Math Matters: 
Education Choices and Wage Inequality

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Abstract

This paper provides an explanation for part of the increasing wage inequality between the mid-1970s and 2010 by showing that the top deciles of college earners’ significant relative wage growth is underpinned by the link between \textit{ex ante} math ability, math-heavy college majors and highly quantitative occupations. This mechanism is further strengthened on the supply-side by a strong and accelerating shift away from math-heavy college majors and occupations for successive cohorts. We develop a general equilibrium model with multiple education options, where occupational outcomes depend on individuals’ \textit{ex ante} math abilities and study preferences. This research shows that a large portion of wage inequality is determined by initial math/quantitative abilities. Furthermore, these results imply that policy measures aimed at increasing college enrollment, to decrease wage inequality, do not address the underlying process and, in some cases, may exacerbate wage inequality.

JEL classification: E20, E24, E25, I20, J24, J31

Keywords: wage inequality, college majors, occupations, mathematics abilities

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1 Introduction

Higher education, broadly defined as college or university education beyond the secondary level, is typically seen as a mechanism driving both social and economic mobility. The completion of higher education is correlated with many positive \textit{ex post} outcomes, including higher wages, better quality of life and longer life expectancy (see Baum et al., 2010). These correlations have prompted significant government intervention to promote higher education, with the goal of extending these benefits to a larger section of the population (Baum et al., 2010). However, this paper shows that this conclusion is erroneous due to the heterogeneity both intra- and inter-education groups. The US has seen a large rise in wage inequality. Figure 1 plots the 90th-to-10th (90-10), 90-50 and 50-10 percentiles of hourly log wages for full-time, full-year\(^1\) males aged 25 to 59 from the Current Population Survey (CPS).\(^2\) Inequality has risen faster at the upper half of the wage distribution.

\(^1\)At least 40 weeks and 35 hours per week.
\(^2\)CPS data is obtained from the IPUMS-CPS project (King et al., 2010).
This points to the well established fact that there is increasing wage inequality amongst college graduates. Figure 2 plots the same percentiles by education group. Here college graduates are all individuals with at least a university degree (i.e., bachelor or graduate degree holders).

Source: Current Population Survey

Figure 2: US Wage Inequality by Education

While the 90-10 wage differential increased by 33 percentage points for individuals with no-college degree, it increased by 40 percentage points within the group of college graduates. Moreover, the rise in the 50-10 percentile wage inequality was twice as large for college graduates, with 16 percentage points (versus 8 percentage points), while the 90-50 percentile wage inequality increased by the same 24 percentage points. While the rise was larger in the bottom half, the inequality was larger initially for college graduates in the top half of the distribution and equal for the bottom half. The seminal research by Kambourov & Manovskii (2009) explains a large part of within group wage inequality by focusing on occupational mobility and the cost of switching occupations. They find that occupational mobility accounts for a significant portion of wage inequality. Their approach differs from this paper by concentrating on the given conditions at the time of occupational choice (i.e., educational choices are not modeled). Thus, we expand on their research by modeling the initial conditions that proceed labor force choices that are central to their
results. We believe this to be important, since we show that education choices matter for occupation decisions later in life. We further show that by separating the population strictly by educational attainment alone, some very significant points are missed. That is, the following six facts result from a decomposition of the within and between group heterogeneity previously discussed.

1. The top earners without college degrees earn substantially more than the bottom earners with college degrees (see Figure 3).
2. The top earners with college degrees primarily drive wage inequality. (see Figure 4).
3. Occupation-specific math skills have become (increasingly) correlated with higher wages (see Figure 5).
4. Occupation-specific math skills are highly correlated with college-level math credits by major (see Figure 6).
5. *Ex ante* math abilities are highly correlated with college-level math credits. (see Figure 7).
6. When ranking college majors by their math requirements, there has been a dramatic shift toward the tails (i.e., an increase in the bottom quintiles and the top quintiles). This shift is depicted in Figure 8.

Thus, by concentrating on the “average” college graduate alone, policy makers may design policies that are inconsistent with a thorough analysis across the wage distribution conditional on educational attainment. By basing education policy on blunt and broad measures of the benefits of college education, there will be a misallocation of both human and physical capital. Therefore, the aim of this paper is to show the importance of combining the six facts above in an analysis aimed at framing wage inequality. To our knowledge, there is no prior research that has explored these six facts together. The recent paper by Altonji et al. (2012) is the first to establish point three above, while the other four facts are new contributions to the literature. Altonji et al. (2012) are “motivated by the large
discrepancies in labor market outcomes across college majors noting that, “there is much less work on why individuals choose between different types of education,” despite the, “returns across college majors rival(ing) the college wage premium.” They note that the 2009 American Community Survey (ACS) shows a 56.1-percent gap between male electrical engineering and male general education majors, compared to a college wage premium of 57.7 percent. Due to the empirical focus of their research, the primary results are detailed estimated returns to college majors with associated math and verbal SAT scores. The authors note that this area of research is relatively unexplored and is important for understanding the structural mechanisms underpinning the ex post outcomes of higher education. A crucial difference between our research and Altonji et al. (2012) is, by using the information on mathematic skill requirement within occupations from the Dictionary of Occupational Titles (DOT) and associated Occupational Information Network (O*net) data, we show that mathematics-focused majors are highly correlated with ex post wage outcomes through the occupational choices available to these majors.

Addressing similar wage discrepancies, Silos & Smith (2012) look at the trade-off between acquiring specific and targeted human capital. They concentrate on individuals’ choices between education paths leading to specific occupations versus those that have broader applicability, and thus more occupational choice. This research fits well within the broader educational transition described above. The authors show that policies directed at occupation-specific human capital accumulation lead to lower income growth and lower inequality. In this research, we emphasize the importance of the skill types accumulated, with particular attention given to mathematics as either a specific necessary ability or as a strong indicator of associated abilities. Specifically, those who have majored in math-intensive areas may initially sort into high-wage occupations, with little incentive of switching to (alternative high-wage) occupations.

The aim here is to explicitly model the education decision based on the five facts out-

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3 A comprehensive review of the existing empirical studies on the returns to college major can be found in Table 2 of Altonji et al. (2012).
5 Occupational Information Network, 2002
line above, and presented in detail in section 2, with math abilities being the main driver of wage inequality. Specifically, we hypothesize that wage inequality is driven to a large extent by individuals’ initial abilities, which limits an individual’s educational choices. Therefore, that part of the within group wage inequality is explained by individuals’ education choices, which are largely dependent on individuals’ innate mathematics abilities. A secondary concern of this research is highlighting the effects of currently enacted policy goals directed toward increasing college attendance rates. Lastly, this research suggests that individuals maximize their expected wages based on their initial abilities, with the learning of mathematics skills in school being the most important initial determinant.

The model in this paper is loosely based on Hendricks & Schoellman (2012). In that paper the authors look at the discrete education choices of individuals (i.e., high school, some college, and college), focusing on \textit{ex ante} abilities as measured by IQ scores. Their results show that one-third of the college wage premium and one-fourth of its growth is driven by ability (“ability premium”). While looking at ability as a driver of wage outcomes, Hendricks & Schoellman (2012) define broad education categories that mask the subset mainly driving wage inequality: the top earning college graduates, who exhibit strong mathematical abilities.

As wage inequality across different education groups, mathematics requirements of college majors and quantitative occupation requirements are the key features within this study, Section 2 provides a summary of the data facts related to wage inequality, occupations and college majors over time and across cohorts. The general equilibrium model is outlined in Section 3, and Section 4 provides analytical results. Section 5 concludes.

2 Data

This research relies on six facts listed in the introduction and expanded upon in this section. These six points together present a coherent story of \textit{ex ante} mathematical ability dictating initial college major options, from which occupations and, ultimately, wages are determined. I.e., those with higher mathematics abilities pursue math-heavy majors and occupations. These particular occupations also enjoy the highest wages. Furthermore, the
math intensive majors that lead to higher wage occupations are increasingly shunned by each subsequent generation of college degree holders. This shift away from math-heavy majors is further exacerbating wage inequality.

2.1 Who is Driving Wage Inequality?

To illustrate which education group subsets are driving wage inequality, we use data from the CPS from which the residual of a Mincer wage regression is derived from log hourly wages of full-time, full-year males aged 25-59. The regression controls for age, age-squared, race, marital status, and state of residence (using CPS weights). The unexplained residual for various education-wage groups are compared in Figures 3 and 4. These cross-education wage-group comparisons highlight the importance of high-earning college graduates in driving wage inequality, especially since the mid-1980s (see Figure 4).

![Figure 3: Bottom Wage College Graduate Performance](image_url)

Figure 3 compares the residual wages of the bottom 10th and 20th percentile of college graduates with the middle and upper non-college wage groups (50, 80 and 90th percentiles)
with the average individual, the 50th percentile in the total sample (A50). It is important to note that the bottom earning college graduates have significantly lower wages than the middle and upper non-college wage groups. All comparisons show a flat or mild divergence, with continuing relative wages favoring non-college graduates. The final comparison within Figure 3 shows that the bottom college-wage decile has lost ground against the average individual. To put the average into perspective, college-graduates account for about 30 percent of the sample. That is the average individual is likely a non-college graduate, just above the average non-college graduate (see Figure 4 bottom line). In contrast to Figure 3, Figure 4 compares the residual wages of the middle and top college and non-college wage groups with the average individual (A50). The 90th percentile college-wage group has outpaced the median non-college-wage group by more than 50 percent since the mid-1970s. This is a remarkable performance considering the top college-wage groups have also increased their wage premium against the top non-college-wage groups by approximately 35 percent. The implications of this figure are summarized in two points: (1) the top
college-wage groups are outpacing all other groups, but (2) are sprinting ahead of the median non-college earners. Thus, a large part of wage inequality growth is driven by the top college-wage groups, while the bottom college-graduates are behind compared to a large share of non-college graduates.

2.2 Occupations

Figure 5 links college majors with the skill requirements of occupations for two cohorts born between 1945-1950 (“1950”) and 1980-1985 (“1985”) at age 25-30 (1975 and 2010, respectively). Here, the DOT and O*net numerical job requirements labeled “General Educational Development in Mathematics” (GED math) associated with occupations are applied to the ACS. The ACS contains individual-level observations with college major and occupation information. Figure 5 shows that the math requirements for occupations was important for both cohorts, but more-so for the 1985 cohort. This increasing importance of math requirements in occupations is a central empirical fact of this research. Not only is the math requirement of occupations strongly linked with wages, but this link appears to be strengthening with a correlation coefficient of 0.24 for the 1950 cohort versus 0.51 for the 1980 cohort.

Figure 6 takes the next step and illustrates how the math requirements of occupations are distributed across college majors and the college-level math credits associated with these majors. In this figure, all individuals are first grouped by their college major and the average occupation math requirement is commuted, as there are multiple occupation outcomes within each college major. This figure shows that occupation-specific math skills are highly correlated with college-level math credits by college major, with a 0.71 correlation coefficient.

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6The American Community Survey 2010 is used throughout. The ACS 2009 is the first year in which college major is included. Note that the trends observed in the ACS 2010 are virtually identical in the ACS 2009.
Figure 5: Wages and Math Requirements for Occupations

(a) Occupations by Math 1950

(b) Occupations by Math 1980

Source: ACS, DOT and O*net

Figure 6: Occupation Math Requirements and College-Level Math Credits by College Major
2.3 College Majors

The previous subsection linked occupation math requirements to college-level math credits. From here, we look at understanding the initial characteristics that lead to college major sorting. Figure 7 takes individuals in the ACS and the college-level math credits and mean SAT math score by college major from the NCES to illustrate that \textit{ex ante} math abilities are highly correlated with college-level math credits. With a correlation coefficient of 0.77, those with high math abilities prior to college, as measured by the average SAT math scores of those graduating within a specific college major, are very likely to graduate from math intensive college majors, as measured by the college-level math credits.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{SAT Math Scores and College-Level Math Credits by College Major}
\end{figure}

The ACS offers a cross-sectional snapshot in 2010 of college major by individuals from which we construct a measure of how individuals’ college major choice has evolved. This assumes that most individuals do not go back to school at age beyond 30, and that the ACS sample is representative of the population at every age group. Figure 8 illustrates the
changes in degrees obtained, as measured by the mean SAT math score by college majors in 2008-2009 from the NCES. To do this, we plot smoothed changes in the share of college graduates between the 1955 cohort and subsequent cohorts at each percentile of the SAT math score associated with the college degrees obtained by each cohort as reported in the ACS. Figure 8 highlights the declining share of high math majors compared with the 1955 cohort. Only the 1960 cohort displays growth in this area compared to the 1955 cohort. All cohorts, except 1960, show a significant decline in all majors except those ranked in the bottom decile by SAT math scores. Furthermore, the three middle quintiles experienced the largest declines, leading to a pronounced U-shaped change in graduation shares for all cohorts.

ACS 2010 is used here, but the results from the ACS 2009 are similar. The smoothing method used is similar to Autor et al. (2003).
2.4 Other Measures

Table 1 shows how other measures of *ex ante* ability, such as the SAT verbal score or GED general intelligence measure, are correlated with log wages, college math credits and the usual measures of *ex ante* math ability. The results presented here are only for the 1985 cohort, the results are similar for older cohorts. The correlation between all math measures are roughly double those focused on other measures. While this is possibly due to noise within the non-math measures, the correlation between SAT math and verbal is 0.81. This high correlation between math and verbal scores despite the much higher correlation between log wages and math scores further highlights the special significance of math as either a direct or indirect measure of high return skills in the labor market. Moreover, while the results for older cohorts are similar, with every successively older cohort the correlation between verbal measure and wages becomes stronger, suggesting that selection of individuals into college could really be driving the results, rather than noise.

### Table 1: Ability Measures and Wages

<table>
<thead>
<tr>
<th></th>
<th>log(w)</th>
<th>GEDM</th>
<th>GEDV</th>
<th>Col. Math Cred.</th>
<th>SATM</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEDM</td>
<td>0.711***</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEDV</td>
<td>0.360***</td>
<td>0.735***</td>
<td>1</td>
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<td>Col. Math Cred.</td>
<td>0.597***</td>
<td>0.708***</td>
<td>0.435***</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>SATM</td>
<td>0.536***</td>
<td>0.737***</td>
<td>0.594***</td>
<td>0.771***</td>
<td>1</td>
</tr>
<tr>
<td>SATV</td>
<td>0.229**</td>
<td>0.420***</td>
<td>0.479***</td>
<td>0.489***</td>
<td>0.808***</td>
</tr>
</tbody>
</table>

* p < 0.05, ** p < 0.01, *** p < 0.001

3 Model

As this research is focused on time-series of wage inequality, a general equilibrium model is required to provide the time dynamics that would invariably be ignored running a regression model on the cross sections of data available. This overlapping generations model will feature a unit mass of finitely lived agents and a single representative firm.
3.1 Individuals

The model is loosely based on Hendricks & Schoellman (2012). In addition to analyzing the schooling decision in a general equilibrium set-up, the model is also modified to incorporate a choice between college majors and the fact that individuals will face constraints based on their ability. The majority of the action within this model occurs in two periods, period 0 and period 1. As such, a brief overview of the individual agent model timing is first presented, followed by additional detailed functional forms.

**Period 0.** Individuals must choose a level of education, $M_j$, where $M_0$ is high school and college when $j > 0$. Thus, individuals choose to attend college or not at age 0 (i.e., age 18 in the data), and which major to specialize in. The college majors each require different levels of math achievement, $\lambda_j$, where $\lambda_j < \lambda_{j+1}$. This can be thought of as an increasing cost of studying more difficult college majors. Individuals know their innate math ability, $\theta^i$, in period 0 (e.g., SAT math scores). We assume $\theta^i$ will directly influence their education choice, with higher math abilities denoted by a higher $\theta$. Specifically, we assume that higher ability individuals face a lower cost of completing more difficult majors, $\frac{\lambda_j}{\exp(\theta')} < \frac{\lambda_j}{\exp(\theta)}$, where $\theta' > \theta$. This assumption, in general, leads to high ability individuals selecting more difficult majors, and very low ability individuals forgoing a college-level education. While there is positive sorting by ability into more math intensive schooling types in the data (see Altonji et al., 2012), the sorting is not perfect. Therefore, we introduce a random component, $\eta^i$, which is iid across individuals, and summarizes the individual-specific preferences for studying. The random component could be interpreted either as the tolerance for studying (e.g., “love” of studying) or preferences across majors in school. I.e., a high ability individual with a negative $\eta^i$ will make a less math-intensive education choice potentially because he prefers the subject matter of another major. We will be agnostic about the exact nature of this component, and calibrate it to match the data on ability and college attendance in the data. Average wages for each education choice

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8Empirical research has found men sort into majors by ability and wages (), while women sort based on other characteristics as well ().
in period 0, $w_{j0}$, are also known. The initial education choice is depicted in Figure 9, after which individuals age.

![Figure 9: Period 0: Individuals’ Decision Tree](image)

**Period 1.** In period 1 (i.e., $18 < \text{age} < 23$), individuals who choose $M_0$, high school education only, work. Individuals who choose a major, $M_j$, with $j > 0$, enter college and graduate from their education choice, $M_j$ only if $\theta^i + \epsilon^i \geq \theta_j$, where $\theta^i + \epsilon^i$ is an individual’s realized college math ability. If $\theta^i + \epsilon^i < \theta_j$, then the individual graduates from the highest $M_k$ major such that $\theta^i + \epsilon^i \geq \theta_k$. Thus, individuals only move downward from their initial choice in terms of math content. These cutoffs ensure that all individuals graduating within any specific college major have a minimum math ability. This assumption also implies that some individuals will dropout and only attain a high school education, but bear the cost of attending college through missing one period of work. Figure 10 details the possible education choices and paths. Note that period 2 onward is trivial in that individuals who chose to pursue college education will enter the labor market in period 2 and inelastically supply labor in each subsequent period. As stated earlier, those who do not pursue college education start work in period 1 and continue working in each subsequent period.
Maximization Problem. Figures 9 and 10 are summarized in the following objective function with individuals maximizing expected lifetime income:

$$
\max_j \left\{ \sum_{t=g^i}^{A} \beta^{t-1} E \left( \sum_{k=0}^{j-1} (p_{k}^i - p_{k+1}^i) w_{kt}^i \right) - \frac{\lambda_j}{\exp(\theta^i)} + 1_{(j>0)}\eta^i \right\}^{n}_{j=0}
$$

, where $\beta$ is the discount factor, $g^i$ is the period in which an individual begins working (i.e., 1 for high school, 2 for college) and $A$ is the total working life. $\theta^i$, $\lambda^i$ and $\eta^i$ are defined above, with the probability of graduating within any college major, $p_j^i$, and wages, $w_t^i$, described in-detail below. Maximizing life-time income is equivalent to maximizing consumption utility assuming complete markets.

Wages. Actual wages, drawn after graduating, depend on an individual’s math ability, schooling choice, and the aggregate supply of college graduates with a given skill set (i.e., “college major”). In the context of wage inequality and schooling choice, the general equilibrium effects cannot be ignored. Thus, wages are determined in general equilibrium, with the wage for an individual with college major $M_j$ calculated as,

$$
\ln(w_{jt}^i) = w_{jt} + \rho \theta^i + \mu_t^i
$$

. The log wage is, thus, the college major premium, $w_{jt}$, an individuals’ efficiency units of labor, $\rho \times \theta^i$, and a residual term, $\mu_t^i$. In modeling individuals’ efficiency units of labor
we follow Hendricks & Schoellman (2012), where an individual’s efficiency units of labor depend on their innate ability $\theta_i$. The individual’s wage is composed of the product of the schooling-specific price in the labor market multiplied by their labor units. The schooling-specific prices are determined in general equilibrium. Note that, while in school, individuals forgo working and, after graduating, will supply labor inelastically to the labor market at wage $w^i_t$, which will depend on their education choice.

**Probabilities.** The *ex ante* probability, $p^i_j$, of graduating within a specific college major, $M_j$, depends on the agent’s ability and realized college math-ability,

$$p^i_j = p(\theta^i + \epsilon \geq \bar{\theta}_j) = p(\epsilon \geq \bar{\theta}_j - \theta^i).$$

The expected wage from choosing a given major $j$ is,

$$E(w^i_j) = \left( \sum_{k=0}^{j} (p^i_k - p^i_{k+1})w^i_{kt} \right),$$

where $p^i_{k+1} = 0$ and $p^i_0 = 1$. That is, it is the probability of graduating from major $j$ or any subsequent major below $j$.

### 3.2 Firm

A representative firm operates a CES production function to produce the final consumption good in the economy, $Y$. The firm hires labor of all education types, which are imperfect substitutes,

$$Y = \left\{ \sum_{j=0}^{n} A_{jt}L^t_j \right\}^{1/\nu},$$

where labor shares are defined as,

$$L_j = \int_{i}^{\left(1_{(s^i=j)}e^{\theta^i}\right)} di, \forall j = 0, \ldots, n$$

$A_{jt}$ is skill-biased technical change (SBTC) over time. In this paper, SBTC can be redefined as math-biased technical change (MBTC), given the focus on the importance of math skills. The elasticity of substitution between education types is $\frac{1}{1-\nu}$. 

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The relative labor demand can be defined in terms of $A_{jt}$ and wages,

$$\left( \frac{L_{kt}}{L_{jt}} \right)_{\text{demand}} = \left( \frac{A_{kt}}{A_{jt}} \right) \left( \frac{w_{jt}}{w_{kt}} \right)^{1-\nu} \forall k,j. \quad (7)$$

Firms demand relative more labor of type $k$ the lower the wage rate or the higher the labor’s productivity $A_{kt}$.

To compute actual wage rates, we normalize the final goods’ price to one, $p_t = 1$. Using the unit cost of producing one unit of output, it is straightforward to derive all wages. Labor demand to produce one unit of output is,

$$\frac{L_{jt}}{Y_t} = \left( \frac{A_{jt}}{w_{jt}} \right)^{\frac{1}{1-\nu}} \left( \sum_{j=0}^{n} A_{jt} w_{jt} \right)^{\frac{1}{1-\nu}} \forall j, \quad (8)$$

while the cost of producing one unit of output in equilibrium must equal prices,

$$p_t = \sum_{j=0}^{n} w_{jt} \frac{L_{jt}}{Y_t}. \quad (9)$$

### 3.3 Equilibrium

The general equilibrium conditions are dependent on the the individuals’ and firm maximization problems.

An equilibrium, given college math premiums $\{w_{jt}\}_{j=0}^{n}$, exists and is defined by,

1. The education choice, $j$, that maximizes the individual problem

   $$\max_j \left\{ E(u_j^i) \right\}_{j=0}^{n}, \quad (10)$$

   subject to the graduation constraint $\theta^i + \epsilon^i \geq \bar{\theta}_j$;

2. The demand for labor $\{L_{jt}\}_{j=0}^{n}$, that minimizes the firm’s problem

   $$\max_{\{L_{jt}\}_{j=0}^{n}} \left\{ P_t Y_t - \sum_{j=0}^{n} w_{jt} L_{jt} \right\}; \quad (11)$$

   and

3. Labor markets clear,

   $$\left( \frac{L_{kt}}{L_{jt}} \right)_{\text{demand}} = \left( \frac{L_{kt}}{L_{jt}} \right)_{\text{supply}}. \quad (12)$$
3.4 Dynamics

Using Equation substituting Equation (8) into (5) and using the equilibrium pricing condition Equation (9), wages must satisfy,

\[ w_{0t} = A_{0t} \left( \sum_{j=0}^{n} A_{jt} \left( \frac{L_{jt}}{L_{0t}} \right)^{\nu} \right)^{\frac{1-\nu}{\nu}} \quad \forall j, \]

and

\[ \left( \frac{w_{jt}}{w_{0t}} \right) = \frac{A_{jt}}{A_{0t}} \left( \frac{L_{0t}}{L_{jt}} \right)^{1-\nu} \quad \text{for } j > 0. \]

*Ceteris paribus* a rise in \( \frac{A_{jt}}{A_{0t}} \) for \( j > 0 \), MBTC, results in a relative wage increase for college educated individuals of major \( j \), high school wages \( w_{0t} \) fall over time. However, a rise in the relative share of college major \( j \), leads to a relative fall in college major \( j \) wages (compared to high school graduates). In general, an increase in the supply of a certain college major results in a fall of their relative wages, assuming \( 0 < \nu < 1 \), i.e., they are imperfect substitutes. With MBTC, more individuals find college attractive. However, if the new college entrants have, on-average, lower ability levels, after accounting for the ability constraint on graduating, an excess supply of low-math majors builds-up. Therefore, wage inequality within college majors widens. As a consequence, the larger the absolute number of college entrants, the larger the post-education wage inequality.

4 Calibration

The model estimated parameters estimated can be grouped into four categories: (1) general- \( \{ \beta, A \} \); (2) college major- \( \{ \{ M_j, \lambda_j, \theta_j \} \}_{j=1}^{n}, \sigma_j \} \); (3) firm- \( \{ \{ A_{jt}, \delta_j \} \}_{j=0}^{n}, \nu \} \); and (4) individual-related \( \{ \theta, \rho, \sigma_\eta \} \}. The estimation procedure for these for groups is discussed in the following subsections. The calibration procedure estimates all parameter values jointly, however, below, we point about which moment we believe to be particularly informative of a given parameter.
4.1 Calibration: General

We set $A = 9$ where each period is assumed to cover five years. Thus, the modeled working life-time of individuals covers the equivalent of 45-years. As the period 0 decision is assumed to take place around the age of 18, this means the model covers an age range of 18 to 63, approximately.

The model uses a standard discount factor of $\beta = 0.9$ per period. This implies a discount rate of approximately two percent per year, given the assumptions on $A$.

4.2 Calibration: College Majors

There are four parameters associated with college majors that must be defined:

- $M_j$: college majors;
- $\theta_j$: college major cutoffs;
- $\lambda_j$: college major math credits; and
- $\epsilon^i$: revealed math ability in college (assumed $\sim N(0, \sigma^2_\epsilon)$).

College major choices, $M_j$: We first discretize the possible number of college majors. As the correlation between college-level math credits and occupation-specific math requirements is large, we translate the DOT/O*net GED math measure of math requirements for occupations to represent the spectrum of college majors. This definition implies a one-to-one match between college majors and occupations in terms of math content ranking. Thus, the definition of GED math, a measure of math requirements ranging from 1 to 6, is used to create four levels of college majors or three college-level cut-offs. The GED math range is defined as:

1. Add and subtract two-digit numbers, multiply and divide 10s and 100s;
2. Four operations on all units, including common and decimal fractions;

$^9$The discount rate satisfies the equation $\beta = \frac{1}{1 + r}$
3. Basic algebra (polynomials), geometry (area, volume);

4. Algebra (real number systems), geometry (axiomatic geometry);

5. Calculus (differentiation/integration), algebra (mathematical induction); and

6. Adv. calculus (implicit \( f \) theorem), algebra (differential equations).

The four college majors are defined in terms of the GEDM: \( M_1: 1-2; M_2: 3; M_3: 4; M_4: 5-6. \) It should be noted that the 1-6 GED math measure covers all occupations. This is consistent with college-educated individuals occupying all levels of employment, with respect to math content.

**College Major Ability Cut-offs, \( \bar{\theta}_j \):** This parameter is a hard cutoff in terms of \( \theta \) such that individuals can only graduate from their college major choice \( M_j \) if \( \theta^i + \epsilon^i \geq \bar{\theta}_j \) holds. With four college majors, there are four hard cutoffs. We compute the value of these cutoffs to match the college major shares in 1975.

**Cost of Studying Math \( \lambda_j \):** Each college major has a unique level of math content, with more difficult college majors containing more math content. Using the ACS individual-level observations, we sort all individuals into the four college major bins from above defined by GED math (\( M_j \)). With college majors sorted into four broad groups, consistent with the definition of \( M_j \), college-level math credits from the NCES for each college major are averaged across the four modeled college majors. This allows a comparison of the log average college-level math credits across the four modeled college majors. The results of this computation are: \( \lambda_j = \{1.00, 1.18, 2.31, 3.00\} \). Normalizing \( \lambda_1 = 1 \) means that \( M_2, M_3 \) and \( M_4 \) have 18\%-131\%-200\% more college-level math credits, respectively, than \( M_1 \). These are relative cut-offs. To compute the actual cut-offs within the model, we calibrate a multiplicative constant to match the average dropout rate of \( \theta \) of college graduates obtained from the NLSY in 1979. Thus, the final vector is \( \lambda_j = 0.09 \times \{1.00, 1.18, 2.31, 3.00\} \), with 0.09 being equal to the fixed cost of studying the major with the lowest math content.
**Realized Math Ability, $\epsilon_i$:** We assume that the revealed math ability in college is $\theta^i + \epsilon^i$, where $\epsilon^i$ is $\sim N(0, \sigma^2_\epsilon)$). Thus, the variance of $\epsilon^i$, $\sigma^2_\epsilon$, is calibrated to match a college dropout rate of 54 percent, as reported in ?. In the model presented, this means that approximately 41 percent of individuals in 1975 will attempt to obtain a college degree ($M_j$ where $j > 0$), but ultimately only attain a high school level of education, $M_0$.

### 4.3 Calibration: Firm

There are two parameters associated with the firm that must be pinned down:

- $A_{jt}$: math biased technical change;
- $\nu$: labor substitution

Using the firm’s problem (Equation (14)), we can derive a standard wage equation to estimated, the substitution between labor types,

$$\log(\frac{w_{jt}}{w_{0t}}) = d_j + \delta_j t + (\nu - 1) \log(\frac{L_{jt}}{L_{0t}}) + \epsilon_{jt}, \text{ for } j > 0,$$

where MBTC takes the form $\log\left(\frac{A_{jt}}{A_{0t}}\right) = d_j + \delta_j t$. To compute the elasticity we run stacked regressions for $j > 0$, using the time-series data from the CPS merged with the GED math measure from the DOT/O*net. Table 2 summarizes the results.

**Table 2: SBTC and Elasticity of Labor**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_1$</td>
<td>0.0154***</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.0136***</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>0.0170***</td>
</tr>
<tr>
<td>$\delta_4$</td>
<td>0.0171***</td>
</tr>
<tr>
<td>$(\nu - 1)$</td>
<td>-0.336***</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.997</td>
</tr>
</tbody>
</table>

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
**Elasticity of substitution** $\nu$: The resulting elasticity of labor is $\frac{1}{1-\nu} = 2.98$, that is the elasticity is towards the upper estimates standard college to non-college labor elasticities, (see Autor et al., 1998, for elasticities for college to non-college labor).

**Initial labor shares, $A_{jt}$**: Given the definition of MBTC, $\log \left( \frac{A_{jt}}{A_{0t}} \right) = d_j + \delta_j t$, it is necessary to pin down the initial weights in production, $d_j$. We do this by matching labor shares for $M_j$ in 1975, with the weight on high school labor normalized to one each period, $A_{0t} = 1$. Note, that is equivalent to calibrating $A_{j0}$ for all $j > 0$. The growth rates, $\delta_j$ is of $A_{jt}$ are taken directly from table 2.

### 4.4 Calibration: Individual

There are three parameters associated with individuals that must be defined:

- $\theta_i$: *ex ante* math ability;
- $\eta_i$: preference for studying; and
- $\rho$: the return on math ability.

All the above individual parameters are estimated from the NLSY 1979, Armed Forces Qualification Test, math section ($AFQT_{math}$).

**Innate ability, $\theta_i$**: We assume that $\theta_i \sim N(0,1)$ as this generally fits the normalized NLSY 1979 $AFQT_{math}$ scores seen in Figure 11.

**Taste for college, $\eta_i$**: While individuals sort into college education based on their innate math ability, we do not observe perfect sorting (see Figure 12). Thus, we assume $\eta_i \sim N(0, \sigma_\eta^2)$ and we calibrate $\sigma_\eta$ to match the average $\theta$ of individuals who go to college. Note that this target should not be confused with the proportion of individuals graduating with a college degree.
Figure 11: Ability Distribution

Figure 12: Ability Distribution by Education Type
Returns to innate ability, $\rho$: The return to math ability is found by regressing $\log(w^t)$ on the standardized armed forces test score in mathematics, verbal, an indicator for college graduates, age, age squared, race, region, an indicator for being married and whether the individual leaves in a metropolitan statistical area. The results of this regression are found in Table 3. The results of two regression model specifications are included to highlight the separation of the verbal and math AFQT measures. Specification (1) uses only the combined general AFQT variable, which can be thought of as the returns to ability. The return for ability in specification (1) is 0.168, whereas separating the AFQT into math and verbal measures seen in specification (2) shows that the majority of the returns to ability is captured solely by $AFQT_{math}$. That is 79 percent of the returns to ability is captured by the $AFQT_{math}$ variable. Not only does the math component capture all the wage differences, the verbal component is statistically insignificant and magnitudes smaller. Furthermore, our estimate for the return to ability closely matches the value estimated by Hendricks & Schoellman (2012), who use the NLSY97.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CG</td>
<td>0.391***</td>
<td>0.385***</td>
</tr>
<tr>
<td>$AFQT$</td>
<td>0.168***</td>
<td></td>
</tr>
<tr>
<td>$AFQT_{math}$</td>
<td>0.133***</td>
<td></td>
</tr>
<tr>
<td>$AFQT_{verb}$</td>
<td>0.044</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>1379</td>
<td>1379</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.201</td>
<td>0.203</td>
</tr>
</tbody>
</table>

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
4.5 Calibration Summary

Table 4 summarizes the estimated and calibrated parameters, with the central line dividing estimated parameters at the top and calibrated parameters at the bottom.

Table 4: Calibration Summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source / Targets (1975)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9</td>
<td>Standard</td>
</tr>
<tr>
<td>$A$</td>
<td>9</td>
<td>Retirement at 65 (5 year periods)</td>
</tr>
<tr>
<td>$M_j$</td>
<td>{1-2, 3, 4, 5-6}</td>
<td>DOT/O*net GEDM</td>
</tr>
<tr>
<td>$\delta_j$</td>
<td>{0, 0.0154, 0.0136, 0.0170, 0.0171}</td>
<td>CPS</td>
</tr>
<tr>
<td>$\theta^i$</td>
<td>$\sim N(0, 1)$</td>
<td>NLSY</td>
</tr>
<tr>
<td>$\lambda_j$</td>
<td>$\lambda_1 \times {1.18, 2.31, 3.00}$</td>
<td>College math credits (NCES, ACS)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.66</td>
<td>CPS</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.13</td>
<td>NLSY</td>
</tr>
<tr>
<td>$A_{0t}$</td>
<td>1.00</td>
<td>Normalized</td>
</tr>
<tr>
<td>$A_{jo}$</td>
<td>{0.45, 0.47, 0.61, 0.60}</td>
<td>Relative wages: (w_j/w_0) (CPS)</td>
</tr>
<tr>
<td>$\bar{\theta}_j$</td>
<td>{0.76, 0.99, 1.34, 2.35}</td>
<td>College major shares (CPS)</td>
</tr>
<tr>
<td>$\sigma_{\epsilon}$</td>
<td>1.68</td>
<td>College dropout rate</td>
</tr>
<tr>
<td>$\sigma_{\eta}$</td>
<td>0.24</td>
<td>Average $\theta_{go to college}$ (NLSY)</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.09</td>
<td>$\theta_{college graduate}$ (NLSY)</td>
</tr>
</tbody>
</table>

The 1975 data targets used in pinning down the calibrated parameters are summarized in Table 5. The model does well in matching all targets, except for slightly underestimating the average ability of college graduates.

5 Results [Preliminary]

The model accurately captures a variety of inequality dynamics, including both the general wage trends and a high degree of the relative wage evolution between 1975 and
Table 5: Targets Summary

<table>
<thead>
<tr>
<th>Target</th>
<th>Data (1975)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1/w_0$</td>
<td>0.169</td>
<td>0.161</td>
</tr>
<tr>
<td>$w_2/w_0$</td>
<td>0.253</td>
<td>0.236</td>
</tr>
<tr>
<td>$w_3/w_0$</td>
<td>0.319</td>
<td>0.321</td>
</tr>
<tr>
<td>$w_4/w_0$</td>
<td>0.413</td>
<td>0.424</td>
</tr>
<tr>
<td>$M_1$ share</td>
<td>0.038</td>
<td>0.039</td>
</tr>
<tr>
<td>$M_2$ share</td>
<td>0.043</td>
<td>0.043</td>
</tr>
<tr>
<td>$M_3$ share</td>
<td>0.089</td>
<td>0.091</td>
</tr>
<tr>
<td>$M_4$ share</td>
<td>0.064</td>
<td>0.065</td>
</tr>
<tr>
<td>College dropout rate</td>
<td>0.540</td>
<td>0.541</td>
</tr>
<tr>
<td>Average $\theta_{go to college}$</td>
<td>0.530</td>
<td>0.575</td>
</tr>
<tr>
<td>$\theta_{college graduate}$</td>
<td>0.960</td>
<td>0.882</td>
</tr>
</tbody>
</table>

2010. In particular, the 90-50 percentile wages are matched well throughout the modeled time period. This can be seen in figure 13, where the actual US inequality trends and the modeled results are compared.

A detailed numerical comparison of the data and modeled results is presented in table 6. The relative wage loss (gain) against the median wage from 1975 to 2010 is compared to the equivalent model output. The model does well in matching the C90 and C10 education groups, but the discrete major choice over-predicts the rise in C50 wages. The NC50 and NC90 results are less accurate, as the model does not employ a specific theory to match non-college wages. Overall, the results point to math ability as a substantial component of wage inequality, even when considered in the simple framework presented here.

6 Counterfactual [Preliminary]

To assess the importance of the returns to math, a counterfactual is explored where all college majors earn the average college math premium. The results are presented in table 7.
as the relative loss (gain) to the median wage from 1975 to 2010. The results indicate that math matters for wage inequality, in that the counterfactual cannot explain the C10 wages. I.e., the C10 education group would earn substantially higher wages if the math premium were applicable. Thus, the counterfactual suggests that if lower ability students could earn the average college (math) premium, then college would be a good choice. However, since ability matters for what students are able to learn in college this is not the case. Therefore, simply increasing college enrollment leads to larger (not less) wage inequality.
7 Conclusion [Preliminary]

The connection between \textit{ex ante} math abilities, math heavy college majors, and high math occupations provides a simple and powerful mechanism explaining a large component of wage inequality in the US. The model and results presented within this paper have strong policy implications. To the extent that math abilities are determined in-advance of the education choice (e.g., college and/or field of study), then policies aimed at extending college education to those with less strong math skills to address wage inequality are unlikely to succeed. To address inequality at a fundamental level, policies aimed at increasing math skills, at a young age, across all ability levels would likely achieve better results. The college education choice is not necessarily the driving force behind wage inequality, as intra-college graduate wage inequality is significant.

\begin{table}[h]
\centering
\caption{Wage Inequality Counterfactual: Equal Math Premium}
\begin{tabular}{lcccccc}
\hline
 & C10 & C50 & C90 & NC50 & NC90 \\
\hline
Data (\%) & -12.4 & 6.5 & 26.8 & -13.3 & 5.0 \\
Model (\%) & -16.5 & 27.9 & 32.6 & -8.2 & -6.2 \\
Counterfactual (\%) & 0.3 & 29.2 & 29.9 & -8.3 & -6.4 \\
Explained of Model (\%) & -1.8 & 104.8 & 91.8 & 101.9 & 102.4 \\
\hline
\end{tabular}
\end{table}
References


