to \( m(p) \), where for the sake of simplicity \( p \) is the probability of control (encompassing arrest, conviction and punishment for an illegal behavior). As is usual in the literature, we assume that this cost is financed through a lump sum tax \( t \) plus the expected fine levied on the fraction of the population which is seen as criminal (either for whom the harmful activity entails the harm, or is not deterred from committing the crime). However, we assume that expenditures in the monitoring of criminals’ behavior are exogenously set throughout in the text.

We focus here on punishment, i.e. the choice of a monetary sanction (penalty or fine) \( f > 0 \). We assume that the management costs associated with the monetary penalty are negligible. We also assume that the maximal fine is the legal wealth of the population \( w \), i.e. \( f \in [0, w] \): throughout of the paper, we will consider that \( w \) is large enough in order that (almost) all equilibria are defined as interior solutions.

The game (sequence of moves) between the public authority and the citizens is the usual one: after Nature move (choosing the type of citizens, not observable for public authorities) at stage 0, the authority makes at stage 1 its announcement regarding the relevant regime of sanction and the level of fine applied; at stage 2, citizens decide whether or not they abide the law; at stage 3, the law is enforced.

### 2.2 act-based sanction

We first analyze the behavior of citizens, and then describe the enforcement strategy of the authority when it is assumed to behave according to a perfectly benevolent planner. We assume that the cost a crime imposes to the society is defined as a externality term affecting individuals’ utility level, with a very simple formulation: \( E = -qh \), where \( q \in [0, 1] \) is the probability of crime. As usual in the literature on crime enforcement, we will show that \( q = \Pr(b \geq \hat{b}) \), with \( \hat{b} \) denoting the deterrence threshold (to be more specifically defined therein). Hence \( q = \int_{\hat{b}}^{B} \frac{1}{\pi} db = \left( 1 - \frac{\hat{b}}{\pi} \right) \).

Under the act-based regime, let us denote the utility level of a criminal as:

\[
\begin{align*}
u_c^a &= w + b - t - pf - q\sigma h \\
&= w + b - t - pf - \left( 1 - \frac{\hat{b}}{B} \right) \sigma h
\end{align*}
\]

where \( \hat{b} \) is for the level of deterrence, while for an honest individual, we have:
\[ u_h^a = w - t - q\sigma h \]
\[ = w - t - \left( 1 - \frac{\hat{b}}{B} \right) \sigma h \]

Hence as usual, \( \hat{b} \) is defined by \( v_c = v_h \); under the act-based regime, a potential criminal decides to undertake the activity if the benefit he retrieves from doing so is higher that the expected punishment, i.e. if \( b \geq pf = \hat{b} \).

Regarding the behavior of the public enforcer, the public budget constraint writes as:

\[ m(p) = t + \left( 1 - \frac{\hat{b}}{B} \right) pf \] (1)

In a sense, we consider here only balanced-budget policies.

Assume that the policy maker acts as a benevolent planner, and uses a pure utilitarian criterion: the social welfare function is the sum of \( u_h^a \) and \( u_c^a \) given the structure of the population. Then, the social function of a benevolent planner writes as:

\[
S^a = \frac{1}{B} \int_0^\hat{b} u_h^a db + \frac{1}{B} \int_\hat{b}^B u_c^a db
\]
\[ = w - t + \frac{1}{B} \int_\hat{b}^B (b - pf - \sigma h) db \] (2)

and substituting with (1) yields:

\[
S^a = w - m(p) + \frac{1}{B} \int_{pf}^B (b - \sigma h) db \] (3)

The integral term in \( S^a \) corresponds to the expected private benefit net of the external cost associated with the illegal activity. The other terms (exogenous here) are the citizens’ wealth net the cost of monitoring for public authorities. In the case of a act-based sanction, the fine paid by the criminal when arrested is a mere transfer (the perceived probability of paying the fine, is equal to the perceived probability of collecting it). Note that the problem of the enforcer does not depend on \( \pi \), its own probability that the harm occurs.

**Proposition 1** (Act-based sanctions under a utilitarian contact)

The social welfare-maximizing sanction is \( f_u^a = \frac{\sigma}{p} h \), and is associated with a probability of crime equal to \( q_u^a = 1 - \frac{\sigma}{B} h \).
Proof. The derivative of $S^a$ with respect to $f$ is given by:

$$\frac{\partial S^a}{\partial f} = -(b - \sigma h)p = (\sigma h - pf)p$$

implying that $\frac{\partial S^a}{\partial f} |_{f=0} > 0$, and thus $f > 0$. Hence, assuming that $\frac{\partial S^a}{\partial f} |_{f=w} < 0 \Leftrightarrow \sigma h < pw \Leftrightarrow h < h_1 \equiv \frac{p}{\sigma}w$, and solving the FOC for the optimal sanction $f^a_u$ yields:

$$f^a_u = \frac{\sigma}{p} h$$

The associated probability of crime is $q^a_u = 1 - \frac{pf}{B} = 1 - \frac{\sigma}{p} h$. ■

We consider now a regime based on a harm-based sanction, such that the observation of the illegal act is not enough to punish an individual; in contrast, this requires the observation of the wrongful, undesirable consequences of the acts.

### 2.3 harm-based sanction

Under the harm-based regime, the utility level of a criminal is:

$$u^h_c = w + b - t - p\sigma f - \left(1 - \frac{b}{B}\right) \sigma h$$

where $b$ is the level of deterrence, while for an honest individual, we have:

$$u^h_h = w - t - \left(1 - \frac{b}{B}\right) \sigma h$$

Using the conditions $u_c = u_h$, we obtain for a deterrence level equal to $b = pf$. A straightforward noticeable implication is that all else equal, i.e. for a given level of $f$ the sanction, $\bar{b} < b$: the harm-based regime allows less deterrence than the act-based one. But the analysis has to be completed, taking into account how the enforcer sets the punishment.

The budget constraint of the enforcer writes now as:

$$m(p) = t + \left(1 - \frac{b}{B}\right) p\pi f$$

In this case, the government has to choose a fine $f$ in order to maximize the social welfare function:

$$S^h = \frac{1}{B} \int_0^\bar{b} u^h_c db + \frac{1}{B} \int_{\bar{b}}^B u^h_c db$$

$$= w - t + \frac{1}{B} \int_{\bar{b}}^B (b - p\sigma f - \sigma h) db$$

$$= w - t + \frac{1}{B} \int_{\bar{b}}^B (b - p\sigma f - \sigma h) db$$

8
taking into account for the budget constraint (4). Substituting (4) in (5) yields:

\[ S^h = w - m(p) + \frac{1}{B} \int_{\rho \sigma f}^{B} (b - \sigma h + (\pi - \sigma) p f) dB \]  

(6)

As long as \( \sigma \neq \pi \), the fine is no longer a mere transfer between the (risk neutral) criminal and the government.

We investigate the choice by the utilitarian regulator of the optimal monetary sanction \( f^h > 0 \) (we do not consider the issue of a corner solution with \( f = 0 \) which is of a weaker interest here).

**Proposition 2** (Harm-based sanctions under a utilitarian contract)

i) If \( \pi \leq \frac{\sigma}{2} \), the social welfare maximizing sanction satisfies \( f^h = w \), with a probability of crime equal to \( q^h = 1 - \frac{\sigma}{B} w \).

ii) If \( \pi > \frac{\sigma}{2} \) and \( \frac{\pi}{2} B > \sigma h > (1 - \frac{\pi}{2}) B \), the social welfare maximizing sanction is \( f^h = \frac{1}{2\pi - \sigma} \left( \frac{\alpha}{p} h + \left( \frac{\pi - \sigma}{\rho \sigma} \right) B \right) \), with a probability of crime equal to \( q^h = \frac{\sigma}{2\pi - \sigma} \left( \frac{\pi}{2} h - \frac{\sigma}{2} h \right) \).

**Proof.** According to the derivative of \( S^h \), we have:

\[ \frac{\partial S^h}{\partial f} = p \left[ \frac{\sigma^2}{B} h + (\pi - \sigma) - (2\pi - \sigma) \frac{\rho \sigma}{B} f \right] \]

Let us assume that \( \frac{\partial S^h}{\partial f} \big|_{f=0} > 0 \Leftrightarrow \frac{\sigma^2}{B} h + (\pi - \sigma) > 0 \Leftrightarrow \frac{\sigma}{p} h + \left( \frac{\pi - \sigma}{\rho \sigma} \right) B > 0 \); as a result, it comes that:

- \( \pi \leq \frac{\sigma}{2} \Rightarrow \frac{\partial S^h}{\partial f} > 0 \forall f \); hence: \( f^h = w \). Note that the condition \( B > \rho \sigma w \) is required in order that \( q^h = 1 - \frac{\sigma}{B} w > 0 \).

- or consider that \( \pi > \frac{\sigma}{2} \), and let us assume that \( \frac{\partial S^h}{\partial f} \big|_{f=w} < 0 \Leftrightarrow h < h_5 \equiv (2\pi - \sigma) \frac{\sigma}{2} w - \left( \frac{\pi - \sigma}{\rho \sigma} \right) B \). Checking for the FOC in \( f \), \( \frac{\partial S^h}{\partial f} = 0 \), and solving for the optimal \( f^h \) yields:

\[ f^h = \frac{1}{2\pi - \sigma} \left( \frac{\alpha}{p} h + \left( \frac{\pi - \sigma}{\rho \sigma} \right) B \right) \]

which is positive under the condition \( \frac{\alpha}{p} h + \left( \frac{\pi - \sigma}{\rho \sigma} \right) B > 0 \Leftrightarrow \sigma h > (1 - \frac{\pi}{\sigma}) B \). Note that the condition \( \frac{\sigma}{p} h + \left( \frac{\pi - \sigma}{\rho \sigma} \right) B > 0 \Leftrightarrow \frac{\pi}{2} B > \sigma h \) is required in order that \( q^h = 1 - \frac{\rho \sigma f^h}{B} = \frac{\sigma}{2\pi - \sigma} \left( \frac{\pi}{2} - \frac{\sigma}{2} h \right) > 0 \).

The misalignment of beliefs between the public enforcer and the population affects the policy in this harm-based regime, and thus it is useful to have a broader look at this issue here. It can be verified that in the case where the expected value of the external cost for public enforcers is larger than those of the citizens because \( \pi > \sigma \), a solution with \( f > 0 \) always exists. The intuition is that (see in (6) the term \( (\pi - \sigma) pf > 0 \) in the integral) punishing the deviant
behaviors is socially beneficial: raising $f$ has a positive (direct) effect of the social welfare. However, the authority is facing a trade-off, since on the other hand, raising $f$ has an indirect effect on the population (the lower bond of the integral –level of deterrence – increases) which is negative. Now, to the converse, assuming that the expected value of the external cost for public enforcers is smaller than those of the citizens (since $\pi < \sigma$), it may be better not to punish at all the offenders. The intuition is now that punishing the offenders may socially wasteful (since $(\pi - \sigma) pf < 0$). Then, in the limit case where citizens’ belief is small in the sense that $\sigma h < (1 - \frac{\pi}{\sigma}) B$, it is not socially worth to punish the offenders ($f = 0$). Thus punishing the illegal behaviors should not be used except when the external cost of crime is large enough in the sense that $\sigma h > (1 - \frac{\pi}{\sigma}) B$ – in a sense, the trade-off is now that reducing the sanction increases the social welfare (direct effect) but at a cost which is that the number of crimes not deterred increases (indirect effect).

2.4 the optimal timing under the utilitarian contract

Since the objective of the benevolent planner is the maximization of the social welfare, we now investigate the endogenous choice of the regime of sanctions. First, we compare the level of deterrence obtained in each case:

Proposition 3 (act versus harm based sanction, and deterrence)

i) If $\pi \in [0, \frac{\pi}{\sigma}] \cup [\sigma, 1]$, then the act-based sanction is smaller than the harm-based one: $f_u^a < f_u^h$.

ii) If $\pi \in [\frac{\pi}{\sigma}, \sigma]$, then the act-based sanction may be higher or smaller than the harm-based one.

iii) Assume $\pi < \frac{\pi}{\sigma}$; then $q_u^a < q_u^h$ if $\frac{b}{p} > w$, whereas $q_u^a > q_u^h$ if $\frac{b}{p} < w$.

iv) Assume $\pi > \frac{\pi}{\sigma}$; then $q_u^a < q_u^h$ if $(\pi - \sigma) (\sigma h - \frac{B}{\pi}) > 0$, but $q_u^a > q_u^h$ if $(\pi - \sigma) (\sigma h - \frac{B}{\pi}) < 0$.

v) Assume $\pi = \sigma$; then $f_u^a < f_u^h$ and $q_u^a = q_u^h$.

Proof. Note that the conditions $\sigma h < pw \text{ and } \sigma h > (1 - \frac{\pi}{\sigma}) B$ are required by definition of $f_u^a$ and $f_u^h$ respectively.

i) When $\pi < \frac{\pi}{\sigma}$ the result is direct since $f_u^h = w$. In contrast, assuming that $\pi > \frac{\pi}{\sigma}$, we have:

$$f_u^h - f_u^a = \frac{1}{2\pi - \sigma} \left( \frac{\sigma}{p} h + \left( \frac{\pi - \sigma}{p\sigma} \right) B - \frac{\sigma}{p} h \right)$$

$$= \left( \frac{1 - \pi}{2\pi - \sigma} \right) \frac{\sigma}{p} h - \left( \frac{\pi - \sigma}{2\pi - \sigma} \right) \left( \frac{\sigma}{p} h - \frac{B}{p\sigma} \right)$$

In the appendix 1, we compare the level of sanctions, and show that $f_u^a - f_u^h$ has generally an ambiguous sign, and is more demanding in terms of qualifications as compared to the levels of deterrence.
where \( \sigma h - \frac{B}{\sigma} < 0 \), since using assumption 1: \( \frac{B}{\sigma} > B > \sigma h \). Hence \( \pi - \sigma > 0 \) implies \( f_u^h > f_u^a > 0 \).

ii) Consider now that \( \pi \in ]\frac{\sigma}{2}, \sigma[ \), and assume for example that \( f_u^h < f_u^a \); then it comes:

\[
\left( \frac{\sigma}{p} h + \left( \frac{\pi - \sigma}{p \sigma} \right) B \right) \frac{p}{\sigma h} < 2\pi - \sigma
\]

\[
\Downarrow
\]

\[
\pi \left( \frac{B}{\sigma h} - 2\sigma \right) < \sigma \left( \frac{B}{\sigma h} - (1 + \sigma) \right)
\]

Remark that if \( B \) is large enough, then it is reasonable to assume \( \frac{B}{\sigma h} - (1 + \sigma) > 0 \) which also implies \( \frac{B}{\sigma h} - 2\sigma > 0 \), and thus a sufficient condition for \( f_u^h < f_u^a < 0 \) is \( \pi < \sigma \frac{B}{\sigma h} - (1 + \sigma) \) where \( \frac{B}{\sigma h} - (1 + \sigma) < 1 \). Otherwise, we may have \( f_u^h > f_u^a > 0 \).

iii) Assuming \( \pi < \frac{\sigma}{2} \), we have \( f_u^h = w \) and thus:

\[
q_u^a - q_u^h = \frac{\sigma}{B} (pw - h)
\]

Hence the result.

iv) Assuming now \( \pi > \frac{\sigma}{2} \), it is straightforward that we obtain:

\[
q_u^a - q_u^h = -\frac{2}{B} \left( \frac{\pi - \sigma}{2\pi - \sigma} \right) \left( \sigma h - \frac{B}{\sigma} \right)
\]

and thus, \( \text{sign} \left( q_u^a - q_u^h \right) = -\text{sign} \left( \pi - \sigma \right) \times \left( \sigma h - \frac{B}{\sigma} \right) \). In the next table, we consider the implications for the probability of crime in the different possible cases:

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<th>( \sigma h &lt; \frac{B}{\sigma} )</th>
<th>( \sigma h &gt; \frac{B}{\sigma} )</th>
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<td>( \pi &lt; \frac{\sigma}{2} )</td>
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<td>( \pi &gt; \frac{\sigma}{2} )</td>
<td>( q_u^a &gt; q_u^h )</td>
<td>( q_u^a &lt; q_u^h )</td>
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v) Self-evident.

A main consequence of proposition 3 is that depending on the importance of the disagreement between the citizens and the enforcer (size of the difference \( \pi - \sigma \)), the optimal level of act-based sanction may be higher or lower than the harm-based. This is in contrast to case v), where both agree on the probability of the external harm, implying that the harm-based sanction is the larger, while yielding the same deterrence level (see also the discussion of probabilistic harm in Shavell, 2005).

Remark also that by definition \( \text{sign} \left( \hat{b}_u^a - \hat{b}_u^h \right) = -\text{sign} \left( q_u^a - q_u^h \right) \). Note that proposition 3 means that a harm-based one may induce either more deterrence
(for example, once \( \pi > \frac{\sigma}{2} \), when \( (\pi - \sigma) \times (\sigma h - \frac{B}{2}) < 0 \) or in contrast less deterrence (when \( (\pi - \sigma) \times (\sigma h - \frac{B}{2}) > 0 \)), as compared to the act-based sanction. Under a harm-based sanction, the lower level of deterrence occurs because either the public enforcer overestimates the large expected loss that citizens associate with crime (i.e. \( \pi h > \sigma h > \frac{B}{2} \)), or on the contrary, underestimates the small expected loss that citizens associate with crime (i.e. \( \pi h < \sigma h < \frac{B}{2} \)). On the other hand, a higher level deterrence occurs under a harm-based sanction because the public enforcer’s expected loss in case of crime is closer than citizens’ expectation to the mean of illegal benefits (\( \sigma h < \pi h < \frac{B}{2} \) or \( \pi h > \sigma h > \frac{B}{2} \)). Finally, both regimes entail the same level of deterrence when \( (\pi - \sigma) (\sigma h - \frac{B}{2}) = 0 \).

We now discuss the issue of the optimal sanction, using the results of the comparison of welfare levels associated with each regime.

**Proposition 4** *(act versus harm based sanction, and social welfare)*

i) If \( \pi < \sigma \), then the act-based sanction gives a social welfare higher than the harm-based sanction: \( S^a - S^h > 0 \).

ii) If \( \pi > \sigma \) and \( \sigma h > \frac{B}{2} \), then the act-based sanction gives a social welfare smaller than the harm-based sanction: \( S^a - S^h < 0 \).

iii) If \( \pi = \sigma \), the act-based and the harm-based sanction are equivalent in terms of social welfare: \( S^a - S^h = 0 \).

**Proof.** We have after integrating:

\[
S^a - S^h = \frac{1}{B} \int_{\hat{b}^u_a}^{\hat{b}^h} (b - \sigma h) db - \frac{1}{B} \int_{\hat{b}^h}^{B} (\pi - \sigma) pf_u db
\]

\[
= \frac{1}{2B} (\hat{b}^h_u - \hat{b}^a_u)^2 - \left( \frac{\pi - \sigma}{\sigma} \right) \hat{b}^h_u \left( 1 - \frac{\hat{b}^h_u}{B} \right)
\]

i) Thus, if \( \pi < \sigma \) it comes that \( S^a - S^h > 0 \).

ii) But if \( \pi > \sigma \), we now have \(- \frac{1}{B} \int_{\hat{b}^h}^{B} (\pi - \sigma) pf_u db < 0 \), implying that the sign of \( S^a - S^h \) is ambiguous a priori. Remark that \( \hat{b}^a_u \) does not depend on \( \pi \); we obtain:

\[
\frac{\partial}{\partial \pi} (S^a - S^h) = \frac{1}{B} \left( \hat{b}^h_u - \hat{b}^a_u \right) \frac{\partial \hat{b}^a_u}{\partial \pi} - \frac{\partial}{\partial \pi} \left( \left( \frac{\pi - \sigma}{\sigma} \right) \hat{b}^h_u \left( 1 - \frac{\hat{b}^h_u}{B} \right) \right)
\]

with:
\[
\frac{\partial}{\partial \pi} \left[ \left( \frac{\pi - \sigma}{\sigma} \right) \hat{b}_u \left( 1 - \hat{b}_u \right) \right] \\
= \left( \frac{\hat{b}_u}{\sigma} \right) \left( 1 - \hat{b}_u \right) + \frac{2}{B} \left( \frac{\pi - \sigma}{\sigma} \right) \left( B - \hat{b}_u \right) \frac{\partial \hat{b}_u}{\partial \pi}
\]

Observe that:

\[
\frac{1}{B} \left( \hat{b}_h - \hat{b}_u \right) \frac{\partial \hat{b}_u}{\partial \pi} - \frac{2}{B} \left( \frac{\pi - \sigma}{\sigma} \right) \left( B - \hat{b}_u \right) \frac{\partial \hat{b}_u}{\partial \pi}
\]

\[
= 2 \left( \frac{\pi - \sigma}{2\pi - \sigma} \right) \left( \sigma h - \frac{B}{2} \right) \frac{\partial \hat{b}_u}{\partial \pi}
\]

Moreover, \( \frac{\partial \hat{b}_h}{\partial \pi} = \frac{2}{2\pi - \sigma} \left( \frac{B}{2} - \hat{b}_u \right) \) with \( \frac{B}{2} - \hat{b}_u = \left( \frac{\pi - \sigma}{2\pi - \sigma} \right) \left( B - \sigma h \right) \). Hence, when \( \pi > \sigma \), it comes that \( \sigma h > \frac{B}{2} \implies \frac{\partial \hat{b}_h}{\partial \pi} < 0 \), which is sufficient to obtain \( \frac{\partial}{\partial \pi} \left( S_h - S_h \right) < 0 \).

iii) Self-evident given proposition 3. 

The main conclusion of proposition 4 is that when the enforcer holds a probability of social harm smaller than citizens \( \pi < \sigma \), then the choice of the act-based regime is socially optimal. But when \( \pi > \sigma \), the comparison of the welfare levels associated with both regimes of sanction is generally ambiguous, since the payment of fines based on the observation of harm adds social value under the harm-based regime. Sanctions are no longer a mere transfer between criminals and the government, but have a positive social value. In a sense, whether the act-based sanction allows more deterrence, by itself, does not matter for its superiority as compared to the harm-based sanction. For example, the act-based sanction may allow a higher welfare than the harm-based one despite a smaller level deterrence.

What may be found uncomfortable with the utilitarian approach for the purpose of explaining the structure and content of the penal code is that social preferences are given, and reflect both the preferences of honest and criminal citizens\(^7\). From this point of view, it is astonishing that the debate in the Law & Economics literature regarding the objectives of the law has focused on the issue

\(^7\)Since Stigler (1970), the introduction of illegal gains in the social value function is a controversial issue. Both the significance and the objective of the penal code are still in debate among scholars; see Dau-Schmidt (1990) and Lewin and Trumbull (1990).
of efficiency vs fairness (Dari-Mattiacci and Garoupa (2007), Fleurbaey, Tungodden and Chang (2003) and Kaplow and Shavell (2001)), in a sense ignoring to a large extent the more general debate in Social Choice theory initiated after the seminal work of Arrow (1951). The next section consider the case where social preferences are revealed through the political process, which is relevant in democratic countries. We introduce here a simple model of electoral competition, in the vein of the framework known as Dowsian electoral competition (see Persson and Tabellini, 2000).

3  Deterrence, sanction, and the political market

The general set up previously used is maintained for the benevolent government (i.e. characteristics of the population of citizens), except for the political process leading to the choice of the regime of sanction – the issue here is how citizens vote for a new government (public authority). In this interpretation, social preferences are revealed through a political process which is not perfect, in the sense that the collective preferences do not reflect the preferences of all citizens, but only those of the majority of voters. In our set up, this means that sanctions (and other collective decisions) are the output of an electoral competition.

3.1  a basic model of electoral competition

We focus on electoral strategies based on a policy against criminality, and which allow candidates to maximize their chances to win the elections. Our main focus here is to discuss in a simple framework whether/how democracy and political competition may promote the toughness of both the sanctions and the standards of proof, in the various domains of the penal law.

Assume there exist two candidates $i = l, r$, representative of two political parties, competing for national (presidential or legislative) or local (municipal) elections. Competing for elections here is alike a rent seeking contest, where $V$ the exogenous rent obtained in case of victory is attached to holding offices, ministries and so on. Thus, we do not consider the case for opportunistic politicians, responding to lobbies pressures or adopting Leviathan behavior (following strategies of capture).

A programme or electoral platform $(R, f)$ consists in a regime of sanction $R$ and a fine $f$. We consider for obvious reasons that their exists only two regimes of sanction – the harm-based regime, or the act-based regime: $R \in \{ R^h, R^a \}$. Regarding the monetary sanction, we still assume that the maximal fine is the legal wealth of the population $w$. The objective of politician $i$ is to maximize the expected value of the rent $\alpha_i V$, where $\alpha_i$ is the probability that he wins the elections. To this end, candidate $i$ proposes to electors a electoral platform $(R_i, f_i)$ that will indeed maximizes $\alpha_i$. We consider the (simple) majority rule for voting. All citizens are electors and do participate: each voter simply votes
for the candidate whose platform allows him to reach the highest utility level, and in case where he is indifferent, he tosses a coin to decide for whom he votes.

Let us denote as \( n(R_l, f_l) = n_l \in [0, 1] \) the proportion of the population voting for \((R_l, f_l)\) candidate \(l\) (with \(n_r = 1 - n_l\) the complementary proportion of the population voting for \((R_r, f_r)\)). Then the probability that candidate \(l\) wins the elections can take three values:\(^8\)

\[
\alpha_l = \begin{cases} 
0 & \text{if } n_l < 1 - n_l \\
\frac{1}{2} & \text{if } n_l = 1 - n_l \\
1 & \text{if } n_l > 1 - n_l 
\end{cases}
\]

When both candidates obtain half of the voters, the winner is chosen tossing a coin.

The timing of the electoral competition game between the candidates and the citizens/voters is as follows: after that Nature moves at stage 0 (choosing the type of citizens, not observable for politicians), the electoral competition begins at stage 1, which is a simultaneous move (non cooperative) game between the candidates, where they both choose and announce their platform \((R_l, f_l), (R_r, f_r)\) both satisfying the balanced budget constraint (1); at stage 2, elections take place, and citizens simultaneously choose between the two candidates; at stage 3, the elected candidate implements his policy\(^9\) – it becomes a law; at stage 4, citizens choose to abide or not the law; at stage 5, the law is enforced.

### 3.2 equilibrium timing and the majority rule

We first consider the equilibria\(^10\) obtained when \(\sigma \neq \pi\) and \(\sigma h \neq \frac{B}{2}\); we denote as \(f_h^i\) (\(f_c^i\)) the choice of sanction by the honest (respectively, criminal) population in regime \(i = a, h\).

**Proposition 5** When \(\sigma \neq \pi\) and \(\sigma h \neq \frac{B}{2}\), the unique symmetric equilibrium has the following features:

i) Assume \(\sigma h > \frac{B}{2}\); then:

- If \(\sigma - \pi > 0\), the unique equilibrium consists in both candidates announcing \((R_h, f_h^h)\) where \(f_h^h = \frac{1}{2} \left[ \frac{\sigma h}{p} + \frac{B}{p} \right] \); and the probability of crime is \(q_h^h = \frac{1}{2} \left[ 1 - \frac{\pi^2 h}{\sigma h} \right] \).

- If \(\sigma - \pi < 0\), the unique equilibrium consists in both candidates announcing \((R_a, f_h^a)\) where \(f_h^a = \frac{1}{2} \left[ \frac{\pi h}{p} + \frac{B}{p} \right] \); and the probability of crime is \(q_h^a = \frac{1}{2} \left[ 1 - \frac{\pi h}{\pi h} \right] \).

ii) Assume \(\sigma h < \frac{B}{2}\); then:

\(^8\)When both candidates obtain half of the voters, the winner is chosen tossing a coin.

\(^9\)i.e., we assume that candidates commit to their own electoral platform.

\(^10\)The term "equilibrium" throughout the paper is for "Nash subgame perfect equilibrium". Note that we focus here only on pure strategies equilibria.
If \( \sigma - \pi > 0 \), the unique equilibrium consists in both candidates announcing \((R^h, f^h_c)\) where \( f^h_c = \frac{1}{2} \left( \frac{\sigma h}{p} + \frac{(\pi - \sigma)}{\pi} \frac{B}{p \sigma} \right) \), and the probability of crime is \( q^h_c = \frac{1}{2} \left[ 1 + \frac{\sigma}{\pi} (1 - \frac{\pi}{\sigma} h) \right] \).

If \( \sigma - \pi < 0 \), the unique equilibrium consists in both candidates announcing \((R^a, f^a_c)\) where \( f^a_c = \frac{1}{2} \frac{\sigma h}{p} \); and the probability of crime is \( q^a_c = 1 - \frac{1}{2} \frac{B}{h} \).

The proof is provided in Appendix 1. Remark also that each candidate wins the elections with probability \( \frac{1}{2} \) at equilibrium, in each case considered. Moreover, we show in Appendix 2 that we have both \( f^h_a > f^a_h \) and \( f^h_c > f^a_c \).

In the previous section, we have seen that the sign of \((\pi - \sigma) \times (\sigma h - \frac{B}{2})\) is governing the sign of \( \hat{b}^a_h - \hat{b}^h_c \), although the choice between a act-based or a harm-based sanction mainly relies on the sign of the difference \( \pi - \sigma \). Proposition 5 shows that under political competition, both the sign of \((\pi - \sigma) \times (\sigma h - \frac{B}{2})\) may have their own influence on the electoral equilibrium and its outcome. This also means that whatever the regime of sanctions arising, it may be associated either with a strong enforcement equilibrium (i.e. high level of punishment) or in contrast with a weak enforcement equilibrium (i.e. low level of punishment).

In the case where \( \sigma h > (\pi) \frac{B}{2} \) a strong (weak) enforcement equilibrium occurs in the sense that the high (low) level of sanction preferred by the honest citizens (criminals). Accordingly, a high (low) level of deterrence is observed in a strong (weak) enforcement equilibrium. Now if \( \sigma - \pi > 0 \), then the harm-based sanction arises at equilibrium; in contrast, if \( \sigma - \pi < 0 \), then the act-based sanction arises at equilibrium.

The next proposition compares the severity of the sanction and the probability of crime (level of deterrence) in the strong and weak enforcement equilibria.

**Proposition 6** When \( \sigma \neq \pi \) and \( \sigma h \neq \frac{B}{2} \):

i) In a strong enforcement equilibrium, the harm-based regime gives a sanction higher than the act-based one: \( f^h_h > f^a_h \), and a smaller probability of crime: \( q^h_h < q^a_h \).

ii) In a weak enforcement equilibrium, the harm-based regime gives a probability of crime higher than the act-based one: \( q^h_h > q^a_h \). Define \( \hat{\pi} = \frac{B - \sigma h}{\sigma \pi - \sigma h} \); if \( \pi < \hat{\pi} \), the harm-based regime gives a sanction smaller than the act-based one: \( f^h_h < f^a_h \); if \( \pi > \hat{\pi} \), then the harm-based regime gives a sanction higher than the act-based one: \( f^h_h > f^a_h \).

**Proof.** i) Let us remark that \( f^h_h \) and \( \hat{b}^h_h \) do not depend on \( \pi \) the politicians’ belief, while \( f^a_h \) and \( \hat{b}^a_h \) do (moreover with \( \pi < \sigma \)). Thus, the results are easy to obtain:

\[
\begin{align*}
    f^h_h - f^a_h &= \frac{1}{2} \left( \frac{\sigma h}{p} \left( \frac{1}{\pi} - 1 \right) + \frac{B}{p} \left( \frac{1}{\sigma} - 1 \right) \right) > 0 \\
    q^h_h - q^a_h &= \frac{\sigma h}{2B} \left( 1 - \frac{\pi}{\sigma} \right) < 0
\end{align*}
\]
ii) Let us remark that now \( f^h_c \) and \( \hat{b}^c \) do not depend on \( \pi \) the politicians’ belief, while \( f^h_a \) and \( \hat{b}^h \) do with \( \pi < \sigma \). Thus, we obtain now:

\[
f^h_c - f^a_c = \frac{1}{2} \left( \frac{1}{\pi} \left( \frac{\sigma h}{p} + \frac{(\pi - \sigma)}{p\sigma} B \right) - \frac{\sigma h}{p} \right) \geq 0
\]

The sign depends on the sign of \( \frac{1}{2} \left( \frac{\sigma h}{p} + \frac{(\pi - \sigma)}{p\sigma} B \right) - \frac{\sigma h}{p} \). We have:

\[
\frac{1}{\pi} \left( \frac{\sigma h}{p} + \frac{(\pi - \sigma)}{p\sigma} B \right) - \frac{\sigma h}{p} > 0
\]

\[\Downarrow\]

\[
\frac{\pi}{\sigma} \left( \frac{B}{\sigma} - \sigma h \right) > \frac{B}{\sigma} - h
\]

\[\Downarrow\]

\[
\pi > \frac{\frac{B}{\sigma} - h}{\frac{B}{\sigma} - \sigma h}
\]

with: \( \frac{B}{\sigma} - h < \frac{B}{\sigma} - \sigma h \); thus \( \frac{B - \sigma h}{\frac{B}{\sigma} - \sigma h} = \tilde{\pi} < 1 \). As a consequence, if \( \pi > \tilde{\pi} \) then \( f^h_c - f^a_c > 0 \). But if \( \pi < \tilde{\pi} \) we obtain the opposite result. On the other hand, we have:

\[
q^h_c - q^a_c = \frac{1}{2} \left( 1 + \frac{\sigma}{\pi} \left( 1 - \frac{\sigma h}{B} \right) \right) - \left( 1 - \frac{1}{2} \frac{\sigma h}{B} \right)
\]

\[= \frac{1}{2} \left( \frac{\sigma}{\pi} - 1 \right) \left( 1 - \frac{\sigma h}{B} \right) > 0\]

under assumption 1, and since \( \sigma > \pi \) here. ■

A main consequence of proposition 6 is that the strongest enforcement equilibrium corresponds to the one associated with \( (R^h, f^h_h) \). In this case, electoral competition yields the smallest probability of crime: \( q^h_h = \frac{1}{2} \left[ 1 - \frac{\sigma^2 h}{\pi \cdot B} \right] \).

Thus, in cases where \( \sigma h > \frac{B}{2} \), the outcome that results from political competition works depends on whether politicians under-estimate or in contrast over-estimate citizens’ belief. In the first case (\( \sigma - \pi > 0 \)), political competition minimizes the probability of crimes; in the second (\( \sigma - \pi < 0 \)), political competition damps the impact on deterrence.

It is useful to have a quick look at the issue of multiplicity of equilibria. \(^{11}\) Let us consider all equilibria obtained when \( \sigma h = \frac{B}{2} \) – which may be understood as

\(^{11}\)Separating cases, it can be shown that:

i) when \( \sigma h = \frac{B}{2} \), then any combination of announcements \( (R^h, f^h_h), (R^h, f^h_h) \) is an equilibrium if \( \sigma - \pi > 0 \); and any combination of announcements \( (R^h, f^h_h), (R^h, f^h_h) \) is an equilibrium if \( \sigma - \pi < 0 \).

ii) when \( \sigma = \pi \), then any combination of announcements \( (R^h, f^h_h), (R^h, f^h_h) \) is an equilibrium if \( \sigma h - \frac{B}{2} > 0 \); and any combination of announcements \( (R^h, f^h_h), (R^h, f^h_h) \) is an equilibrium if \( \sigma h - \frac{B}{2} < 0 \).
cases where the expected harm is close to the mean of the distribution of criminal benefits – it can be shown that the harm-based regime arises at equilibrium when $\sigma - \pi > 0$, whereas the act-based regime arises at equilibrium when $\sigma - \pi < 0$. Now let us consider all equilibria obtained when $\sigma = \pi$ – which may be understood as cases where there is an agreement between citizens and politicians regarding the expected harm associated to crimes – it can be shown that a strong enforcement equilibrium, associated with a large level of deterrence arises when $\sigma h > \frac{B}{2}$, a weak enforcement equilibrium, associated with a low level of deterrence arises when $\sigma h < \frac{B}{2}$. As a consequence, when $\sigma = \pi = \theta$ and $\sigma h = \frac{B}{2}$, there exists a multiplicity of equilibria, either symmetric or asymmetric, i.e. any of (the $4^2$ possible) profiles of strategies where candidate $l$ and candidate $r$ announce either $(R^a, f^a_h)$, or $(R^b, f^h_h)$, or $(R^a, f^c_c)$, or $(R^b, f^c_c)$, is an equilibrium of the electoral game.

3.3 inefficiencies in law enforcement and the majority rule

A general property of the equilibria obtained under the majority rule, is that they yield inefficient law enforcement policies: as far as social preferences are truncated towards the preferences of the majority of citizens, the policies that politicians propose are not socially optimal. The next propositions discusses more accurately the inefficiencies associated with the different policies that may emerge in an electoral equilibrium, both from the point of view of the severity of sanction, and of the level of deterrence (probability of crime).

First, we compare the properties of a strong enforcement equilibrium to the utilitarian contract:

**Proposition 7** Whatever the strong enforcement equilibrium arising from political competition, it is associated with a sanction which is larger than the social welfare maximizing one, and a probability of crime which is smaller (i.e. $f^a_h > f^a_u$ and $q^a_h > q^a_u$; $f^h_h > f^h_u$ and $q^h_h < q^h_u$).

**Proof.** A/ The proof is straightforward, since when $f^a_h$ is enforced, we have:

$$f^a_h - f^a_u = \frac{1}{2} \left( \frac{B}{p} - \frac{\sigma h}{p} \right) > 0$$

under assumption 1.

B/ Consider now that $f^h_h$ is enforced at equilibrium under political competition, with $\pi > \sigma$. We have:

$$f^h_h - f^h_u = \frac{1}{2} \left[ \frac{\sigma h}{p} + \frac{\pi}{p} B \right] \frac{1}{\pi} - \frac{1}{2} \left[ \frac{\sigma h}{p} + \left( \frac{\pi - \sigma}{p} \right) B \right] \frac{1}{\pi - \frac{\sigma}{\pi}}$$

Hence:

$$f^h_h - f^h_u = \frac{1}{2} \frac{1}{p} \frac{1}{2\pi - \sigma} \left[ B - \frac{\sigma^2}{\pi} h \right] > 0$$
given that the condition $B - \frac{\sigma^2}{2} h > 0$ is needed in order that $q^h_u > 0$.

In the act-based case, note that a probability of crime is defined as $q = 1 - \frac{\sigma}{p}$; thus $q^c_a - q^c_u$ has a sign opposite to $f^c_a - f^c_u$. In the harm-based case, note that a probability of crime is defined as $q = 1 - \frac{\sigma}{B}$; hence $q^h_u - q^h_h$ has a sign opposite to $f^h_u - f^h_h$. \qed

Then, we compare the properties of a weak enforcement equilibrium to the utilitarian contract:

**Proposition 8** Whatever the weak enforcement equilibrium arising from political competition, it is associated with a sanction which is smaller than the welfare maximizing one, and a probability of crime which is higher: (i.e. $f^c_a < f^u_a$ and $q^c_a > q^u_a$; $f^h_c < f^h_u$ and $q^h_c > q^h_u$).

**Proof.** A/ When $f^c_c$ is enforced, it comes:

$$f^c_c - f^u_u = \frac{1}{2} \frac{\sigma}{p} h - \frac{\sigma}{p} h < 0$$

B/ Consider now that $f^h_c$ is enforced at equilibrium under political competition. As $\pi \leq \frac{\pi}{2}$ then $f^h_u = w$ and by construction $w > f^h_c$. In contrast, consider the case $\sigma > \pi > \frac{\sigma}{2}$; we obtain:

$$f^h_c - f^h_u = \frac{1}{2} \left[ \frac{\sigma}{p} h + \left( \frac{\pi - \sigma}{p \sigma} \right) B \right] \frac{1}{\pi} - \frac{1}{2} \left[ \frac{\sigma}{p} h + \left( \frac{\pi - \sigma}{p \sigma} \right) B \right] \frac{1}{\pi - \frac{\sigma}{2}}$$

$$= \frac{1}{2} \left[ \frac{\sigma}{p} h + \frac{\pi - \sigma}{p \sigma} B \right] \left( \frac{1}{\pi} - \frac{1}{\pi - \frac{\sigma}{2}} \right) < 0$$

since $\frac{1}{\pi} - \frac{1}{\pi - \frac{\sigma}{2}} = -\frac{\sigma}{\pi - \frac{\sigma}{2}} < 0$.

In the act-based case, note that a probability of crime is defined as $q = 1 - \frac{\sigma}{p}$; thus $q^a_c - q^a_u$ has a sign opposite to $f^a_u - f^a_c$. In the harm-based case, note that a probability of crime is defined as $q = 1 - \frac{\sigma}{B}$; hence $q^h_u - q^h_h$ has a sign opposite to $f^h_u - f^h_c$. \qed

### 4 Concluding remarks

This paper revisits the analysis of public law enforcement and the timing of enforcement measures that relies on differences in information regarding the character of acts, that exist between the enforcer and citizens. We have characterized the optimal timing of sanctions chosen by a benevolent/utilitarian enforcer, and compared it to the equilibrium (decentralized) timing that results from the political market and electoral competition. Critical for our framework is this assumption that a disagreement exists between citizens and politicians...
regarding the expected harm of illegal acts. Up to now, we have not been very specific regarding the source of this disagreement. Without going into details, let us simply argue that advanced theories in decision under risk and uncertainty may provide us with alternative foundations for beliefs and preferences on uncertain events which could be introduced to motivate our assumption.

For example, we may have considered the case with rank dependant evaluation such that the citizens and the public authority distort a known probability reflecting, to be short, a pessimistic evaluation – both are more sensitive to losses/small gains they experience than to large gains. Or we may have introduced a richer context where competing experts give a different assessment on the prevalence of risks associated with the offences; often, judgments of experts take the form of alternative probability distributions, or probabilities intervals, or are expressed in a events/states space which is not directly informative to citizens and so on. In a sense, the existence of several sources of information (and possibly, inconsistent) could yield the same outcome as no information at all: for naïve individuals, this could result in an ambiguous knowledge of those risks. However, the informed judgments of experts are used (summarized, aggregated) by citizens and the public authority to form their own beliefs (Chateauneuf, Eichberger and Grant, 2007).

Remark that the specific models that could be used for this purpose will add technical developments which are not central for our arguments and results. The clue is that this disagreement is not understood as reflecting judgment errors, but rather, reflects that individual beliefs are part of the preferences of both citizens and politicians (and finally, public authorities). Indeed this is the main point of the argument from a pure utilitarian point of view: if the disagreement were to reflect errors in risks perception made by citizens (for example), the public authority should not take care of these errors to built the criterion used to design the optimal policy. On the other hand, there is no reason to believe that the political process, given the general result that it yields socially inefficient decisions, will allow to remove the erroneous beliefs of citizens.

Our paper first analyzes the issue of the optimal timing of sanction chosen by a utilitarian enforcer. The results are collected in the next table:

<table>
<thead>
<tr>
<th>$\sigma &gt; \pi$</th>
<th>$\pi = \sigma$</th>
<th>$\sigma &lt; \pi$ and $\sigma h &gt; \frac{B}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>act-based sanction</td>
<td>act or harm-based sanction</td>
<td>harm-based sanction</td>
</tr>
</tbody>
</table>

The argument runs as follows: when the enforcer and citizens disagree on the expected value of social harm, a harm-based sanction punishing criminal acts would be no longer a mere transfer between criminals and society, but it would have a social value (positive if $\pi > \sigma$; negative if $\pi < \sigma$). Thus, the optimal policy depends on the trade-off between on the one hand, the social value of deterrence, and on the other hand, the social value of punishment...
(fines, monetary sanctions). A sufficient condition in order that the act-based sanction is socially optimal, is given by $\pi < \sigma$. Note that this solution allows to reach a higher level of deterrence only when $\sigma h < \frac{B}{2}$. In contrast, the socially optimal sanction is harm-based when both $\pi > \sigma$ and $\sigma h > \frac{B}{2}$ hold.

Then, the paper considers the effects of electoral competition on the timing of sanctions. The point is that in modern democracies, the structure and content of the penal law is the result of the political process. Our results suggest that citizens’ opinions on the one hand, and the disagreement with politicians’ one on the other hand, have a different effect regarding the design of public law enforcement, and more specifically, regarding both the level of the optimal sanctions and the choice of the regime of sanction. The main results are summarized in the next schedule:

**Table 2 – the equilibrium timing of sanctions**

<table>
<thead>
<tr>
<th>$\sigma h &gt; \frac{B}{2}$</th>
<th>$\sigma &lt; \pi$</th>
<th>$\sigma = \pi$</th>
<th>enforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>harm-based sanction</td>
<td>act-based sanction</td>
<td>act or harm-based sanction</td>
<td>strong enforcement</td>
</tr>
<tr>
<td>weak enforcement</td>
<td>strong enforcement</td>
<td>strong enforcement</td>
<td></td>
</tr>
<tr>
<td>$\sigma h &lt; \frac{B}{2}$</td>
<td>harm-based sanction</td>
<td>act-based sanction</td>
<td>act or harm-based sanction</td>
</tr>
<tr>
<td>weak enforcement</td>
<td>weak enforcement</td>
<td>weak enforcement</td>
<td></td>
</tr>
<tr>
<td>$\sigma h = \frac{B}{2}$</td>
<td>harm-based sanction</td>
<td>act-based sanction</td>
<td>act or harm-based sanction</td>
</tr>
<tr>
<td>strong or weak enforcement</td>
<td>strong or weak enforcement</td>
<td>strong or weak enforcement</td>
<td></td>
</tr>
<tr>
<td>regime</td>
<td>harm-based sanction</td>
<td>act-based sanction</td>
<td>act or harm-based sanction</td>
</tr>
</tbody>
</table>

Interestingly enough, we find that a act-based sanction is the result of the electoral process when $\pi > \sigma$; while a harm-based sanction emerges if $\pi < \sigma$. Simultaneously, the condition $\sigma h > \frac{B}{2}$ ($\sigma h < \frac{B}{2}$) induces harsher (less severe) sanctions than what should be socially optimal.

To conclude, let us highlight the way our work allows to reconcile the Law & Economics view on the criminal law with the legal doctrine. Legal scholars (see Dau-Schmidt, 1990; Levin and Trumbull, 1990) put the diagnosis that the beckerian approach was wrong because of its failure to recognize that, due to its moral references, the main objective of the penal code was not deterrence (ex ante view), but punishment and rehabilitation (ex post view), in order to shape criminals’ preferences and to promote individual behaviors in conformity with the socially valuable acts. It is worth remembering that in his classical text, Shavell (1985) justified that both approaches were more complement rather than concurrent as it seemed to the legal scholars. The purpose of the paradigm...
initiated after Becker’s work (1968) was not to deny the ethical roots of the penal code. But in contrast, and in a sense despite its moral foundations, it was to analyze its rational motivations and to investigate the issue of deterrence\textsuperscript{12}. To go a little further for our purpose, remark that the legal view appears quite tautological as long as it fails to explain how moral and ethical concerns are transplanted into the criminal law. Thus one way to bridge a gap between the legal doctrine and the Law & Economics approach is to consider the role of democratic institutions and mainly the role of the political market in the production and evolution of laws. In this spirit, an extended view of the legal doctrine, consistent with our approach, would explain the heterogeneity of social values and preferences among countries as reflecting the observed differences in their ethical or philosophical references, comprising the weight of religion, the age of democracy, or the quality of democratic institutions and so on. The straightforward prediction afforded by this extended view is that, for a given set of acts, it may exist some heterogeneity between countries in the treatment of these acts according to the penal code, both in terms of its structure and content. Lastly, this approach is useful to explain the evolution of the penal code and administrative law: we have in mind well known examples of offenses which, as time has passed, have been shifted from the penal code to the civil law or sectorial regulatory statutes (divorce, abortion), and other criminal activities for which the existence of an external harm for society may be (have been) disputable (drug consumption, prostitution\textsuperscript{13}).

References


\textsuperscript{12}Miceli (2009a,b) developps a unified theory of criminal punishment that encompasses both the ex ante and ex post views.

\textsuperscript{13}If we disantangle individual behaviors related to casual drug consumption and/or independent prostitutes, from the much larger traffics connected to the organized crime at the international level (drug trafficking, or networks of prostitution).


APPENDIX 1

We first analyze two kinds of subgames where the strategies of candidates to elections are constrained in the sense that it is assumed that they are bound to announce a predefined regime of sanction, the act-based regime (thus, they are
not allowed to enforce a harm-based regime), or the harm-based regime (thus, they are not allowed to enforce a act-based regime), and they are allowed to chose only the severity of the punishment. Finally, we solve the equilibrium for the complete game.

- **The act-based equilibrium.** We consider only the subgames of the complete game beginning after that a act-based platform is enforced, and ruling out all the subgames beginning after that a harm-based platform is enforced.

Consider now a subgame beginning after a platform \((R^a, f)\) is enforced – that is a act-based regime with a monetary sanction \(f\) is enforced.

Using the analysis of paragraph 2.2, it is direct that the level of deterrence at equilibrium is \(\bar{b} = pf\) and the probability that citizens be honest is \(\bar{b} = \frac{pB}{B} f\).

Moreover, it is straightforward that honest people, thereafter characterized with the subscript \(h\), prefer a monetary sanction which is set at a higher level than the citizens who commit a crime/offense (criminals) therein characterized with the subscript \(c\), since according to the definition of their respective satisfaction level, we have for an honest individual \(u_h = w - t - \left(1 - \frac{b}{B}\right)\sigma h\) and thus:

\[
\frac{\partial u_h}{\partial f} = \frac{\partial \left(\frac{b}{B}\right)\sigma h}{\partial f} = \frac{p\sigma}{B} h > 0
\]

while for a criminal, we have \(u_c = w + b - t - pf - \left(1 - \frac{b}{B}\right)\sigma h\) such that:

\[
\frac{\partial u_c}{\partial f} = -p + \frac{\partial \left(\frac{b}{B}\right)\sigma h}{\partial f} = -p \left(1 - \frac{\sigma}{B} h\right) < 0
\]

under assumption 1. These two populations having different attitudes regarding the law, have also different preferences regarding the strategy of vote at the electoral competition stage. For an honest individual, we defined now the preferred sanction as: \(f^a_h = \arg \max_f \{u_h \text{ under (1)}\}\). Substituting (1) in \(u_h\) leads now to:

\[
u_h = w - m(p) + \left(1 - \frac{pf}{B}\right)(pf - \sigma h)
\]

Note that \(\frac{\partial u_h}{\partial f} |_{f=0} = p + \frac{p^2}{B} h > 0\). Assuming that \(\frac{\partial u_h}{\partial f} |_w < 0 \Leftrightarrow w > \frac{1}{2} \left[\frac{p}{B} h + \frac{B}{B}\right]\), the first order condition is given by: \(\frac{\partial u_h}{\partial f} = 0\) or:

\[
-\frac{1}{B} (pf - \sigma h) + \left(1 - \frac{pf}{B}\right) = 0
\]

hence solving for \(f^a_h\):
\[
\begin{align*}
    f_{b}^{a} &= \frac{1}{2} \left[ \frac{\sigma h + B}{p} \right] \\
    \frac{\bar{b}_{b}^{a}}{B} &= \frac{pf_{b}^{a}}{B} = \frac{1}{2} \left[ \frac{\sigma h + 1}{B} \right] > \frac{1}{2}
\end{align*}
\]

For a criminal, let us denote as: \( f_{c}^{a} = \text{arg max}_f \{ u_{c} \text{ under (1)} \} \).

Substituting (1) in \( u_{c} \) yields now:

\[
    u_{c} = w + b - m(p) - pf + \left( 1 - \frac{pf}{B} \right) (pf - \sigma h)
\]

Note that \( \frac{\partial u_{c}}{\partial f} |_{f=0} = \frac{pf}{B} h > 0 \). Assuming that \( \frac{\partial u_{c}}{\partial f} |_{w} < 0 \), the first order condition \( \frac{\partial u_{c}}{\partial f} = 0 \) writes as:

\[
    -1 - \frac{1}{B} (pf - \sigma h) + \left( 1 - \frac{pf}{B} \right) = 0
\]

which yields:

\[
\begin{align*}
    f_{c}^{a} &= \frac{1}{2} \frac{\sigma}{p} h \\
    \frac{\bar{b}_{c}^{a}}{B} &= \frac{pf_{c}^{a}}{B} = \frac{1}{2} \frac{\sigma}{B} h < \frac{1}{2}
\end{align*}
\]

and it is straightforward that \( f_{c}^{a} < f_{b}^{a} \) and that \( \frac{\bar{b}_{c}^{a}}{B} < \frac{1}{2} < \frac{\bar{b}_{b}^{a}}{B} \). This result is used to establish the:

**Lemma 9** Under electoral competition with a act-based sanction:

i) If \( \sigma h > \frac{B}{2} \), then the unique equilibrium is such that both candidates announce the monetary sanction \( f_{b}^{a} = \frac{1}{2} \left[ \frac{\sigma h + B}{p} \right] \), and each candidate wins the elections with probability \( \frac{1}{2} \); the probability of crime is \( q_{a} = \frac{1}{2} \left[ 1 - \frac{\bar{b}_{b}^{a}}{B} h \right] \).

ii) If \( \sigma h < \frac{B}{2} \), then the unique equilibrium is such that both candidates announce the monetary sanction \( f_{b}^{a} = \frac{1}{2} \frac{\sigma}{p} h \), and each candidate wins the elections with probability \( \frac{1}{2} \); the probability of crime is \( q_{a} = 1 - \frac{1}{2} \frac{\bar{b}_{b}^{a}}{B} h \).

iii) If \( \sigma h = \frac{B}{2} \), there exist a multiplicity of equilibria, either symmetric or asymmetric, i.e. the next four profiles of strategies are equilibria of the electoral game:

- candidate r announces \( f_{b}^{a} \), and candidate \( b \) announces \( f_{c}^{a} \);  
- candidate r announces \( f_{b}^{a} \), and candidate \( b \) announces \( f_{c}^{a} \);  
- both candidate \( b \) and candidate \( c \) announce \( f_{b}^{a} \);  
- both candidate \( b \) and candidate \( c \) announce \( f_{c}^{a} \).

In all of these equilibria, each candidate wins the elections with probability \( \frac{1}{2} \), and the probability of crime is \( q_{a} = \frac{3}{4} \).
Proof. Assume that candidates are bound to announce an act-based regime \( R^a \). Note first that the fraction of voters for \( f_h^a \) is \( \frac{b_h^a}{B} > \frac{1}{2} \), while the fractions voting for \( f_c^a \) is \( 1 - \frac{b_c^a}{B} > \frac{1}{2} \). Then, it is easy to verify that:

\[
\frac{b_h^a}{B} - \left( 1 - \frac{b_c^a}{B} \right) = \frac{\sigma}{B} h - \frac{1}{2}
\]

and thus,

\[
\begin{align*}
\frac{\sigma}{B} h - \frac{1}{2} > 0 & \Rightarrow \frac{b_h^a}{B} > \left( 1 - \frac{b_c^a}{B} \right) \\
\frac{\sigma}{B} h - \frac{1}{2} < 0 & \Rightarrow \frac{b_h^a}{B} < \left( 1 - \frac{b_c^a}{B} \right) \\
\frac{\sigma}{B} h - \frac{1}{2} = 0 & \Rightarrow \frac{b_h^a}{B} = \left( 1 - \frac{b_c^a}{B} \right)
\end{align*}
\]

**CASE 1:** \( \frac{\sigma}{B} h - \frac{1}{2} > 0 \).

At the electoral competition stage, note first that both candidates anticipate that, once the elections are held (i.e. considering any subgame beginning once a platform \( (R^a, f) \) is enforced), then the enforced law \( (R^a, f) \) will split the population in two sub-groups: citizens who abide the law, and citizens who commit a crime/offense. Secondly, a situation where the candidates announce a different platform \( (R^a, f_l) \neq (R^a, f_r) \) cannot be an equilibrium, since either the distribution of voters satisfies \( n_l > n_r \) and thus candidate \( r \) would change his platform, otherwise he looses the elections for sure; or to the converse, the associated distribution satisfies \( n_l < n_r \), and now candidate \( l \) has an incentive to make a different announcement. Thus, any subgame perfect equilibrium must be such that both candidates choose and announce the same programme: \( (R^a, f_l) = (R^a, f_r) = (R^a, f) \), and both have a 50% chances to win the elections.

Given that \( f_h^a = \frac{1}{2} \left[ \frac{\sigma}{B} h + \frac{b_h^a}{B} \right] \) is maximizing the number of voters: \( \frac{1}{2} \left[ \frac{\sigma}{B} h + 1 \right] > \frac{1}{2} \), the unique subgame perfect equilibrium is associated with the announcement by both candidates of the monetary sanction \( f_h^a \).

**CASE 2:** \( \frac{\sigma}{B} h - \frac{1}{2} < 0 \).

According to the same argument, any subgame perfect equilibrium must be such that both candidates choose and announce the same programme: \( (R^a, f_l) = (R^a, f_r) = (R^a, f) \), and both have a 50% chances to win the elections. Given that \( f_c^a = \frac{1}{2} \left[ \frac{\sigma}{B} h + \frac{b_c^a}{B} \right] \) is now maximizing the number of voters: \( 1 - \frac{1}{2} \frac{\sigma}{B} h > \frac{1}{2} \), the unique subgame perfect equilibrium is associated with the announcement by both candidates of the monetary sanction \( f_c^a \).

**CASE 3:** \( \frac{\sigma}{B} h - \frac{1}{2} = 0 \). Whatever the announcement, \( f_h^a \) or \( f_c^a \), the associated probability of voters is \( \frac{1}{2} \left[ \frac{\sigma}{B} h + 1 \right] = \frac{3}{4} \). Thus there exist four kind of equilibria: \((f_h^a, f_c^a) ; (f_h^a, f_h^a) ; (f_c^a, f_h^a) ; (f_c^a, f_c^a)\).
The harm-based equilibrium. We consider now only the subgames beginning after that a harm-based platform is enforced, and ruling out all the subgames beginning after that a act-based platform is enforced.

Consider first a subgame beginning after a platform \( (R^h, f) \) is enforced – that is the elected candidate held a platform consisting in a harm-based regime with a monetary sanction \( f \), which is now enforced.

Using the analysis of paragraph 2.3, it is direct that the level of deterrence at equilibrium is \( \bar{b} = p\sigma f \) and the probability that crime be deterred is \( \bar{\beta} = \frac{\sigma}{\beta_f} f \).

Moreover, it is straightforward that honest people, thereafter characterized with the subscript \( h \), prefer a monetary sanction which is set at a higher level than the citizens who commit a crime/offense (criminals) therein characterized with the subscript \( c \), since according to the definition of their respective satisfaction level, we have for an honest individual \( u_h = w - t - \left( 1 - \frac{\bar{b}}{B} \right) \sigma h \) and thus:

\[
\frac{\partial u_h}{\partial f} = \frac{\partial \left( \frac{\bar{b}}{\beta_f} \right) \sigma h}{\partial f} = \frac{p\sigma^2}{B} h > 0
\]

while for a criminal, we have \( u_c = w + b - t - p\sigma f - \left( 1 - \frac{\bar{b}}{\beta_f} \right) \sigma h \) such that:

\[
\frac{\partial u_c}{\partial f} = -p\sigma + \frac{\partial \left( \frac{\bar{b}}{\beta_f} \right) \sigma h}{\partial f} = -p\sigma \left( 1 - \frac{\sigma}{\beta_f} h \right) < 0
\]

These two populations having different attitudes regarding the law, have also different preferences regarding the strategy of vote at the electoral competition stage. A voter who anticipates to be honest will have \( f^h = \arg \max_f \{ u_h \text{ under } (1) \} \) as a preferred platform. Substituting (1) in \( u_h \) leads to:

\[
u_h = w - m(p) + \left( 1 - \frac{p\sigma f}{B} \right) \left( p\pi f - \sigma h \right)
\]

Note that \( \frac{\partial u_h}{\partial f} \big|_{f=0} = p\pi + \frac{p\sigma^2 h}{B} > 0 \). Assuming that \( \frac{\partial u_h}{\partial f} \big|_{w} < 0 \Leftrightarrow w > \frac{1}{2} \left[ \frac{\pi h}{\pi p} + \frac{B}{p\sigma} \right] \), the first order condition is given by: \( \frac{\partial u_h}{\partial f} = 0 \) or:

\[
\left( 1 - \frac{p\sigma f}{B} \right) \pi - \frac{\sigma}{B} (p\pi f - \sigma h) = 0
\]

Solving for \( f^h \) gives:

\[
f^h = \frac{1}{2} \left[ \frac{\sigma h}{\pi p} + \frac{B}{p\sigma} \right]
\]

\[
\bar{\beta}_h \frac{B}{B} = \frac{p\sigma f}{B} \bar{f}_h = \frac{1}{2} \left[ \frac{\sigma^2 h}{\pi B} + 1 \right] > \frac{1}{2}
\]

Now, similarly a voter who anticipates that he will commit an offense will have as a preferred platform: \( f^c = \arg \max_f \{ u_c \text{ under } (1) \} \). Substituting (1) in \( u_c \) yields now:
\[ u_c = w + b - m(p) - psf + \left(1 - \frac{psf}{B}\right) (psf - \sigma h) \]

Note that \( \frac{\partial u_c}{\partial f} \big|_{f=0} = p(\pi - \sigma) + \frac{\sigma}{2} B h \), and thus in order to have \( \frac{\partial u_c}{\partial f} \big|_{f=0} > 0 \) it must be that \( \frac{\sigma}{2} B h + \left(\frac{\pi - \sigma}{p}\right) B > 0 \) (see also proposition 3). Assuming moreover that \( \frac{\partial u_c}{\partial w} \big|_{w} < 0 \), the first order condition \( \frac{\partial u_c}{\partial f} = 0 \) writes as:

\[ -\sigma - \frac{\sigma}{B} (psf - \sigma h) + \left(1 - \frac{psf}{B}\right) \pi = 0 \]

which yields:

\[ f_c^h = \frac{1}{2} \left[ \frac{\sigma h}{\pi p} + \frac{(\pi - \sigma)}{p} \frac{B}{p \sigma} \right] \]

\[ \frac{\sigma h}{B} f_c^h = \frac{p \sigma}{B} \]

since according to assumption 1, \( \frac{\sigma h}{B} < 1 \) \( \Rightarrow \frac{\sigma^2 h}{B^2} < \frac{\sigma}{\pi} \Rightarrow \frac{\sigma^2 h}{B^2} + \left(\frac{\pi - \sigma}{\pi}\right) < 1 \).

Thus, it is straightforward that \( f_c^h < f_h^h \) and \( \frac{\sigma h}{B} < \frac{1}{2} < \frac{\sigma h}{B} \). This result is used to establish the:

**Lemma 10** Under electoral competition with a harm-based sanction:

i) If \( \sigma h > \frac{B}{2} \), then the unique equilibrium is such that both candidates announce the monetary sanction \( f_h^h = \frac{1}{2} \left[ \frac{\sigma h}{\pi p} + \frac{B}{\pi p \sigma} \right] \), and each candidate wins the elections with probability \( \frac{1}{2} \); the probability of crime is \( q^h = \frac{1}{2} \left[ 1 - \frac{\sigma^2 h}{\pi B} \right] \).

ii) If \( \sigma h < \frac{B}{2} \), then the unique equilibrium is such that both candidates announce the monetary sanction \( f_c^h = \frac{1}{2} \left[ \frac{\sigma^2 h}{\pi p} + \left(\frac{\pi - \sigma}{\pi}\right) \frac{B}{p \sigma} \right] \), and each candidate wins the elections with probability \( \frac{1}{2} \); the probability of crime is \( q^h = \frac{1}{2} \left[ 1 - \frac{\sigma^2 h}{\pi B} + \left(\frac{\pi - \sigma}{2 \pi}\right) \right] \).

iii) If \( \sigma h = \frac{1}{2} \), there exist a multiplicity of equilibria, either symmetric or asymmetric, i.e. the next four profiles of strategies are equilibria of the electoral game:

- candidate l announces \( f_h^h \), and candidate r announces \( f_h^h \);
- candidate l announces \( f_c^h \), and candidate r announces \( f_h^h \);
- both candidate l and candidate r announce \( f_h^h \);
- both candidate l and candidate r announce \( f_c^h \).

In all of these equilibria, each candidate wins the elections with probability \( \frac{1}{2} \), and the probability of crime is \( q^h = \frac{1}{2} \left[ 1 + \frac{\sigma}{2 \pi} \right] \).
Proof. Assume that candidates are bound to announce a harm-based regime $R^h$. The fraction of voters for $f^h$ is $\frac{b^h}{B} > \frac{1}{2}$, while the fractions voting for $f^c$ is $1 - \frac{b^h}{B} > \frac{1}{2}$. Then, it is easy to verify once more that:

$$\frac{b^h}{B} - \left(1 - \frac{b^c}{B}\right) = \frac{\sigma}{B} h - \frac{1}{2}$$

and thus,

$$\frac{\sigma}{B} h - \frac{1}{2} > 0 \Rightarrow \frac{b^h}{B} > \left(1 - \frac{b^c}{B}\right)$$

$$\frac{\sigma}{B} h - \frac{1}{2} < 0 \Rightarrow \frac{b^h}{B} < \left(1 - \frac{b^c}{B}\right)$$

$$\frac{\sigma}{B} h - \frac{1}{2} = 0 \Rightarrow \frac{b^h}{B} = \left(1 - \frac{b^c}{B}\right)$$

**CASE 1:** $\frac{\sigma}{B} h - \frac{1}{2} > 0$. At the electoral competition stage, note first that both candidates anticipate that, once the elections are held (i.e. considering any subgame beginning once a platform $(R^h, f)$ is enforced), then the enforced law $(R^h, f)$ will split the population in two sub-groups: citizens who abide the law, and citizens who commit a crime/offense. Secondly, a situation where the candidates announce a different platform $(R^h, f_l) \neq (R^h, f_r)$ cannot be an equilibrium, since either the distribution of voters satisfies $n_l > n_r$ and thus candidate $r$ would change his platform, otherwise he looses the elections for sure; or to the converse, the associated distribution satisfies $n_l < n_r$, and now candidate $l$ has an incentive to make a different announcement. Thus, any subgame perfect equilibrium must be such that both candidates choose and announce the same programme: $(R^h, f_l) = (R^h, f_r) = (R^h, f)$, and both have a 50% chances to win the elections.

Finally, given that $f^h = \frac{1}{2} \left[\frac{\sigma}{\pi} h + \frac{B}{p^2}\right]$ is maximizing the number of voters:

$$\frac{1}{2} \left[\frac{\sigma^2}{\pi^2} h + 1\right] > \frac{1}{2},$$

the unique subgame perfect equilibrium is associated with the announcement by both candidates of the monetary sanction $f^h$.

**CASE 2:** $\frac{\sigma}{B} h - \frac{1}{2} < 0$.

According to the same argument, any subgame perfect equilibrium must be such that both candidates choose and announce the same programme: $(R^h, f_l) = (R^h, f_r) = (R^h, f)$, and both have a 50% chances to win the elections. Given that $f^c = \frac{1}{2} \left[\frac{\sigma}{\pi} h + (1 - \sigma) \frac{B}{p^2}\right]$ is now maximizing the number of voters:

$$\left(1 - \frac{1}{2} \left[\frac{\sigma^2}{\pi} h + \left(\frac{\pi - \sigma}{\pi}\right) \frac{B}{p^2}\right]\right) > \frac{1}{2}$$

the unique subgame perfect equilibrium is associated with the announcement by both candidates of the monetary sanction $f^c$. 

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CASE 3: \( \frac{\sigma}{\pi} h - \frac{1}{2} = 0 \). Whatever the announcement, \( f^h \) or \( f^h_c \), the associated probability of voters is \( \frac{1}{2} \left[ \frac{\sigma}{\pi} + 1 \right] \). Thus there exist four kind of equilibria:
\((f^h, f^a_c)\); \((f^a_c, f^h)\); \((f^c, f^h)\); \((f^c, f^a_c)\).

- **Equilibria of the complete game**. Let us return now to the complete game (propositions 5), where candidates have the opportunity to announce either a harm-based regime: \((R^h, f)\), or a act-based one: \((R^a, f)\). Let us assume that \( \sigma \neq \pi \); then:

CASE 1: \( \frac{\sigma}{\pi} h - \frac{1}{2} > 0 \). The platform which could be announced by both candidates at the electoral competition stage is either \((R^h, f^h)\), or \((R^a, f^h)\), since after any subgame beginning after the election stage, each is subgame perfect. Remark first that:

\[
\frac{\hat{b}^h}{B} - \frac{\hat{b}^a}{B} = \frac{1}{2} \left[ \frac{\sigma^2}{\pi B} h + 1 \right] - \frac{1}{2} \left[ \frac{\sigma}{B} h + 1 \right] \\
= \frac{1}{2} \left[ \frac{\sigma^2}{\pi B} B - \frac{\sigma}{B} B \right] \\
= \frac{1}{2} \frac{\sigma}{B} h \left[ \frac{\sigma}{\pi} - 1 \right]
\]

and thus, it is easy to see that the sign of \( \frac{\hat{b}^h}{B} - \frac{\hat{b}^a}{B} \) depends on the sign of \( \sigma - \pi \).

- either \( \frac{\sigma}{\pi} > 1 \), and the platform \((R^h, f^h)\) gives more voters than the platform \((R^a, f^h)\); when proposed by both candidates, no one wants to deviate, since a deviation means loosing the elections. This is the unique equilibrium.

- or \( \frac{\sigma}{\pi} < 1 \) and now the platform \((R^a, f^h)\) gives more voters than the platform \((R^h, f^h)\); when proposed by both candidates, no one deviates, since it cannot allow to win the elections. This is the unique equilibrium.

CASE 2: \( \frac{\sigma}{\pi} h - \frac{1}{2} < 0 \). Straightforward since we have:

\[
\frac{\hat{b}^h}{B} - \frac{\hat{b}^a}{B} = \frac{1}{2} \left( \frac{\sigma - \pi}{\pi} \right) \left( 1 - \frac{\sigma}{B} h \right)
\]

Thus under assumption 1, \( \sigma - \pi < 0 \) implies \( \frac{\hat{b}^h}{B} - \frac{\hat{b}^a}{B} > 0 \Rightarrow \left( 1 - \frac{\hat{b}^h}{B} \right) - \left( 1 - \frac{\hat{b}^a}{B} \right) < 0 \); and \( \sigma - \pi > 0 \) implies \( \frac{\hat{b}^h}{B} - \frac{\hat{b}^a}{B} < 0 \Rightarrow \left( 1 - \frac{\hat{b}^h}{B} \right) - \left( 1 - \frac{\hat{b}^a}{B} \right) > 0 \).

And thus, the same argument as before leads to the result.