State aids regulation for firms in difficulty: a rationale for production capacity constraints

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Abstract

This article analyses one aspect of the European State aids control for firms in difficulty. Most of the time, State aids are submitted to production and sales constraints. We try to find a rationale for these constraints by building a simple model with two firms, a healthy one and a firm in difficulty. When the aid takes the form of a soft loan and is not submitted to conditions, it can hurt consumers since the healthy firm sometimes stops her investment. We show that, under a set of parameters, we can build a menu of contracts with capacity constraints on production such that the healthy firm is still investing and the firm in difficulty receives an aid and stays in the market.

1 Introduction

The European competition policy strictly controls State aids because of their potentially harmful impact on competition. Most of the time, the European Commission imposes output restrictions when allowing State aids for firms in difficulty. This additional restrictions to the restructuring plan aim at protecting consumers and competition. Many cases can illustrate the constraints imposed on production, such as the aid to Air Malta. This flying company had to give up some planes and some destinations and time slots to receive an aid.

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1. Others examples can be taken to illustrate the restrictions on production imposed to firms for receiving the aid. L'imprimerie Nationale, a French printing firm, was forced to keep its market shares constant during the whole duration of the aid, or FSO, a Polish car manufacturer, received limitation on production and sales in addition to the interdiction of getting new car licence during the aid.
2. “[...]The reduction of frequencies operated and the cancellation of certain routes, [...] landing slots pairs will be surrendered are at European Level. The Commission notes that the surrender of these landing slots will enable other competing airlines to increase their capacity at those coordinated airports [...] and thus represents a reduction of entry barriers on the market. Therefore, this measure can be accepted as a compensatory measure.” COMMISSION DECISION of ON THE STATE AID No SA.33015 which Malta is planning to implement for Air Malta plc., §116
The objective of this paper is to find an explanation to these production restrictions. We show that in some cases, constraining the output allow the survival of a firm in difficulty while preserving investment of the competitors. But production restrictions are not always necessary. Besides, they can be insufficient to imply the investment and survival of all the firms.

Our model sets up a simple framework of imperfect competition and imperfect financial market. A “healthy firm” and a “firm in difficulty” are competing à la Cournot. They both own assets which can be invested to lower costs. The investment success rests on firms’ effort and is then subject to moral hazard issues.

We first present our model and define the type of firms. The firm in difficulty experiences a higher marginal cost than its competitor, that leads to her exit without a cost decrease. Her assets are however insufficient to encourage her effort, although the investment project is profitable. The healthy firm is trying to decrease her cost (investment and effort are done). We hence have a monopoly when there is no aid, which points out the rationale for State aids in our case: a better situation can be reached by reestablishing a duopoly. The characterization of the benchmark leads to assumptions on profit functions and on parameters of the model.

We then study the optimal state aid. The State is able to observe firms and wants to design a mechanism encouraging both firms to invest and make effort. He proposes a contract to the firm in difficulty containing a soft loan and a limitation on her production. The major difficulty of the contract design is to find a mechanism implying the survival of the firm in difficulty while preserving innovation. Indeed, when there is no restrictions on the production following the aid, we can have a crowding out effect: the competitors of the helped firm have less incentive to invest since the competition is higher. But imposing a limitation makes the aid useless if it is too strict to allow the survival of the helped firm.

The main results are the following. We have two instruments (the level of aid and of production limitation) to deal with two problems, which are the negative externalities on the healthy firm and moral hazard. We show that under a set of conditions insuring that our assumptions are satisfied, there exists two types of contracts. The first one happens when externalities are quite weak. There is no need to impose restrictions on the helped
firm’s production because the probability of being in a duopoly for the healthy firm is sufficiently low. We can solve big moral hazard issue in this case by giving a large State aid. This contract leads to production efficiency.

We also have a second type of contract in which there is production restrictions. When there is a high probability of success in case of effort, negative externalities are larger. It’s possible to save the firm in difficulty by giving a large amount of aid, but we must impose restrictions on her production to compensate the healthy firm.

Outside these two contracts, there is no State aid contract allowing the survival of the firm in difficulty and the investment of the healthy firm because of the tension on the production restrictions. To save the firm in difficulty, we must give her a large State aid with soft restrictions, but externalities on the healthy firm are too large to ensure her investment and effort in this case.

Section 2 presents the model, section 3 deals with optimal State aids in case of moral hazard. Section 4 is an extension to adverse selection. The State is no longer able to differentiate firms and to know if they are in difficulty. It hence has to construct a menu of contract to push firm to self-select and take the aid only if they can’t survive without it.

Discussion of the literature

Our work is mostly related to the literature on State intervention on market in difficulty. Philippon & Skreta (2012) and Tirole (2011) are studying the interaction between the State intervention and the financial market while our paper is focusing on the interaction with the good market. In Tirole, the State buys the assets of the less efficient firms so the survival firms have their productivity increased. We don’t have this kind of Akerlof effect in our model since we take the interest rate as exogenous.

This paper is also related to the literature on production efficiency at the optimum. Diamond and Mirrlees (71) showed that there should be aggregate production efficiency at the optimum when there are commodity taxes, even when the Pareto optimum is not reached. Firms are supposed to be on the production frontier even in the second best equilibrium. This is quite different of our results where the production efficiency can’t always be reached with the State aid contract (at the optimum).

Finally, there is not much articles on State aids themselves, but few articles focused empirically on restructuring and rescuing State aids for firms in difficulty. The one of Chin-
dooroy, Muller & Notaro analyses the survival of firms in distress after receiving a State aid. They show that the level and the form of the aid have a very limited impact on the survival of the firm, while the reasons of the difficulty play a crucial role. Glowicka shows that restructuring firms have higher chances to survive, and that State aids should be linked to bailouts procedure to avoid moral hazard and be more efficient.

2 Model

**Firms**

There are two firms \( i = \{d, h\} \) with assets \( A_i \) and respective marginal costs \( b \) and \( a \). We denote \( D^i(x_i, x_{-i}) \) the duopoly profit of a firm \( i \) facing a marginal cost \( x_i \) while its competitor faces a marginal cost \( x_{-i} \). \( M(x_i) \) states for the monopoly profit of a firm \( i \) having a marginal cost \( x_i \).

**Investment project**

Both firms can invest \( I \) in a cost decrease project that leads to a lower marginal cost \( c - a \) in case of success. The success of this project depends on the firm’s effort \( e \) :

\[
P_H = \begin{cases} 
0 & \text{if } e = 0 \\
P_H & \text{if } e = 1 
\end{cases}
\]

and the entrepreneur receives a private benefit \( B \) when there is no effort \( (e = 0) \)

To implement this project, firms can borrow on the private market at the exogenous interest rate \( r \) and we consider that projects have a positive net present value for both firms, which implies :

**Assumption 1**

\[
P_H^2 D^h(c - a, c - a) + P_H(1 - P_H)M(c - a) + (1 - P_H)P_H D^h(c, c - a) + (1 - P_H)^2 M(c) \\
-(I - A_h)(1 + r) \geq P_H D^h(c, c - a) + (1 - P_H)M(c) + A_h(1 + r)
\]

(1)

It guarantees that both firms have a higher expected profit when they invest and make efforts than when they don’t invest.
Definition 1 Firm $d$ is considered in difficulty because she exits the market without a cost decrease and can’t make the necessary effort to do so. This is implied by:

Assumption 2:

$$B \geq P_H^d D^d(c - a, c - a) + P_H(1 - P_H) D^d(c - a, c) - (I - A_d)(1 + r) \quad (2)$$

The private benefit must be higher than the expected profit of $d$ in case of effort.

Assumption 3:

$$P_H D^d(b, c - a) + (1 - P_H) D^d(d, c) \leq 0 \quad (3)$$

The firm in difficulty must exit the market without a cost decrease, i.e. its expected profit should be negative or null if she keeps her original cost.

Definition 2 Firm $h$ is considered as healthy in the sense that she’s making the necessary effort to decrease her cost, and survive even when the investment doesn’t work while its competitors succeed in decreasing its cost. We need that:

Assumption 4

$$B + M(c) - (I - A_h)(1 + r) \leq P_H M(c - a) + (1 - P_H) M(c) - (I - A_H)(1 + r) \quad (4)$$

The healthy firm has a higher expected profit when she makes effort than when she’s not. Contrary to the firm in difficulty, she’s not just receiving $B$ in case of non effort but also an expected profit, because she stays in the market.

Compatibility of assumptions

Rewriting our assumptions, we got the following conditions (see the appendix for the details in the linear Cournot):

\[ \text{Compatibility of assumptions} \]
These conditions are quite logical according to our definition of the two types of firms. Due to a high marginal cost ($b$), the firm in difficulty couldn’t stay in the market without a cost decrease. A medium level of investment ($I(1 + r)$) ensures that the project is feasible and has a positive net present value, but is costly enough for the firm in difficulty, so she needs the aid. If the ailing firm’s assets ($A_d$) were too high or the private benefit ($B$) too low, she would like to make the effort and will not be in difficulty. However, as we also needs the healthy firm to make effort, the private benefit can’t neither be too big. For the same reason, the probability of success ($P_H$) when making effort must be sufficiently high. Finally, the cost when the cost decrease happens ($c - a$) must be relatively small to ensure that the investment project can’t have a negative net present value.

**Contracts**

The State is able to observe firms and proposes a contract to the firm in difficulty which will take the form of a soft loan with an interest rate $s < r$. This contract contains a level of aid $\tilde{A}$ and a restriction on production $\tilde{q}$. When designing the contract, the State searches the lower $\tilde{A}$ and the higher $\tilde{q}$ such that both firms have incentives to decrease their cost. He hence applies the principle of minimal aids imposed by the European Commission to restrain the distortion effect of the aid, and maximizes consumers’ surplus which depends positively on the production.

**Timing of the game**

The timing is the following:

1. The State proposes the contract to firm $d$.
2. Each firm makes its effort and investment choices.
3. Firms compete à la Cournot.
3 The optimal State aid contract

In this part, we only want to study the optimal aid when there is moral hazard. As we previously said, the State wants to design a contract such that both firms invest and make efforts, to prevent the firm in difficulty from exiting the market and avoid a crowding out effect. Two sets of condition must be satisfied.

Remark: We now denote the duopoly profit of firm $d$ facing a limitation on her production $D^d(x_d, x_h, \bar{q})$ and the profit of the healthy firm when the firm in difficulty receives an aid $D^h(x_h, x_d, \bar{q})$.

For the firm in difficulty

**Condition 1:**

$$B \leq P_H^2 D^d(c-a, c-a, \bar{q}) + P_H(1 - P_H)D^d(c-a, c, \bar{q}) - (I - A_d - \tilde{A})(1 + r) - \tilde{A}(1 + s)$$

(5)

To survive, the firm in difficulty needs to decrease her cost, and so, to make effort.

**Condition 2:**

$$P_H^2 D^d(c-a, c-a, \bar{q}) + P_H(1 - P_H)D^d(c-a, c, \bar{q}) - (I - A_d - \tilde{A})(1 + r) - \tilde{A}(1 + s) \geq A_d(1 + r)$$

(6)

The project net present value should still be positive for firm $d$.

For the healthy firm

**Condition 3:**

$$B + P_H D^h(c, c-a, \bar{q}) + (1 - P_H)M(c) - (I - A_h)(1 + r) \leq P_H^2 D^h(c-a, c-a, \bar{q}) + P_H(1 - P_H)D^h(c, c-a, \bar{q}) + (1 - P_H)^2 M(c)$$

(7)

The aid should not deter the healthy firm from making effort. For simplicity, we consider that the healthy firm will always produce even when its competitors makes the effort to
decrease its cost while she doesn’t.

**Condition 4:**

\[
P_H^2 D^h(c - a, c - a\tilde{q}) + P_H(1 - P_H)M(c - a) - (I - A_h)(1 + r) + P_H(1 - P_H)D^h(c, c - a, \tilde{q}) \\
+ (1 - P_H)^2 M(c) \geq P_H D^h((c, c - a, \tilde{q}) + (1 - P_H)M(c) + A_h(1 + r) \quad (8)
\]

Finally, the healthy firm’s project to decrease her cost should still have a positive net present value.

**Results**

**Proposition 1** Under the assumptions $\mathcal{H}$

- When $P_H \leq f_1(a, c), I(1 + r) \leq f_2(P_H, a, c), A_d \leq f_3(P_H, a, c, B)$ and $B \geq f_4(P_H, a)$ there is a state aid contract without restrictive limitation on the production.
- On the contrary, there exists a state aid contract with a restrictive limitation of production when $I(1 + r) \geq f_5(P_H, a, c); P_H \geq f_6(a, c))$ and $1 - c \geq f_7(a)$
- Otherwise, there exists no contracts pushing both firms to invest and make effort.

The type of contract and their occurrence depend (among other things) on the extent of the moral hazard problem and of the externalities of the aid. We focus on the probability of success when firms invest and make effort. We first look at the cases where there is a small probability of success, and hence, a small probability for the healthy firm to face a duopoly but also a small probability for the firm in difficulty to survive. We then study the cases where this probability is higher. Finally, we discuss the case where there is no contract allowing the cost decrease for both firms.

When the probability of success ($P_H$) is weak, the aid induces small externalities. The chances for the healthy firm to face a duopoly situation are quite low and it is easy to preserve her investment. The imposition of production constraints is no longer necessary. This is why we have the first type of contract. In this one, there is a big moral hazard issue, which is solved by a non restrictive aid given to the firm in difficulty. As the probability of being in a duopoly is low, there is no need to compensate the negative effects of the aid on the healthy firm.
On the contrary, when the externalities are larger, the probability to survive for the firm in difficulty is higher, and there are more cases in which the healthy firm is in a duopoly. The State must imposed restrictions on the production of the helped firm to compensate for the negative impact on firm h and preserve her investment. This is exactly what we get in the second type of contract. The firm in difficulty needs an important aid because of a big moral hazard issue or because of a high level of investment. The relatively high probability of success makes it possible to save this firm, but restrictions on her production are necessary because of the large negative impact on healthy firm.

The absence of contracts for certain sets of parameters shows the limitation of the State influence: for some intermediate probability of success, there is no aid allowing cost decrease for both firms. Indeed, the probability is too low to allow the survival of the firm with a restricting contract, but too high to protect the healthy firm without imposing some. This comes from the tension existing on the production restriction. There is a trade off between allowing a high production to the firm in difficulty that ensures she’s able to make the effort and the distorting impact on her competitor, that can’t always be solved. It seems necessary in some cases to make a choice between preserving the competitive structure and protecting innovation.

References

Chindooroy R., Muller P. and Notaro G., 2007, Company survival following rescue and restructuring state aid, European Journal of Law and Economics

Diamond D., Mirrlees J., 1971, Optimal Taxation and Public Production I: Production Efficiency, American Economic Review

Glowicka E., 2006, Effectivness of Bailouts in the EU, CIG working papers


4 Appendix

4. A Compatibility of assumptions within each other

We check here that the assumptions of our model fit together.

- The first thing we do is to study the exit conditions of the firm in difficulty when there is no cost decrease. We know that

\[ P_H D^d(d, c - a) + (1 - P_H)D^d(d, c) \leq 0 \]  

(9)

It is obvious that for a given firm, the duopoly profit when there is no firm decreasing her cost is higher than the duopoly profit when its competitor decreases its cost while she keeps the high cost. Hence, \( D^d(d, c - a) \leq D^d(d, c) \). The necessary and sufficient condition to verify the condition in this case is then \( D^d(d, c) = 0 \). Deriving the Cournot profit, we can get the duopoly quantity that the firm chooses \( q_d = \frac{1+c-2d}{3} \). The profit is null as soon as this quantity is negative, i.e as soon as \( d \geq \frac{1+c}{2} \). □

- The conditions on \( I(1+r) \) are just obtained by replacing the duopoly profit by the ones of a linear Cournot and by rewriting the assumptions 1, 2 and 5 to isolate \( I(1+r) \).

We get:

\[ I(1+r) \in \left[ P_H^2 \left( \frac{1+a-c}{3} \right)^2 + P_H(1-P_H) \left( \frac{1+2a-c}{3} \right)^2 - B + A_d(1+r); \right. \]

\[ \left. \min\{ P_H^2 \left( \frac{1+a-c}{3} \right)^2 + P_H(1-P_H) \left( \frac{1-c+a}{2} \right)^2 - P_H^2 \left( \frac{1-a-c}{3} \right)^2 - P_H(1-P_H) \left( \frac{1-c}{2} \right)^2 \right] \]  

(10)

Proof is left to the reader.

- We now have to check that the conditions on \( I(1+r) \) do not imply a negative value of investment (which is obvious) and are compatible, i.e. that the upper bound of \( I(1+r) \) are higher than the lower limits. It gives us the following restrictions on \( A_d(1+r) \)

\[ A_d \leq \min\{ B; P_H(1-P_H) \left[ \left( \frac{1-c+a}{2} \right)^2 - \left( \frac{1-c}{2} \right)^2 - \left( \frac{1+2a-c}{3} \right)^2 \right] \]  

\[ - P_H^2 \left( \frac{1-c-a}{3} \right)^2 + B \} \]  

(11)
- We also have to check that this previous conditions $A_d$ does not imply negative assets for the firm in difficulty, so we have to check that the upper bounds are positive. It gives us a first condition on $B$:

$$B \geq [P_H(1-P_H)\left[\left(\frac{1-c}{2}\right)^2 - \left(\frac{1-c+a}{2}\right)^2 + \left(\frac{1-c+2a}{3}\right)^2\right] + P_H^2 \left(\frac{1-c-a}{3}\right)^2];$$

(12)

Furthermore, as we consider that the healthy firm never leaves the market even when its competitor receives an aid and make the effort, its condition of efforts is:

$$B + M(c) - (I - A_h)(1+r) \leq P_H M(c-a) + (1-P_H) M(c) - (I - A_h)(1+r)$$

$$\iff B \leq P_H \left(\frac{a^2 + 2a - 2ac}{4}\right)$$

(13)

- The compatibility of the conditions on $B$ implies that $P_H$ must be high enough

$$P_H \geq \frac{2a^2 + 20a - 20ac - 4(1-c)^2}{-3a^2 + 6ac - 6a}$$

(14)

- To finish, we have to check that the lower bound of the probability is lower than 1, which gives us the following condition on the arrival cost:

$$1 - c \leq 6.6869a$$

(15)

4. B Results of the hazard moral part

The results of the moral hazard part are derived from our model, assumptions and conditions. We made an analysis in 4 steps.

1. We start by deriving the optimal amount of aid (as a function of $\bar{\tilde{q}}$, the limitation of the production) from the condition of the firm in difficulty. We want the minimal $\tilde{A}$ such that the two conditions are respected. We hence find:

$$\tilde{A} = \frac{1}{r - s} [(I - A_d)(1+r) + B - P_H^2 D((c-a, \text{Min}\{q_d, \bar{\tilde{q}}\}); (c-a, q_h))$$

$$- P_H(1-P_H) D((c-a, \text{Min}\{q_d, \bar{\tilde{q}}\}); (c, q_h))$$

(16)

The level of aid depends on the level of authorised production and on whether the limitations are restricting for the firm. Since $D((c-a, \text{min}\{q_d, \bar{\tilde{q}}\}); (c-a; q_h))$ and

$D((c-a, \text{min}\{q_d, \bar{\tilde{q}}\}); (c; q_h))$ are not restricted at the same level, we have to envisage three cases: when the maximal production allowed to the firm is always restrictive,
when it only restricts the production in the case the other firm doesn’t decreases her costs, and when the \( q \) is large enough to never constraint the firm in difficulty. We have:

* When \( \bar{q} \leq \frac{1+a-c}{3} \), the firm in difficulty is always constrained in her production, either the healthy firm decreases her cost or not. We have

\[
\hat{A} = \frac{1}{r-s} \left[ (I - A_d)(1+r) + B + \frac{P_H}{2} \bar{q}(P_Ha - 1 + c - 2P_Ha) + \frac{P_H}{2} \bar{q}^2 \right] \tag{17}
\]

* When \( \bar{q} \in \left[ \frac{1+a-c}{3}; \frac{1+2a-c}{3} \right] \), the firm in difficulty is only constrained when its competitor doesn’t decrease its cost. We have

\[
\hat{A} = \frac{1}{r-s} \left[ (I - A_d)(1+r) + B - P_H^2 \left( \frac{1+a-c}{3} \right)^2 - \frac{P_H(1-P_H)}{2} \bar{q}(-c+2a) + \frac{P_H(1-P_H)}{2} \bar{q}^2 \right] \tag{18}
\]

* Finally, when \( \bar{q} \geq \frac{1+2a-c}{3} \), the limitation of the production is never restricting for the firm in difficulty, and the optimal level of aid doesn’t rely on \( \bar{q} \)

\[
\hat{A} = \frac{1}{r-s} \left[ (I - A_d)(1+r) + B - P_H^2 \left( \frac{1+a-c}{3} \right)^2 - P_H(1-P_H) \left( \frac{1+2a-c}{3} \right)^2 \right] \tag{19}
\]

2. In the second stage, we search the \( \bar{q} \) such that the conditions of the healthy firm are verified. As the consumers’ surplus depends positively on the production, we want the maximal \( \bar{q} \) that respects the two conditions and we got

\[
\bar{q} = 2 \left[ \frac{a^2+2a-2ac}{4} - \frac{1}{P_H} \left[ P_H(1-P_H)(2ac-2a-a^2) + I(1+r) + \text{Max} \{ B - I(1+r); 0 \} \right] \right] \tag{20}
\]

We immediately see that the value of \( \bar{q} \) depends on \( B \) compared to \( I(1+r) \). Moreover, as for \( \hat{A} \), we have to take into account the restrictive impact of \( \bar{q} \) on the production of the firm. We obtain four cases When \( B \geq I(1+r) \):

* There is an equilibrium without constraining the firm in difficulty when

\[
B \leq P_H^2 \left[ \left( \frac{1+a-c}{3} \right)^2 - \left( \frac{1-a-c}{3} \right)^2 \right] - P_H(1-P_H)(2ac-2a-a^2) \tag{21}
\]

We hence have \( \bar{q} > \frac{1+2a-c}{3} \)
* And there is an equilibrium with a restricting $\bar{q}$ when

$$B \geq \frac{P_H^2}{6} \left( \frac{4a - 4ac + a^2}{2} \right) - P_H(1 - P_H)(2ac - 2a - a^2) \quad (22)$$

In this case, we have

$$\bar{q} = \frac{2}{a} \left[ \frac{a^2 + 2a - 2ac}{4} - \frac{1}{P_H^2} [P_H(1 - P_H)(2ac - 2a - a^2) + B] \right] \quad (23)$$

When $I(1 + r) \geq B$

* There is no need of a production limitation \( \bar{q} > \frac{1 + 2a - c}{3} \) if

$$I(1 + r) \geq \frac{P_H^2}{6} \left( \frac{4a - 4ac + a^2}{2} \right) - P_H(1 - P_H)(2ac - 2a - a^2) \quad (24)$$

* We can have an equilibrium when $\bar{q}$ is constrained if

$$I(1 + r) \geq \frac{P_H^2}{6} \left( \frac{4a - 4ac + a^2}{2} \right) - P_H(1 - P_H)(2ac - 2a - a^2) \quad (25)$$

The limitation of production in this case is

$$\bar{q} = \frac{2}{a} \left[ \frac{a^2 + 2a - 2ac}{4} - \frac{1}{P_H^2} [P_H(1 - P_H)(2ac - 2a - a^2) + I(1 + r)] \right] \quad (26)$$

Remark: The conditions imposed to be in each cases verify that the $\bar{q}$ is in the correct interval. For example, the firm is always restricted when $\bar{q} \leq \frac{1 + a - c}{3}$. We checked that the $\bar{q}$ we obtain in the restrictive case belongs to the interval $[0, \frac{1 + a - c}{3}]$

3. The third stage controls under which conditions each case respects the conditions linked to our assumptions. For example, in the case where $B \geq I(1 = r)$, we need to have $B \leq \frac{P_H^2}{6} \left( \frac{(1 + a - c)^2 - (1 - a - c)^2}{3} \right) - P_H(1 - P_H)(2ac - 2a - a^2)$ to have an equilibrium without conditions on the production. According to our assumptions, we can only have this equilibrium if

$$P_H^2 \left[ \left( \frac{1 + a - c}{3} \right)^2 - \left( \frac{1 - a - c}{3} \right)^2 \right] - P_H(1 - P_H)(2ac - 2a - a^2) \geq P_H(1 - P_H) \left[ \left( \frac{1}{2} \right)^2 - \left( \frac{1 - c + a}{2} \right)^2 + \left( \frac{1 - c + 2a}{3} \right)^2 \right] + P_H^2 \left( \frac{1 - c - a}{3} \right)^2$$

(27)

We develop this condition and obtain the following additional limitation on the parameter to reach the equilibrium:

$$P_H \leq \frac{4(1 - c)^2 - 29a^2 - 74a(1 - c)}{-50a(1 - c) - 33a^2} \quad (28)$$

On the other cases, the additional conditions are the following:
* When \( B \geq I(1 + r) \), there is an equilibrium with a restrictive \( \bar{q} \) when

\[
P_h \geq \frac{18a(1 - c) + 9a^2}{20a(1 - c) + 11a^2}
\]

* When \( I(1 + r) \geq B \), we need that

\[
\begin{align*}
(a) & \quad -\frac{P_H^2}{9}[6a^2 + 12a(1 - c)] + \frac{P_H}{9}[14a(1 - c) + 5a^2 - (1 - c)^2] + B \geq A_d(1 + r) \\
(b) & \quad B \leq \frac{P_H^2}{9}[6a^2 + 12a(1 - c)] - \frac{P_H}{9}[14a(1 - c) + 5a^2 - (1 - c)^2]
\end{align*}
\]

* To finish, when \( I(1 + r) \geq B \) and there’s a restrictive \( \bar{q} \), we need that

\[
\begin{align*}
(a) & \quad P_H \geq \frac{27a^2 + 54a(1 - c)}{58a(1 - c) + 24a^2} \\
(b) & \quad 1 - c \geq 0, \quad 75a
\end{align*}
\]

4. The last step consist in replacing the value of \( \bar{q} \) in \( \tilde{A} \). As we previously said, we have to differentiate the case where \( \bar{q} \) is restricting and the case where it is not.

* In the equilibrium where \( \bar{q} \) is restricting, we have

\[
\tilde{A} = f_9(r, s, I, A_d, B, P_H, a, c)
\]

* When \( \bar{q} > \frac{1 + a - c}{3} \), \( \bar{q} \) is never constraining, and there is no interest in fixing a \( \bar{q} > \frac{1 + 2a - c}{3} \) since the firm will not choose a higher production. We hence have

\[
\tilde{A} = f_{10}(r, s, I, A_d, B, a, c, P_H)
\]