Unfunded Pension and Labor Supply: 
Characterizing the Nature of the Distortion Cost

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Abstract

This article develops a simple benchmark OLG model with endogenous labor supply. We determine the analytical expression of the distortion cost of the labor supply induced by an unfunded pension scheme where the pension amount is not fully linked to social contribution. First, we express the monetary valuation of this cost by estimating what surplus of tax revenue with respect to the implicit public debt the public authority could accumulate with a lump-sum tax, without altering the profile of intertemporal welfare. Second, we compute its welfare valuation by assessing what surplus of welfare a lump-sum tax could induce, while fully repaying the implicit pension debt. Both monetary and welfare costs are expressed as explicit functions that can be straightforwardly interpreted and numerically simulated. JEL codes: D91, H55, J26. Keywords: OLG, unfunded pensions, labor supply, tax distortion cost.

Introduction

The financing of the mandatory and unfunded pension schemes (Feldstein and Samwick, 1992; Feldstein, 2005a and 2005b; Browning, 1975; Barr and

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Diamond, 2006) can be considered as a specific taxation of labor income. Payroll tax and calculus rule (Cigno, 2008) of retirement income modify the life-cycle gross gain of labor effort and the resulting net tax effect is not obvious. This might be due to the fact that the pension link to earnings is not strong enough. Moreover, according to the principle of dynamic efficiency, the rate of return on unfunded pension is generally admitted to be lower than that on funded pension.

Assessing the distortion cost is more complex to appreciate in dynamic context. In effect, it rests upon two dimensions:

- a temporal dimension: workers pay contributions first then receive pensions;
- a generational dimension: the whole of contributions are used to finance current pensions.

From a practical point of view, the incidence of this net tax effect on labor supply can be contemplated either from the point of view of professional career duration (education effort, workforce entry age, retirement age choice), or the work intensity (working hours choice and productive effort). De facto, the reforms of pension systems are often designed to make them more user-contributory orientated in order to increase the incentives to work more and longer. Though difficult to be empirically assessed (Liebman et al., 2009), this tax incidence causes distortion, which might be costly, socially.

From a policy point of view, a strong distortion cost induced by unfunded pension schemes, should be a necessary condition to design pension reforms which would aim at Pareto improvement.

The concept of distortion cost or so-called "deadweight loss" has been popularized by Harberger’s triangle (Hines, 1999). However, it has to be stressed that with the exception of Feldstein (1996, 2005b), who uses the Harberger (1964) formula to approximate the static distortion cost related to unfunded pension, little has been done to estimate it. According to Feldstein (1996), "the deadweight loss due to the net Social Security Tax is about 2.35% of the Social Security payroll tax base" in 1995. As to Kotlikoff (1996), he gives a lower-bound estimate of the welfare cost by finding a Pareto-improving tax reform which would induce a 4.5% welfare gain to future generations.

This paper relates to Feldstein (1996, 2005b) and Kotlikoff (1996) by seeking to estimate analytically the distortion cost. In turn, this analytical estimation makes it possible to study precisely the sensitivity of the distortion cost with respect to the parameters, notably the labor supply elasticity.
the payroll tax and the pension-link-to-earnings coefficient. We introduce explicit endogenous labor supply in an overlapping generations model and we suppose the unfunded pension scheme partially links pensions to contributions. Our model may be interpreted as a simplified specification of OLG computable general equilibrium models which are characterized by more realistic demographic structure and life-cycle profile. We assess the efficiency cost according to two metrics. The first one is monetary and it gives the fiscal surplus induced by a non distortionary taxation, without loss of welfare for all generations (present and future). The other one evaluates the welfare gain inferred by a non distortionary taxation, which refunds the implicit public debt due to the retirement pension forecasts.

In a computable OLG model calibrated on the US economy, Kotlikoff (1996) finds a Pareto-improving tax reform of Social Security. The reform consists of a privatization associated to a tax compensating transition generations. However, Brunner (1996) shows that assuming ex-ante heterogenous agents might compromise such a Pareto improving tax reform. Kotlikoff et al. (1999, 2000) confirm this result by simulations. The contemplated tax reforms induce welfare losses to transition generations. Furthermore, heterogeneity can be due to uncertainty on labor productivity (wage, employment, etc) or lifetime. Hence, the redistributive properties of the Social Security pension rules act as an insurance (Imrohoroglu et al., 1995; Conesa and Krueger, 1999). In this framework, Nishiyama and Smetters (2007) conclude that the efficiency gains of a reform are much reduced. Along the same line, Imrohoroglu and Kitao (2009) study how the labor supply elasticity affects the results induced by Social Security privatization. These works on tax reform are completed by Erosa and Gervais (2002). Relying on Chamley (1986), they give conditions for optimal taxation (second best approach) in an intertemporal framework with OLG. Also, the corresponding optimal taxes are numerically computed. Their approach is normative since they consider as the planner’s objective the maximization of the sum of discounted welfares. However, Kotlikoff only focuses on a positive approach (existence of Pareto-improving tax reforms). A similar computational optimal growth model is applied to the US economy by Conesa and Garriga (2008) who contemplate the Social Security policy in a global tax reform.

The paper is organized as follows. First section presents the model. Section 2 is dedicated to the two distortion cost measures. Section 3 gives the results of sensitivity analysis. The last section concludes.
1 Endogenous labor supply in an OLG model

We consider an economy with two generations of households living in each period (one generation of workers and one generation of retirees), a productive sector (linear technology) and a tax administration. Contrasting with Samuelson (1958) and Diamond (1965) basic models, labor supply is assumed endogenous. This means that it is sensitive to taxes and particularly to the pension rule.

1.1 Tax administration

Tax administration is composed of four components:

- $\tau_w$: Tax rate on the labor income ($w$ denoting the wage) to finance the unfunded pension promises.
- $T$: Lump-sum tax\(^1\) which is only paid by the work force.
- $P$: Amount of unfunded pension which is paid to each retired agent.
- $D$: Public debt per worker.

We denote $\tau = \frac{T}{w l}$ the (average) lump-sum tax rate. The interest factor is denoted $R = 1 + r$ where $r$ is the rate of return. The exponents $+$ or $+(j)$ and $-$ are (respectively) used to mark the future periods ($t+1$) or ($t+j$) and the past period ($t-1$). For the current period ($t$), no temporal index is used.

The intertemporal budget constraint for the tax administration is given by:

$$D^+ = R \cdot D - T + (P - \tau_w \cdot w \cdot l).$$

We suppose $P = \tau_w \cdot w \cdot l$. Hence, $D^+ = R \cdot D - T$. Retired workers receive a pension $P^+$ whose a fraction $\alpha$ is fully contributive by relying on the personal labor supply ($l$), the remaining fraction $(1 - \alpha)$ being lump-sum. The pension rule writes:

$$P^+ (l) = \left( \alpha \cdot \frac{l}{l} + (1 - \alpha) \right) \cdot \frac{\tau^+_w \cdot w^+ \cdot \bar{l}^+}{\text{Pension rule}} \cdot \frac{\text{Tax revenue per pensioner}}{\text{Tax revenue per pensioner}}$$

\(^1\)On the basis of equity, such a tax cannot be implemented in practice.
where $\bar{l}$ and $l$ are respectively the average and individual labor supplies. At the current period, the implicit debt of unfunded pension is equal to the amount of pension promises.

1.2 Households

Each household has a life-span of two periods. The first period is dedicated to work ($l$) whose income is shared between consumption ($c$) and savings ($S$). Notice that $l$ can be interpreted as a measure of work intensity. During the second period of their lives, people are retired and consume $z^+$ made of their interest accrued savings ($R^+ \cdot S$) and their retirement pension ($P^+$).

We suppose an additively separable utility between gains from life-cycle consumption and effort cost as in Jullien and Picard (1998) for a discrete effort choice and Saez (2001) for a static model with continuous labor supply. There are no income effects. $u(c, l)$ exhibits no risk aversion on $w$ resulting from an uncertainty or an unequal distribution of skills. In comparison, Saez supposes $\log (c - v(l))$ (see also Best & Kleven, 2012). Then, the individual utility function writes: $U(c, z^+, l) = u(c, z^+) - v(l)$ where $u(c, z^+)$ is the level of welfare induced by the life-cycle consumption and $v(l)$ the effort cost of the labor supply. We assume that $v(l) = \beta \cdot \frac{\mu + \frac{1}{\varepsilon}}{1 + \frac{1}{\varepsilon}}$ with $\beta, \varepsilon > 0$ and $u$ is an homothetic function w.r.t. $c$ and $z^+$: $u(c, z^+) = c^s \cdot z^{1-s}$ with $s > 0$. Notice that parameter $\varepsilon$ is also known as the Frisch elasticity of labor supply (Imrohoroglu and Kitao, 2009).

The household’s intertemporal budget constraint writes:

\[
\begin{align*}
&\begin{cases}
  c = (1 - \tau_w) \cdot w \cdot l - T - S \quad \text{(when young)} \\
  z^+ = R^+ \cdot S + P^+(l) \quad \text{(when old)}
\end{cases} \\
\end{align*}
\]

Each household maximizes his intertemporal welfare w.r.t savings and labor supply. The F.O.C. are respectively:

\[
\begin{align*}
&\begin{cases}
  S : u_c = R^+ \cdot u_z \\
  l : v'(l) = (1 - \tau_w) \cdot w \cdot u_c + P'^+(l) \cdot u_z
\end{cases} \\
\end{align*}
\]

where the LHS and the RHS are respectively the marginal cost and the marginal benefit of the two components of utility. The S.O.C. can be checked easily.

These two F.O.C. lead to:
\[
\begin{align*}
S^* &= s \cdot (1 - \tau_w) \cdot w \cdot l - (1 - s) \cdot \frac{P^+}{R^+} \\
l^* &= v'^{-1} \left( A(R^+, w) \cdot \left( 1 - \tau_w \cdot \left( 1 - \frac{P^+(-l)/(\tau_w \cdot w)}{R^+} \right) \right) \right),
\end{align*}
\]

where \(s\) is the marginal propensity to save the after-tax labor income and \((1 - s)\) is the marginal propensity to consume the future income (pension); \(A(R^+, w) = (1 - s)^{1-s} \cdot s^s \cdot R^+ \cdot w\) is the gross marginal – and average– utility gain induced by one unit of labor; \(\tau_w \cdot \left( 1 - \frac{P^+(-l)/(\tau_w \cdot w)}{R^+} \right)\) can be interpreted as the marginal tax rate of labor income; \(v'^{-1}(x) = \left( \frac{1}{\beta} \cdot x \right) \varepsilon\) is the inverse of the marginal effort cost of labor supply.

The indirect welfare function \(W\) associated with a tax system \((\tau_w, \alpha, T)\) and marginal factor productivities \((R^+, w)\) is then given by:

\[
W(\tau_w, \alpha, T, R^+, w) = \frac{A(R^+, w)}{v'^{-1}(x)} \cdot l^* \cdot \begin{bmatrix}
1 - \tau_w \cdot \left( 1 - \frac{P^+(-l)/(\tau_w \cdot w)}{R^+} \right) - \frac{T}{w \cdot l^*} \varepsilon
\end{bmatrix}.
\]

### 1.3 Productive sector

The technology of production is assumed to be linear. Let \(R\) and \(w\) be the marginal productivities of capital and labor, respectively. \(R\) is certain and stationary and denoted thereafter \(\bar{R}\). \(w\) is assumed to be increasing, with a constant growth factor \(\Gamma\).

### 1.4 Welfare characterization at the general equilibrium

We turn now to the characterization of welfare at the general equilibrium. We first identify the stationary growth factor of labor income in proposition 1. Then we determine the corresponding labor supply and the level of welfare.

**Proposition 1:** If the tax system is stationary then at the intertemporal general equilibrium, the growth factor of labor income \(\frac{w^+ \cdot l^+}{w \cdot l}\) is equal to \(\Gamma^{1+\varepsilon}\).
Proof: See appendix.

From proposition 1 and FOCs, we deduce:

\[ l^* = l \left( \tau_w, \alpha, w, \tilde{R} \right) = \left( \frac{1}{\beta} \cdot A \cdot \left( 1 - \tau_w \cdot \left( 1 - \alpha \cdot \frac{r^{1+\varepsilon}}{R} \right) \right) \right)^\varepsilon, \tag{7} \]

and after rearranging terms, we obtain:

\[ W^* = W \left( \tau_w, \alpha, \tau, w, \tilde{R} \right) = \beta^{-\varepsilon} \cdot A^{1+\varepsilon} \cdot \left( 1 - \tau_w \cdot \left( 1 - \alpha \cdot \frac{r^{1+\varepsilon}}{R} \right) \right)^\varepsilon \]

\[ \cdot \frac{1}{1+\varepsilon} \cdot \left[ 1 - \tau_w \cdot \left( 1 - \frac{r^{1+\varepsilon}}{R} \cdot (1 + \varepsilon \cdot (1 - \alpha)) \right) - \tau \cdot (1 + \varepsilon) \right]. \tag{8} \]

At the optimum, the welfare level decomposes in three terms:

- \( \beta^{-\varepsilon} \cdot A^{1+\varepsilon} \) can be interpreted as a scale factor. The effect of an increase in \( \beta \) is negative whereas an increase in \( \varepsilon \) has a positive effect iff \( A/\beta > 1 \).

- \( \left( 1 - \tau_w \cdot \left( 1 - \alpha \cdot \frac{r^{1+\varepsilon}}{R} \right) \right)^\varepsilon \) expresses the impact of labor income tax rate and pension rule on the labor supply. \( \tau_w \) directly affects the marginal tax rate and it has a negative effect on the labor supply, whereas \( \alpha \) and \( \varepsilon \) have both opposite effects. \( \frac{r^{1+\varepsilon}}{R} \) is the returns ratio between unfunded and funded pension. Its impact is positive.

- \( \frac{1}{1+\varepsilon} \cdot \left[ 1 - \tau_w \cdot \left( 1 - \frac{r^{1+\varepsilon}}{R} \cdot (1 + \varepsilon \cdot (1 - \alpha)) \right) - \tau \cdot (1 + \varepsilon) \right] \) measures the life-cycle income per unit of gross labor income, minus the cost/benefit ratio of labor supply. The impact of \( \tau_w \) may be positive depending on the sign of the expression \( \left( 1 - \frac{r^{1+\varepsilon}}{R} \cdot (1 + \varepsilon \cdot (1 - \alpha)) \right) \). If \( \varepsilon \) is high enough, then \( 1 + \varepsilon \cdot (1 - \alpha) \) may be greater than \( \frac{r^{1+\varepsilon}}{R} \). It results that \( \tau_w \) has a positive net effect. As to \( \alpha \), it impacts positively the labor supply and consequently its associated welfare cost. But it has no effect on the life-cycle income per unit of labor income. Then, its net impact is negative. As to \( \frac{r^{1+\varepsilon}}{R} \), its net impact is positive, because its income effect is higher than its effect on labor supply cost. Finally, the impact of \( \varepsilon \) is twofold. The first effect is positive via \( \tau_w \) as said before and the second effect is negative via \( \tau \).
Without distortionary taxation \((\tau_w = 0 = \alpha, \tau \geq 0)\), we have:

\[
l^* = l_{\text{max}} (w, \bar{R}) = l (0, 0, w, \bar{R}) = \left( \frac{1}{\beta} \cdot A \right)^{\varepsilon}, \tag{9}
\]

and

\[
W^* = W_{\text{max}} (\tau, w, \bar{R}) = \beta^{-\varepsilon} \cdot A^{1+\varepsilon} \cdot \frac{1}{1+\varepsilon} \cdot [1 - \tau \cdot (1 + \varepsilon)]. \tag{10}
\]

2 Measuring the efficiency cost induced by unfunded pension

2.1 From static to dynamic distortion cost


Dynamic distortion cost=> main challenge = identifying dynamic compensatory lump-sum transfers.

We propose two ways of estimating the intertemporal efficiency cost induced by the unfunded pension scheme.

Firstly, we assess the monetary cost. It is defined by the amount of fiscal surplus that a virtual lump-sum tax might generate, by paying back the implicit debt of unfunded pension sustaining a Pareto-equivalence principle. This approach is similar to that adopted by Diamond and MacFadden (1974), who estimate the deadweight burden of taxation from the dual consumer’s program. The dual consumer’s program gives an explicit expression of the compensated or hicksian demand function. Green and Sheshinski (1979) use this approach to compute the exact distortion cost of taxation in a standard two-period life-cycle model with labor supply. This allows them to compare it with the Harberger’s approximation.

Here, for a given level of the welfare optimal level \(W (\tau_w, \alpha, \tau, w, \bar{R})\), the expenditure function is handled through the lump-sum tax \((\tau)\). Hence, it is needless to compute explicitly the optimal solution of the dual program.

**Definition 1:** A lump-sum tax is Pareto equivalent (PE) iff it sustains the same welfare dynamics as the unfunded pension scheme. Hence, its corresponding rate \(\tau^{PE}\) must check:
Secondly, we compute the welfare cost. This estimation is based on the conventional (or primal) consumer’s program. It is similar to Levhari and Sheshinski (1972) and Chamley (1981), who calculate the intertemporal welfare gain induced by a capital income tax suppression. Let us stress the fact that this gain is maximum, since it supposes that the distortive tax does not finance any public expenditure. In this model, the distortive tax must finance the implicit public debt induced by the pension promises. The welfare loss is obtained by differentiating the two levels of welfare calculated respectively, with the virtual first best lump-sum tax and with the so-called "n\textsuperscript{th} best" current tax\textsuperscript{2}. Auerbach et al. (1987) estimate numerically this dynamic efficiency cost of taxes by introducing a Lump Sum Redistributive Authority (LSRA). Kotlikoff (1996) uses a similar method to assess the efficiency gains from Social Security reform. He seeks an alternative least distortive tax ensuring a welfare gain for all generations, identifying another "n\textsuperscript{th} best" tax which Pareto-dominates the current tax induced by Social Security.

In this paper, we define the welfare cost as the additional welfare induced by a virtual lump-sum tax refunding fully the implicit debt of unfunded pension. But, such an estimation is not unique. In effect, as mentioned in the seminal paper by Auerbach et al. (1987), there exists "an infinite set of welfare paths". To address this problem, they use a maximin criteria to select a stationary welfare path. Here, we opt for another criteria which guarantees dynamic equity in tax payment. Hence, we search a virtual lump-sum tax, such that the ratio \( \tau = \frac{T}{w} \) is constant for all generations. We obtain (see below) an identical result in terms of welfare path as Kotlikoff (1996), who finds efficiency gains of Social Security reform as a constant relative welfare gain.

**Definition 2:** A stationary lump-sum tax is tax revenue equivalent (TRE) iff it refunds exactly the implicit amount of public debt induced by the unfunded pension scheme. Hence, its corresponding rate \( \tau_{\text{tre}} \) must check:

\[
\tau_{\text{tre}} \cdot w \cdot l_{\text{max}}(w, \bar{R}) \cdot \frac{\bar{R}}{R_{-\Gamma^{\text{tre}}}} = \tau_{w} \cdot w \cdot l(\tau_{w}, \alpha, 0, w, \bar{R}).
\]

\(\text{(12)}\)

\textsuperscript{2}See Harberger (1964) for a discussion on this concept in the very first lines of his introduction.
2.2 Monetary distortion cost

Let us now determine the Pareto equivalent lump-sum tax rate. As previously defined, this rate is determined by equating the level of benchmark welfare with the obtained level without distortionary taxation. Proposition 2 gives the exact analytical expression of this rate.

**Proposition 2:** The PE lump-sum tax rate is stationary and equal to:

\[
\tau^{pe} = \frac{1}{1+\varepsilon} \cdot \left[ 1 - \left( 1 - \tau_{w} \cdot \left( 1 - \alpha \cdot \frac{R^{1+\varepsilon}}{R} \right) \right) \varepsilon \right. \\
\left. \cdot \left( 1 - \tau_{w} \cdot \left( 1 - (1 + \varepsilon \cdot (1 - \alpha)) \cdot \frac{R^{1+\varepsilon}}{R} \right) \right) \right].
\]

**Proof:** See appendix.

Let us consider the impacts of variations on the parameters.

An increase in \(\varepsilon\) leads to a higher relative life-cycle income and it reduces the income that would prevail without distortionary taxation. Then the net effect on \(\tau^{pe}\) is unambiguously negative.

An increase in \(\frac{R^{1+\varepsilon}}{R}\) reduces the marginal tax rate induced by pension which has a positive impact on the benchmark labor supply. The effect on \(\tau^{pe}\) is unambiguously negative.

If \(\frac{R^{1+\varepsilon}}{R} \cdot (1 + \varepsilon \cdot (1 - \alpha)) < 1\), an increase in \(\tau_{w}\) reduces directly the life-cycle income per unit of labor income minus the cost/benefit ratio of labor supply, and indirectly the labor supply. It results that the benchmark welfare decreases and the Pareto equivalent lump-sum tax rate is lower. If \(\frac{R^{1+\varepsilon}}{R} \cdot (1 + \varepsilon \cdot (1 - \alpha)) > 1\), the direct effect is positive. Then, the change in the PE lump-sum tax rate is ambiguous.

A higher \(\alpha\) increases the benchmark labor supply and its associated welfare cost. The net effect on \(\tau^{pe}\) is ambiguous.

The monetary distortion cost (MDC) can be expressed as a tax revenue surplus \(w.r.t.\) the implicit public debt. Proposition 3 expresses this cost as a linear expression \(w.r.t.\) the difference between the two virtual lump-sum tax rate \(\tau^{pe} - \tau^{tre}\).

**Proposition 3:** The monetary distortion cost (MDC) of unfunded pension writes:
\[ MDC = \frac{R}{R^{-1} + \varepsilon} \cdot w \cdot l_{\text{max}}(w, \bar{R}) \cdot [\tau^{pe} - \tau^{tre}] . \]  

\textbf{Proof:} See appendix.

Notice that the MDC can be expressed as a simple relative cost (RMDC) when divided by the labor income observed in an economy without distortionary taxation \( (w \cdot l_{\text{max}}(w, \bar{R})) \):

\[ RMDC = \frac{R}{R^{-1} + \varepsilon} \cdot (\tau^{pe} - \tau^{tre}) . \]  

(15)

The sensitivity of the RMDC \textit{w.r.t.} the parameters is mainly linked to the difference \( (\tau^{pe} - \tau^{tre}) \) and the change in \( \frac{R^{1+\varepsilon}}{R} \). Sensitivity of \( \tau^{tre} \) is discussed hereafter.

\section{2.3 Welfare distortion cost}

The tax revenue equivalent lump-sum tax rate can be deduced from the equalization between the implicit debt of unfunded pension and the discounted sum of lump-sum tax. Proposition 4 gives the exact expression of this rate.

\textbf{Proposition 4:} The stationary TRE lump-sum tax rate refunding the implicit public debt is given by:

\[ \tau^{tre} = \tau_w \cdot \left(1 - \frac{R^{1+\varepsilon}}{R}\right) \cdot \left(1 - \tau_w \cdot \left(1 - \alpha \cdot \frac{R^{1+\varepsilon}}{R}\right)\right)^{\varepsilon} . \]  

(16)

\textbf{Proof:} See appendix.

Consider now the impacts of variations in the parameters on \( \tau^{tre} \). An increase in \( \varepsilon \) is ambiguous. First, it increases the labor supply (higher labor supply elasticity and better unfunded pension return) which implies a higher implicit public debt and higher taxable labor income, in an economy without distortionary taxation. Second, it reduces the discount factor \( \frac{R^{1+\varepsilon}}{R} \) used to calculate the perpetual lump-sum tax rate. An increase in \( \frac{R^{1+\varepsilon}}{R} \) has two impacts. First, it boosts the benchmark labor supply, which means a higher implicit public debt. Second, it reduces the discount factor \( \frac{R^{1+\varepsilon}}{R} \) used to sum the lump-sum taxes at the infinite horizon. The net effect on \( \tau^{tre} \) is ambiguous. An increase in \( \tau_w \) has two impacts. First, it entails a higher
tax rate. Second, it reduces the labor supply, which has a negative impact on the taxable labor income. The net effect is ambiguous. An increase in $\alpha$ encourages labor supply. It results in a higher implicit public debt. As a consequence, the TRE lump-sum tax rate raises.

**Proposition 5:** The welfare distortion cost (WDC) of unfunded pension writes:

$$WDC = \beta^{-\varepsilon} \cdot A^{1+\varepsilon} \cdot \frac{1}{1+\varepsilon} \cdot (\tau^{pe} - \tau^{tre}).$$  \hspace{1cm} (17)

**Proof:** See appendix.

Notice that the WDC can be expressed as a simple relative cost (RWDC), when divided by the level of welfare observed in an economy without tax ($W_{max}(0, w, \tilde{R})$). The RWDC then reduces as a simple difference:

$$RWDC = (\tau^{pe} - \tau^{tre})$$

$$= \frac{1}{1+\varepsilon} \cdot \left[1 - \left(1 - \tau_w \cdot \left(1 - \alpha \cdot \frac{1+\varepsilon}{R}\right)\right)^{\varepsilon} \cdot \left(1 + \varepsilon \cdot \tau_w \cdot \left(1 - \alpha \cdot \frac{1+\varepsilon}{R}\right)\right)\right].$$  \hspace{1cm} (18)

If the marginal tax rate $\tau_w \cdot \left(1 - \frac{1+\varepsilon}{R}\right)$ is small enough, the difference $(\tau^{pe} - \tau^{tre})$ is correctly approximated by the following simple expression:

$$(\tau^{pe} - \tau^{tre}) \simeq \varepsilon^2 \frac{1}{1+\varepsilon} \cdot \left(\tau_w \cdot \left(1 - \alpha \cdot \frac{1+\varepsilon}{R}\right)\right)^2.$$  \hspace{1cm} (19)

### 2.4 Comparing with the Harberger’s triangle approximation (Feldstein, 1996 and 2005b)

At each period, the monetary distortion cost affecting labor supply (in per cent of $w \cdot l_{max}$) is measured by $(\tau^{pe}_{f} - \tau^{tre}_{f})$ (see Eq. (19) above).

Notice that it can be compared with Harberger’s triangle approximation. As showed by Feldstein (1996), this approximation (in per cent of $w \cdot l_{max}$) is given by:

$$\frac{1}{2} \cdot \varepsilon \cdot \left(\tau_w \cdot \left(1 - \alpha \cdot \frac{1+\varepsilon}{R}\right)\right)^2.$$  \hspace{1cm} (20)

The relative gap between the two expressions (19) and (20) evolves $w.r.t.$ $\varepsilon$ as follows:
\[ \frac{1}{2} \cdot \left( \frac{1}{\varepsilon} - 1 \right). \]

For very small values of \( \varepsilon \), the relative difference is positive and very high. It cancels when \( \varepsilon = 1 \) and it admits a lower bound of \(-50\%\) for high values of \( \varepsilon \).

3 Addressing the heterogeneity issue

Pension rules have a complex redistributive incidence.

Now, we suppose that a parameter of productivity \( \theta \in \Omega \) is continuously distributed. Heterogeneity can result from an individual risk (uncertainty on the level of average wage during the work period) or a social inequality (difference of skill at the birth). \( F(\theta) \) denotes the cdf. We suppose \( F(\theta) \) is stationary. The wage is \( w(\theta) = \theta \cdot w \) with \( \int_{\Omega} \theta dF(\theta) = 1 \). When \( \theta \) is known, the optimal level of labor supply is:

\[ l_\theta = \theta^\varepsilon \cdot l \left( \tau_w, \alpha, w, \bar{R} \right), \]

and the ex post welfare is given by:

\[
W_\theta \left( \tau_w, \alpha, T, R^+, w \right) = \theta^{1+\varepsilon} \cdot A \left( R^+, w \right) \cdot l \\
\cdot \left( \left( \left( 1 - \tau_w \cdot \left( 1 - \frac{\Gamma^{1+\varepsilon}}{R} \cdot \alpha \right) \right) \cdot \frac{\varepsilon}{1+\varepsilon} + \tau_w \cdot \frac{\Gamma^{1+\varepsilon}}{R} \cdot (1 - \alpha) \cdot \frac{E(\theta^{1+\varepsilon})}{\theta^{1+\varepsilon}} - \frac{\tau_\theta}{u_\theta} \right)^{23}
\]

(1 - \alpha) > 0 contributes to increase the welfare of workers with a productivity index lower than 1. The PE lump-sum tax rate is stationary and equal to:

\[
\tau_{\theta}^{pe} = 1 - \left( 1 - \tau_w \cdot \left( 1 - \frac{\Gamma^{1+\varepsilon}}{R} \cdot \alpha \right) \right)^\varepsilon \cdot \left( \left( 1 - \tau_w \cdot \left( 1 - \frac{\Gamma^{1+\varepsilon}}{R} \cdot \alpha \right) \right) \cdot \frac{\varepsilon}{1+\varepsilon} + \tau_w \cdot \frac{\Gamma^{1+\varepsilon}}{R} \cdot (1 - \alpha) \cdot \frac{E(\theta^{1+\varepsilon})}{\theta^{1+\varepsilon}} - \frac{\tau_\theta}{u_\theta} \right)^{23}
\]

\[
= \tau_{\theta}^{pe} + \left( 1 - \tau_w \cdot \left( 1 - \frac{\Gamma^{1+\varepsilon}}{R} \cdot \alpha \right) \right)^\varepsilon \cdot \tau_w \cdot \frac{\Gamma^{1+\varepsilon}}{R} \cdot (1 - \alpha) \cdot \left( 1 - \frac{1}{\theta} \right).
\]

We deduce the total monetary welfare cost (TMWC):

\[
TMWC = MDC \cdot \int_{\Omega} \theta^{1+\varepsilon} dF(\theta). \]

13
The stationary TRE lump-sum tax rate refunding the implicit public debt. There is a problem of unicity and a difficulty to determine a lump-sum tax rate which guarantees an increase of welfare to all workers. It is necessary to adopt a strategy. Idea of strategy: a part of $\tau_{TRE}$ depends on $\theta$ and reimburses the contributive pension debt and the other part is common to all workers and reimburses the non-contributive pension debt. In terms of income, the pension rule induces:

$$P_\theta = \theta \cdot \frac{l_\theta}{t} \cdot P + (1 - \alpha) \cdot \left(1 - \theta \cdot \frac{l_\theta}{t}\right) \cdot P$$  \hspace{1cm} (26)$$

where $P$ is the average pension.

All workers with a productivity $\theta$ reimburse in the future $\theta \cdot P$ and receive a lump sum transfer $(1 - \alpha) \cdot (1 - \theta) \cdot P$ to compensate a potential welfare loss or an excessive gain induced by the supression of the redistributive property of the pension rule. We obtain:

$$\tau_{TRE}^{\text{tre}} = \tau_{TRE} + \left(1 - \tau_w \cdot \left(1 - \frac{\Gamma^{1+\epsilon}}{R} \cdot \alpha\right)\right) \cdot \tau_w \cdot \frac{\Gamma^{1+\epsilon}}{R} \cdot (1 - \alpha) \cdot \left(1 - \frac{1}{\theta}\right).$$  \hspace{1cm} (27)$$

This lump sum tax induces a welfare gain for all workers because the redistributive aspect of the pension rule is fully compensated. The welfare cost for each worker is measured by:

$$WDC_\theta = \beta^{-\epsilon} \cdot A^{1+\epsilon} \cdot \theta^{1+\epsilon} \cdot \frac{1}{1+\epsilon} \cdot (\tau^{\text{tre}}_\theta - \tau_{TRE}^{\text{tre}})$$

$$= \beta^{-\epsilon} \cdot A^{1+\epsilon} \cdot \theta^{1+\epsilon} \cdot \frac{1}{1+\epsilon} \cdot (\tau^{\text{tre}} - \tau_{TRE}^{\text{tre}})$$  \hspace{1cm} (28)$$

and we deduce the total welfare distortion cost (TWDC):

$$TWDC = \int_\Omega WDC_\theta dF(\theta)$$

$$= WDC \cdot \int_\Omega \theta^{1+\epsilon} dF(\theta).$$  \hspace{1cm} (29)$$

4 Numerical simulations

This section computes numerical values to illustrate the sensitivity of distortion costs according to the parameters. We assume the following benchmark set of parameters: $\varepsilon = 1; \tau_w = 0.2; \alpha = 0.8; s = 0.5; R = (1 + 6\%)^{40}; \Gamma = (1 + 1.5\%)^{40}; w_0 = 1.$
Figure 1: Sensitivity to labor supply elasticity.

4.1 Sensitivity to $\varepsilon$ (labor supply elasticity)

In our simulations, the tax revenue equivalent and Pareto equivalent lump-sum tax rates monotonically decrease with $\varepsilon$. The tax gap reaches a maximum for $\varepsilon \approx 1.8$. Notice that the relative welfare cost also reaches its maximum when $\varepsilon \approx 1.8$, since it is captured by this tax gap. On the contrary, for $\varepsilon \in [0, 2.2]$, the relative monetary cost is monotonically increasing in $\varepsilon$, because the rise of $\frac{\Gamma^{1+\varepsilon}}{R}$ offsets the decreasing impact of $\varepsilon$. 
4.2 Sensitivity to $\Gamma$ (labor productivity growth factor)

An increasing $\Gamma$ reduces the marginal tax rate induced by unfunded pension. This results in higher labor supply. This effect dominates the other effect as discussed above, then the resulting net effect is a reduction in both Pareto and tax revenue equivalent lump-sum tax rates. As $\varepsilon = 1$, the gap expresses as a second-degree polynomial in $\frac{r^2}{R}$:

$$
\frac{1}{2} \cdot \tau_w^2 \cdot \left(1 - \alpha \cdot \frac{r^2}{R}\right)^2
$$

(30)

When $\frac{r^2}{R} < 1$, the relative welfare cost is unambiguously decreasing in $\Gamma$. Also, for a given $R$, the discount factor for monetary cost, $\frac{1+r}{R}$, increases with $\Gamma$. Finally, the relative monetary cost increases unambiguously.
4.3 Sensitivity to $\bar{R}$ (capital return factor)

An increasing $\bar{R}$ has opposite effects with respect to $\Gamma$, except for the relative monetary cost for which the discount factor moves in the opposite way. Hence, the variation of the relative monetary cost w.r.t $\bar{R}$ is non monotonic. For values of $\bar{R}$ less than 4.39, the discount factor effect dominates the increasing lump-sum tax gap.

Figure 3: Sensitivity to capital return factor.
Figure 4: Sensitivity to Social Security tax rate.

### 4.4 Sensitivity to $\tau_w$ (Social Security tax rate)

Since $\frac{1 + \varepsilon}{R} \cdot (1 + \varepsilon \cdot (1 - \alpha)) < 1$ is always checked, the Pareto equivalent lump-sum tax rate is a monotonic decreasing function of $\tau_w$. An increasing $\tau_w$ leads to a higher marginal tax rate. So, corresponding lump-sum tax rates increase, and their gap grows as a square power. Since $\varepsilon = 1$, the gap $(\tau_{pe} - \tau_{tre})$ expresses as a proportion of $\tau_w^2$. This gap admits a minimum for $\tau_w = 0$, hence welfare and monetary distortion costs are increasing for any value of $\tau_w$. 
Figure 5: Sensitivity to contributive pension share.

4.5 Sensitivity to $\alpha$ (contributive pension share)

Again, since we have $\varepsilon = 1$, the gap between lump-sum tax rates evolves as a quadratic polynomial, whose minimum is attained for a value of $\alpha$ greater than 1. Then, for any $\alpha < 1$, the gap $(\tau^{pe} - \tau^{fre})$ is decreasing with $\alpha$. When $\alpha = 1$, the distortion is only caused by the gap between capital return and labor income growth.

Conclusion

Relating to the works of Feldstein (1996, 2005b) and Kotlikoff (1996), this paper proposes an analytical characterization of the distortion cost induced by unfunded pension, based on endogenous labor supply in a simple OLG framework. We propose an alternative assessment to Harberger’s approximation of the intergenerational social deadweight loss induced by unfunded
pension. This allows us to provide new analytical insights. We find tractable expressions of welfare and monetary costs. These are expressed as linear functions of the difference between "Pareto equivalent" and "tax revenue equivalent" lump-sum tax rates. The former sustains the same welfare dynamics as the unfunded pension scheme, while the latter refunds the implicit public debt induced hereby by the unfunded pension scheme.

Our evaluation of the distortion cost is limited to a single flat tax on labor income. It could be extended to a multi-tax framework (capital income, consumption, etc) or progressive taxation.

In this model, the linear production technology leads to the independence of factor productivities with respect to capital accumulation. That means we do not measure the impact of a lower capital accumulation on the distortion cost. Developing a pure analytical approach seems difficult. However, our approach could be easily implemented in a computational framework.

Moreover, the objective of the paper does not directly focus on tax reform which would consider equity linked to heterogeneity of agents. However, whenever a strong distortion cost is evidenced, tax reform and the adoption of a more efficient tax system are at stake. This issue is treated implicitly by Erosa and Gervais (2002) and Conesa and Garriga (2008) in a context of optimal growth for a given social discount rate.

References


Appendix - Proofs

Proposition 1: Let $g = \frac{w^{+}d^{+}}{w^{d}}$ the growth factor of labor income. At the optimum, we have:

$$g = \Gamma^{1+\varepsilon} \cdot \left( \frac{1-\tau_{ww} \cdot \left(1-\alpha \cdot \frac{g^{+}}{\pi} \right)}{1-\tau_{w} \cdot \left(1-\alpha \cdot \frac{g^{+}}{\pi} \right)} \right)^{\varepsilon}.$$ 

The stationary solution $\bar{g}$ is unique and entails:

$$\bar{g} = \Gamma^{1+\varepsilon}.$$ 

This dynamic system can be characterized in the phase diagram $(g^{+}, g^{++})$ as follows:

[C1] $g - g^{+} > (>) 0$ 

$$\Leftrightarrow g^{++} > (>) 0 \Leftrightarrow g^{+} > (>) g^{++}.$$ 

[C2] $g^{+} - g^{++} > (>) 0 \Leftrightarrow g^{+} > (>) g^{++}.$

These conditions are sufficient to guarantee that the rational expectations solution $g = \bar{g}$ is stable. ■

Proposition 2: We search $\tau$ s.t. $W_{\text{max}} (\tau, R, w) = W (\tau_{w}, \alpha, R, w)$. From previous computations, we deduce $\tau$ from:
\[
\left(1 - \tau_w \cdot \left(1 - \frac{\Gamma^{1+\varepsilon}}{R} \cdot (1 + \varepsilon \cdot (1 - \alpha))\right)\right) \cdot \left(1 - \tau_w \cdot \left(1 - \alpha \cdot \frac{\Gamma^{1+\varepsilon}}{R}\right)\right) ^\varepsilon \\
= 1 - \tau \cdot (1 + \varepsilon). \blacksquare
\]

**Proposition 3:**

\[
MDC \quad = \quad \tau_{pe} \cdot \sum_{j=0}^{+\infty} \frac{w^{+}(j) \cdot l_{max}(w^{+}(j), R)}{R^j} - \tau_w \cdot w \cdot l \left(\tau_w, \alpha, w, \bar{R}\right) \\
= \quad \tau_{pe} \cdot w \cdot l_{max} \left(w, \bar{R}\right) \cdot \frac{\bar{R}}{R-1}\varepsilon - \tau_w \cdot w \cdot l \left(\tau_w, \alpha, w, \bar{R}\right).
\]

By definition we have, \(\tau_w \cdot w \cdot l \left(\tau_w, \alpha, w, \bar{R}\right) = \tau_{tre} \cdot w \cdot l_{max} \left(w, \bar{R}\right) \cdot \frac{\bar{R}}{R-1}\varepsilon\). Then we conclude on the value of the MDC. \(\blacksquare\)

**Proposition 4:** We search a lump-sum tax \(\tau\) s.t.:

\[
\tau \cdot \sum_{j=0}^{+\infty} \frac{w^{+}(j) \cdot l_{max}(w^{+}(j), R)}{R^j} = \tau_w \cdot w \cdot l \left(\tau_w, \alpha, 0, w, \bar{R}\right).
\]
We deduce:

\[
\tau_{tre} \quad = \quad \tau_w \cdot w \cdot l \left(\tau_w, \alpha, \tau_f, w, \bar{R}\right) / \sum_{j=0}^{+\infty} \frac{w^{+}(j) \cdot l_{max}(w^{+}(j), R)}{R^j} \\
= \quad \tau_w \cdot \frac{l \left(\tau_w, \alpha, w, \bar{R}\right)}{l_{max}(w, \bar{R})} / \sum_{j=0}^{+\infty} \left(\frac{\Gamma^{1+\varepsilon}}{R}\right)^j = \tau_w \cdot \frac{l \left(\tau_w, \alpha, w, \bar{R}\right)}{l_{max}(w, \bar{R})} \cdot \left(1 - \frac{\Gamma^{1+\varepsilon}}{R}\right). \blacksquare
\]

**Proposition 5:**

\[
W_{DC} = W \left(\tau_{pe}, w, \bar{R}\right) - W \left(\tau_w, \alpha, w, \bar{R}\right).
\]

By definition, \(W \left(\tau_{pe}, w, \bar{R}\right) = W \left(\tau_w, \alpha, w, \bar{R}\right)\). Then, we find:

\[
W_{DC} = \beta^{-\varepsilon} \cdot \alpha^{1+\varepsilon} \cdot (\tau_{pe} - \tau_{tre}). \blacksquare
\]