Market-based pay for long-term CEO performance*

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Abstract

We provide a model in which market-based pay addresses the problem of compensating a CEO when the impact of his actions extends well beyond his tenure at the firm. Using a standard noisy rational expectations model, we examine the optimal design of market-based pay when the firm’s stock price is the outcome of self-interested, asymmetrically informed trading in presence of risk-neutral market makers. We show that even if the stock price is efficient for valuation, it is not efficient for providing incentives. The reason is that the stock price does not react one-to-one to the actual effort decision of the CEO. This property alters the comparative statics of the textbook trade-off between risk and incentives. Our results yield a number of novel empirical predictions on the impact of market conditions, and in particular stock price volatility and noise trading volume, on optimal pay structures.

Keywords: market-based executives compensation; informational efficient prices; market liquidity; pay structure.

JEL Classification Codes: G30; D86.

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1 Introduction

The design of pay for the executives of publicly traded companies is an ongoing matter of debate both in the academic and public domain. In the presence of informationally efficient markets it is widely believed that giving managers stock-based incentive pay has the advantage of using the information content of stock prices. The use of market-based pay seems particularly appropriate when the CEO’s horizon, typically limited to his tenure, is shorter than the time length of the projects he manages. In these circumstances the design of CEO pay requires to reward him well before his strategic decisions come to fruition.

In this paper we analyze a public firm whose long-term value is affected by the unobservable CEO effort but it cannot be contracted upon since it realizes after the CEO departure. We first study the properties of the stock price as an information signal for incentive problems and then show how financial market conditions such as liquidity, trade volume, price informativeness and volatility affect the structure of managerial compensation.

Our first result is that the stock price, even informationally efficient in the semi-strong sense, is an imperfect monitoring device since it is inherently not good to detect shirking. The reason is that the stock price generated by competitive, risk-neutral market makers does not react one-to-one to the actual effort decision of the CEO. Due to this limited sensitivity an efficient price provides a statistically biased signal of the CEO’s contribution to firm value, in the sense that the expected price does not equal the actual CEO’s effort when she deviates from the equilibrium.

Moreover, the stock price necessarily contains a noise term due to liquidity trading which is orthogonal to the firm value. Our second main result is that this noise term is more important precisely when the stock price is more sensitive to the CEO’s actual effort decision.

We proceed analyzing how the above results affect the comparative statics of the classic risk-incentive trade-off. Explicitly constructing the market equilibrium clarifies that the comparative statics of market-based pay with respect to liquidity can be misleading since liquidity
is itself an endogenous variable in a market model. Measuring liquidity with the standard Kyle’s lambda (Kyle 1985), a liquid market is one where the price reacts little to the order flow because this latter contains a relatively high amount of noise trades compared to the informative ones. On the one hand, a reduced sensitivity of the stock price with respect to trades lowers its sensitivity to effort. This gives the CEO high incentives to shirk since he knows that even if the informed traders who monitor the firm would start selling the stock, this would not have a strong negative impact on the equilibrium price. On the other hand, high liquidity allows the informed traders to aggressively trade on their private information making the price more informative about the long-term firm value, and it reduces the impact of noise trading on the price. These latter effects suggest instead that a highly liquid market causes more market based pay. Overall, we confirm a positive correlation between liquidity and market-based pay as in several empirical contributions (Garvey and Swan (2002), Jayaraman and Milbourn (2011). Our paper adds on these since it allows to explicitly disentangle the effects of the determinants of liquidity on market based pay.

Contrarily to the results in standard principal-agent models we find that market based pay increases with the volatility of the stock price. The standard result in the principal-agent framework says that volatility is bad for incentives provision since it exposes the risk-averse agent to uncontrolled risk. In informationally efficient markets volatility is actually good for incentive provisions because a highly volatile price is a very informative one. Such a price provides then a more precise signal about the long-term firm value.

We then turn to the effects of exogenous market characteristics on market based pay.

Interestingly, we find that market-based pay-for-performance sensitivity (PPS) decreases when noise trade contained in the order flow increases. The reason is that semi-strong efficient prices are less reactive to orders when the ratio of noise to informed trade in the order flow increases. But this means that the price is less sensitive to the actual CEO’s contribution to firm value, hence a worse signal to solve the moral hazard. Our finding is confirmed in
Jayaraman and Milbourn (2011) who show that the addition of a firm stock to the S&P 500 Index, which increases stock liquidity but it reduces its informativeness through the increase in noise trade, makes firms relying less on stock prices for PPS. Notice that the above result differs from Holmstrom and Tirole (1993) where the value of the stock market as a monitor increases with the number of liquidity traders because this gives more incentive to an informed speculator to gather private information about the firm value. The key difference between our model and Holmstrom and Tirole (1993) is that we consider a market with a continuum of asymmetrically informed and competitive informed traders, while they analyze a market with only one potential informed trader.

Our analysis of the stock price as an aggregator of dispersed information provides new theoretical content to the empirical evidence on the extent of stock-based CEO pay and stock market conditions in the United States where ownership is widely dispersed. Jayaraman and Milbourn (2011) find that greater stock liquidity is associated with a higher proportion of equity-based pay and with higher pay-for-performance sensitivity (PPS) with respect to the stock price. As in our and other papers (e.g. Kyle (1985), Holmstrom and Tirole (1993), Easley and O’Hara (2004) and Chordia, Roll and Subrahmanyam (2008)) this is motivated by the fact that in efficient markets stock liquidity determines the extent of stock price informativeness. Similarly, Garvey and Swan (2002) find a positive link between the extent of market-based pay and turnover as well as bid-ask spreads, where both turnover and the bid-ask spread are used as measures of stock liquidity, while Kang and Liu (2005) show that managerial pay is more sensitive to stock price changes when the stock price incorporates more information. Interestingly, Jayaraman and Milbourn (2011) also show some evidence that following the addition of the firm’s stock to the S&P 500 Index, the proportion of equity-based compensation to total compensation increases while the PPS decreases. Including a stock in the S&P 500 Index increases its liquidity while reducing stock price informativeness due to a higher volume of noise trading.
Several other costs of using stock prices to provide CEO incentives have been suggested in the literature. One cost arises from the difference between information that is useful for trading and information that is useful for evaluating CEO’s actions (Paul (1992)). Paul (1992) argues that a trader would care about future exogenous shocks to firm value, e.g. the possible entry of other firms, that are irrelevant to the evaluation of past managerial performance. Other papers in the accounting literature also distinguish the valuation and contracting role of stock prices (see Gjesdal (1981), and Lambert (1993)) but none of them models trading explicitly and thus cannot point to the friction on information based trading we highlight in the current paper.

Another kind of cost occurs when the CEO must be given incentives to perform several tasks. The stock price only conveys information about the total value of the firm but not necessarily about the value-added of each individual task undertaken by the manager. Kim and Suh (1993) argue that using the "raw" price to construct market measures is problematic since the stock price impounds public information from earnings reports in addition to private information. As a result the information content of stock prices about management performance may be overstated. If one of management’s activities can be the exaggeration of performance, then Goldman and Slezak (2006) show how stock based performance contracts induce CEOs to waste resources by manipulating the information transmitted to investors. Bolton et al. (2006) show that a CEO has an incentive to wastefully increase the risk of his firm to play up the speculative component of the stock price if the market is inefficient. A distinguishing feature of our model is that even without any of the costs mentioned above, i.e. in a setting where i) the stock market is efficient, ii) the aggregate market information (i.e. the sum of all informed traders signals) perfectly reveals the future value of the firm and iii) the aggregate market information is a sufficient statistic for CEO effort, managerial compensation should not be entirely market based.

The paper closest to our is Holmstrom and Tirole (1993). We base our analysis of a standard moral-hazard problem between the owners and the management of a publicly traded firm on
their model. Holmstrom and Tirole (1993) are, however, interested in a different question. They show the benefit for a firm to go public in terms of more market monitoring, which in turn is due to the incentive for an informed trader to collect more precise information over the firm value, information then impounded into the stock price. There are several differences between their framework and ours. In their set-up there is no horizon mismatch between the firm and the CEO. Moreover, their information structure and trading model are designed to capture the idea of insider trading, since they assume that only one potentially informed trader operates on the market (as in Kyle (1985)). In contrast, we base our market model on Vives (1995), which is a standard model to analyze aggregation of information (see also Vives (2009)). Also, we want to analyze the role of trading condition for the design of CEO pay for a firm that is already publicly traded, where the CEO’s tenure is shorter than the time horizon of the owners of the firm, and where no trader observes special information about the firm or the CEO’s decisions.

Finally, Edmans (2009) considers institutions with an endogenous investment horizon, who can decide to hold their stake in the firm for the long-term based on their private signal. The signal the blockholder receives by monitoring the firm allows him to learn whether weak earnings result from bad management decisions or from desirable long-term investment. In both cases a more liquid market induces the blockholder to trade on his private signal and this in turns causes the stock price to reflect the fundamental value, correcting for managerial myopia.

The paper is organized as follows. Section 2 presents the model. Section 3 shows its solution and collects our main results. Section 4 generalizes them allowing for endogenous information collection and limited risk-tolerance in the market. Section 5 discusses the empirical implications of the model and section 6 concludes. All formal proofs are in the Appendix.
2 Model set-up

The model assumes a standard moral-hazard problem between the owners and the management of a publicly traded firm as in Holmstrom and Tirole (1993). We depart from their set-up by introducing active trading of the firm’s shares in a large competitive market where a continuum of informed traders have heterogenous, dispersed and imperfect information about the future value of the firm. Their self-interested trading leads to an aggregation of information in the stock price that is useful for incentivizing management.

Agents. The firm is run by a risk-averse manager. Its shares are held by two groups: insiders (owners) and outsiders (traders). The owners constitute a well-diversified (i.e., risk-neutral), passive, value oriented collective whose investment horizon coincides with the life of the firm. They hold a constant fraction of the shares of the firm. The remaining fraction - normalized to one share - is traded freely in the stock market by the outsiders.\(^1\) Outsiders buy and sell shares of the firm based on their private information about the future value of the firm and based on the information publicly revealed by stock prices (informed traders). In addition to informed traders, on the market operate also uninformed (noise) traders who trade for exogenous liquidity reasons, e.g., stochastic life cycle motives, and risk-neutral market makers who stand ready to buy (sell) when the price is low (high).

Manager tenure and information for contracting. There are four dates, \(t = 0, 1, 2, 3\). The model begins with the owners hiring the manager and offering him an incentive contract at \(t = 0\) that gives him income \(I\) at the end of his tenure. After signing the contract, the manager exerts an unobservable, costly effort \(e\) that determines the expected long-run value of the firm. The actual value of the firm realizes at the last period \(t = 3\), when the firm is liquidated. It is given by \(v = e + \theta\), where \(\theta \sim N(0, \tau^-1)\) so that \(E[v] = e\). The shock \(\theta\) is a reduced form description of factors that affect firm value but are outside the control of the

\(^1\)Our analysis therefore considers a public firm that is up and running. There is a large literature on optimal ownership structure.
manager. To capture the idea that the manager’s actions affect firm value beyond his tenure, we assume that the manager quits the firm at \( t = 2 \). The manager cannot wait to be paid until \( t = 3 \) when the full consequences of his actions materialize. Instead, he must be paid when he leaves the firm at the latest. As a consequence, the long-term value of the firm \( v \) is not available for incentive contracting. The principal is then bound to contract upon different signals. We assume there are two contractible variables in the model. First, the trading of the firm’s shares occurring at \( t = 1 \) results in a stock price \( p \) that can be used in the incentive contract, which we describe next. The power incentives provided through \( p \) define the market-based pay. In order to allow for a simple alternative to market-based pay, we assume that an unbiased public signal \( y \) about the future liquidation value of the firm arrives when the manager quits the firm at \( t = 2 \): \( y = v + \eta \), where \( \eta \sim N(0, \tau_\eta^{-1}) \) is an observational error independent of all other random variables in the model. The signal \( y \) can be interpreted for example as accounting information. According to standard practice, we limit our analysis to contracts that are linear in the two performance measures.

Managerial contract. The manager’s income contains a fixed wage \( W \), a market based element contingent on the stock price \( p \), denoted by \( S \), and a non-market based part \( A \) contingent on the signal \( y \). The manager’s income is then given by

\[
I = W + Ay + Sp. 
\] (1)

We follow Holmstrom and Tirole (1993) and normalize the price and the incentive contract in order to separate the pricing and the incentive problem. In terms of actual implementation of such a contract, the proposed normalization assumes that the long-term incentives \( S \) are paid in cash when the manager leaves the firm. One could think of \( S \) as a long-term incentive plan whose value depends on the stock price. Thus, the manager is paid \( W + Sp \) in cash while the amount \( Ay \) is paid in shares transferred from long-term inside owners to the manager. The
amount $Ay$ can be considered as an accounting bonus paid in the form of shares that the CEO can liquidate at his departure. The fraction of shares $\alpha$ that must be transferred to the manager to pay the accounting bonus is given by $Ay = \alpha E[v - W - Sp|y, p]$ since this is the fair price of the firm’s shares given public information at the moment the manager leaves.

This normalization leaves all payoffs unchanged while the net liquidation value of the firm becomes $\pi = v - W - Sp$. Defining $\hat{p}$ as

$$\hat{p} = W + (1 + S)p$$

we can write the manager’s income in terms of the normalized share price:\(^2\)

$$I = a_0 + a_\hat{p}\hat{p} + a_y y$$  \hspace{1cm} (3)

where $a_0 = W(1 - a_\hat{p})$, $a_\hat{p} = \frac{S}{1 + S}$ and $a_y = A$.\(^3\)

The optimal incentive contract owners give to the manager at $t = 0$ maximizes the expected net value of the firm, $E[v - I]$, subject to the risk-averse manager acting in his own interest, $e = \arg \max_e E[U(e')]$ and subject to the manager participating, $E[U(e)] \geq 0$, where we have normalized his reservation utility to zero. The manager’s utility is given by $U(e) = -\exp[r_m(I - \frac{2}{2}e^2)]$ where $r_m$ is the coefficient of absolute risk-aversion of the manager and $\frac{2}{2}e^2$ is his (monetary) cost of effort.

**Competitive stock market.** The one outside share of the firm is traded in a competitive stock market between a continuum $i \in [0, 1]$ of risk-averse informed traders, uninformed noise

\(^2\)Note that the prices $p$ and $\hat{p}$ are informationally equivalent.

\(^3\)Alternatively, we can express

$$W = \frac{a_0}{1 - a_\hat{p}}$$

$$S = \frac{a_\hat{p}}{1 - a_\hat{p}}$$
traders and risk-neutral market-makers. We follow Vives (1995) to examine the aggregation of dispersed information through trading. At the opening of the stock market, at $t = 1$, each informed trader $i$ privately receives a different, imperfect and unbiased signal about the future value of the firm, $s_i = v + \varepsilon_i$, where $\varepsilon_i$ are i.i.d. random variables, $\varepsilon_i \sim N(0, \tau_v^{-1})$. The market as a whole “knows” the liquidation value of the firm $v$ since traders’ individual and possibly very large errors $\varepsilon_i$ cancel out in the aggregate due to the law of large numbers: $\int_0^1 \varepsilon_i di = 0$. The information about the liquidation value of the firm is, however, not accessible since it is dispersed among many traders. Instead, the stock price aggregates this information via decentralized, self-interested trading.\(^4\)

An informed trader $i$ maximizes the CARA utility of the return from trading the firm’s shares at price $p$. He has a constant absolute risk aversion coefficient $r$, and rational expectations, i.e. he use all the information available. He submits then demand schedules that condition the amount he buys or sells on his private information $s_i$ as well as the observable stock price. Noise traders’ demand $u$ is random, normally distributed, $u \sim N(0, \tau_u^{-1})$ and independent of all other random variables of the model.

The price of the share at $t = 3$ is simply the liquidation value of the firm net of any cash payment that was made to the manager, $p_3 = \pi = v - W - Sp$. A competitive, risk-neutral market making sector ensures that the stock price at $t = 1$ is semi-strong efficient and reflects all publicly available information.\(^5\) The market maker observes the aggregate limit order book

\(^4\)Notice that in our set-up there is no difference between information that is useful for trading and information that is useful for incentive contracting (see Paul 1992). Speculators value information about shocks to final asset value while incentive contracting values information about the shock that garbles the impact of effort on the value of the firm. Both shocks are identical here and are given by $\theta$.

\(^5\)This means that informed traders will only speculate (trading on information) and withhold from market making (selling/buying when the price is above/below the prior expectation of the asset value). One can show that the price becomes semi-strong efficient when in addition to the continuum of risk-averse informed traders there is a continuum of uninformed traders whose risk aversion tends to zero (instead of a risk-neutral market making sector; see for example, Kyle, 1989, or Vives, 2007).
resulting from the joint demand of informed and noise traders,

\[ L(p) = \int_0^1 x_i(s_i, p)di + u \]

and sets the price efficiently conditional on the information contained in this limit order book:

\[ p = E[v - W - Sp|L(p)] \quad (4) \]

Using the normalized price \( \hat{p} \) allows a simpler expression for the equilibrium price:

\[ \hat{p} = W + p(1 + S) = E[v|L(\hat{p})] \quad (5) \]

given that \( \hat{p} \) is informationally equivalent to \( p \).\(^6\)

Figure 1 summarizes the sequence of events and the different horizons of the firm, the manager and the stock market. The red-arrows indicate the shorter horizon of the manager relative to the horizon of the firm’s owners and of traders.

[Insert Figure 1 here]

### 3 Solution

In this section we characterize the market equilibrium and the optimal contract in the class of linear contracts described above.

\(^6\)To see this, start developping \( E[v - W - Sp|L(p)] = E[v|L(p)] - W - Sp \). Then using (4) one gets \( E[v|L(p)] = W + (1 + S)p = \hat{p} \) given (2). Hence the result in the text, noticing that \( \hat{p} \) is a linear transformation of \( p \), hence informationally equivalent.
3.1 Pricing

The standard CARA-normal framework admits equilibria in which the demand of informed trader \(i\) is linear in prices and in the private signal \(s_i\). Hence, let

\[
x_i(s_i, \hat{p}) = \beta s_i + f(\hat{p})
\]

be the demand schedule chosen by informed trader \(i\) at \(t = 1\). The variable \(\beta\) denotes how aggressively a trader uses his private information \(s_i\). The function \(f\) is linear in prices. The aggregate limit order book then is

\[
L(\hat{p}) = \beta v + f(\hat{p}) + u = \beta(e + \theta) + u + f(\hat{p}) = z + f(\hat{p})
\]

where \(z = \beta(e + \theta) + u\) is the part of the order book which is informative about the long-term value of the firm. The price setting condition (5) can then be written as

\[
\hat{p} = E[v|z]
\]

Calculating this conditional expectation gives the following standard result (see, for example, Vives, 2009):

**Proposition 1 (Pricing)** The normalized stock price \(\hat{p}\) is given by:

\[
\hat{p} = (1 - \lambda \beta) e^* + \lambda \beta(e + \theta) + \lambda u
\]

where \(e^*\) is the hypothesized equilibrium effort of the manager, \(e\) is the actual effort, \(\beta = \frac{\tau_s}{\tau}\) the informed traders aggressiveness, \(\tau \equiv (Var[\hat{p}|z])^{-1} = \tau_\theta + \beta^2 \tau_u\) is the informativeness of the equilibrium price, and \(\lambda \equiv \beta \tau_u\) the inverse of the stock liquidity measure (as in Kyle (1985)).
The stock price equals the weighted average of expected and actual firm value plus a term due to uninformed noise trading. Note that in order to provide incentives one has to distinguish between the anticipated equilibrium effort $e^*$, and the actual effort $e$ in the price. In equilibrium they coincide but to derive the equilibrium they have to be kept separate. The equilibrium effort $e^*$ determines the market’s prior expectation of firm value whereas an (off-equilibrium) deviation $e$ determines actual value and thus drives the signals traders use to update their information. Rewriting (7) as

$$\hat{p} = e^* + \lambda \beta (e - e^*) + \lambda \beta \theta + \lambda u$$

one can see that in case the manager deviates from equilibrium, as long as $\lambda \beta \neq 1$, the stock price is not equal to the actual firm value $v = e + \theta$ plus an orthogonal noise term with fixed volatility, unlike those performance signals on which the standard risk-incentive trade-off is based (see, for example, Prendergast 1999). Alternatively, denoting with $H_0$ the public information available at $t = 0$, off-equilibrium we have

$$E[\hat{p} | H_0] = e^* + \lambda \beta (e - e^*) \neq e = E[v | H_0]$$

whenever $\lambda \beta \neq 1$, hence $\hat{p}$ is a biased signal for incentive purposes. Given that the sensitivity of the price to actual managerial effort is equal to $\lambda \beta < 1$, the price is inherently weak for detecting shirking. The reason is that the information signals that traders receive contain both managerial effort $e$ and the realized shock $\theta$: an effort lower than expected does not necessarily triggers a large amount of sales by the informed traders. Crucially, a more liquid stock, i.e. one with lower $\lambda$, is less powerful to detect shirking since it absorbs orders without much price impact.

Rewriting (7) at equilibrium $e = e^*$ as
\[ \hat{p} = e^* + \lambda (\beta \theta + u) \]

one can see that the sensitivity of the price to actual effort and the amount of noise contained in the price are determined simultaneously. The equilibrium price contains a higher amount of noise \( \lambda (\beta \theta + u) \) exactly when it is more sensitive to managerial shirking (high \( \lambda \beta \)). The market characteristics \( \beta \) and \( \lambda \) simultaneously affect the sensitivity of the price to effort and the noise term that blurs the managerial effort, the two parameters affecting its quality as an incentive signal.

Finally, both the informativeness of the price \( \tau \) and the liquidity of the market \( (\lambda)^{-1} \) are endogenous and depend in particular on the aggressiveness \( \beta \) with which traders use their information. The aggressiveness in turn depends on traders’ risk-aversion and on the precision of their information: a higher \( \beta \) certainly improves the quality of \( \hat{p} \) as an incentive signal since it increases its sensitivity to the actual effort without increasing the noise it contains.

### 3.2 Incentive contracting

The optimal contract \((a_0, a_y, a_{\hat{p}})\) maximizes the value of the firm at \( t = 0 \) net of the manager’s income subject to his incentive compatibility and his participation constraints,

\[
\max_{a_0, a_y, a_{\hat{p}}} \mathbb{E}[v - I] \quad (8)
\]

subject to

\[ e = \arg\max \mathbb{E}[I] - \frac{r_m}{2} \text{Var}[I] - \frac{k}{2} \varepsilon^2 \]

and

\[ \mathbb{E}[I] - \frac{r_m}{2} \text{Var}[I] - \frac{k}{2} \varepsilon^2 \geq 0, \]

where we have used the certainty equivalent for the CARA utility of the manager.

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\(^7\) Vives (1995) shows that this same static equilibrium holds when the informed traders receive an information signal once and for all at the opening of the market and have long investment horizon, i.e. they liquidate their positions when \( v \) realizes. In such a case, the informed traders enter a position after receiving their private signal \( s_i \) and keep it until the end. There is no information trading in the subsequent periods \( t > 1 \) in which the market is infinitely liquid, \( \lambda_t = 0 \).
The next Proposition provides the solution to problem (8).

**Proposition 2:** The optimal weights respectively on the normalized stock price \( \hat{p} \) and on the accounting signal \( y \) are:

\[
\begin{align*}
a_{\hat{p}} &= \frac{\sum \tau}{1 + kr_m \left( \tau^{-1}_\theta + \Sigma \right)} \quad (9) \\
a_y &= \frac{\sum \tau_y}{1 + kr_m \left( \tau^{-1}_\theta + \Sigma \right)} \quad (10)
\end{align*}
\]

where \( \Sigma = (\beta^2 \tau_u + \tau_\eta)^{-1} \). The optimal effort is

\[
e^* = \frac{1}{k} \frac{1}{1 + kr_m \left( \tau^{-1}_\theta + \Sigma \right)} \quad (11)
\]

The coefficients \( a_{\hat{p}} \) and \( a_y \) measure the total market-based and non-market based pay-performance sensitivity (PPS).

In order to interpret the weights \( a_{\hat{p}} \) and \( a_y \) it is useful to view the optimal contract in Proposition 2 as the outcome of a two-step procedure (see Prendergast, 1999). The first step aggregates the information contained in the stock price \( \hat{p} \) and in the public signal \( y \) to obtain an estimate of the future liquidation value of the firm. The second step incentivizes the manager to exert effort using the estimate obtained at the the first step.\(^8\)

Using \( \hat{p} \) and \( y \), the best (linear) estimator of the long-term value \( v \), that we denote by \( \hat{v} \) is unbiased, \( E[\hat{v}|v] = v \) and it minimizes the Mean Square Error \( E[(\hat{v} - v)^2] \). This estimator is given by

\[
\hat{v} = \Sigma (\tau \hat{p} + \tau_\eta y). \quad (12)
\]

The weights the estimator \( \hat{v} \) gives respectively to the stock price \( \hat{p} \) and to the signal \( y \) are

\(^8\)As for comparison, consider the hypothetical case in which the future liquidation value is available for contracting. There is then no need to estimate it and the first step is redundant. The optimal contract must only provide incentives. The manager’s income would simply be \( I = a_0 + a_v v \). The optimal contract would have \( a_v = \frac{1}{1 + kr \tau_\theta} \), which incentivizes the manager to exert an effort level \( e = \frac{1}{k(1 + kr \tau_\theta)} \).
proportional to their respective precisions $\tau$ and $\tau_\eta$. In the Appendix (see Result 1) we show that $\Sigma = Var(\hat{v} - v)$ measures the estimation error. The manager is then given optimal incentives using the estimator $\hat{v}$,

$$I = a_0 + \frac{1}{1 + kr(\tau_\theta^{-1} + \Sigma)} \hat{v}. \quad (13)$$

Combining (12) and (13) gives the same compensation contract as in Proposition 2. Interpreting the solution with this two-step procedure shows that the principal suffers an additional estimation error $\Sigma$ when trying to infer the actual effort of the manager from $(\hat{p}, y)$ than if he could contract on the long-term value $v$. Not only the stock price is worse than $v$ as a signal to provide incentive, but the accounting information $y$ is not redundant for contracting even if it contains an additional noise term and if the price is semi-strong efficient.\(^9\)

\(^{10}\) We now analyze how the market-based (resp. non market-based) PPS (9) (resp. (10)) change with the fundamentals of the model. As in the standard principal-agent model, both (9) and (10) decrease with the noise term $\theta$ that hides the manager contribution to firm value, with his degree of risk-aversion $r_m$ and with the cost parameter $k$.

An increase in the volatility of uninformed trade in the order book, i.e. lower noise trade precision $\tau_u$, both decreases the sensitivity of $\hat{p}$ to effort, due to higher liquidity $\lambda^{-1}$, and increases the estimation error $\Sigma$: overall the effect on the market based PPS then is negative,

\(^9\)The same signal $y$ would be redundant however if $v$ was contractible.

\(^{10}\)A further interpretation of the optimal pay structure is obtained by writing

$$\frac{a_\hat{p}}{a_y} = \frac{\beta \tau_u}{\lambda \tau_\eta} = \frac{\tau}{\tau_\eta}. \quad (14)$$

The ratio of the weights on performance measures is given by the ratio of their “signal-to-noise” ratios (Banker and Datar (1989) etc...??). The signal-to-noise ratio of non-price information $y$ is $\tau_\eta$ since the sensitivity of the performance measure to effort is one. The sensitivity of the stock price to managerial effort is $\lambda \beta$ and its noise is $Var[\lambda u] = \lambda^2 \tau_u^{-1}$; thus, the signal-to-noise ratio of the stock price $\hat{p}$ is given by its informativeness $\tau$. The ratio of market to non-market based pay is then equal to the ratio of the precisions of two exogenous shocks contained in the respective signals, i.e. the noise trading $\tau_u$ and the noise of non-price information $\tau_\eta$, times an endogenous market factor.
i.e. the more the noise trade, the lower the market-based PPS. This result contrasts with Holmstrom and Tirole (1993) where an increase in noise trade, due to a more dispersed, public ownership, leads to more informative prices. In Holmstrom and Tirole (1993) more uninformed trade leads to a more informative price since, as in Kyle (1985), the unique informed speculator operating in the market trades with more aggressiveness on his own private signal.

If the informed traders’ aggressiveness $\beta$ increases, the price $\hat{p}$ becomes more precise in estimating effort (i.e. lower $\Sigma$) and more sensitive to it. Thus the market-based PPS $a_{\hat{p}}$ increases.

Since the variable $\beta$ cannot be easily observed, several empirical studies have tested the effect of stock liquidity and price informativeness and volatility on PPS. If one considers market liquidity $\lambda^{-1}$ as a fundamental parameter of the model, our Proposition 2 predicts that the market-based PPS increases with liquidity (i.e. $a_{\hat{p}}$ decreases with $\lambda$) as in Garvey and Swan (2009), Jarayman and Milbourn (2011). Thanks to the explicit market model, we can clearly disentangle the effects that each determinant of market liquidity, i.e. the noise trade volume, the informed traders’ aggressiveness, and the noise $\tau_{\theta}$, has on the optimal contract.

Also price informativeness, measured by the price precision $\tau$, is an endogenous variable of the model but its effect on market-based PPS has been directly investigated in Garvey and Swan (2009). From the ANOVA decomposition (Vives 1999), due to risk-neutral pricing,

$$Var(\hat{p}) = Var(v) - Var(v | \hat{p})$$

and $Var(v) = \tau_{\theta}^{-1}$. Hence, in efficient markets with risk-neutral pricing, higher price precision $\tau = (Var(v | \hat{p}))^{-1}$, corresponds to higher ex-ante price volatility $Var(\hat{p})$. Risk-neutral pricing allows maximum possible depth in the order book since the market maker has infinite risk tolerance; hence ex-ante price variance comes from an expected arrival of private information that will be compounded in the price. Proposition 2 predicts that the higher the ex-ante price
volatility, the higher the market-based PPS.

### 3.3 Contract Implementation

We are left to compute how the principal can implement the contract \((a_0, a_\hat{p}, a_y)\) illustrated in Proposition 2 through a long-term incentive plan based on the stock price and an accounting bonus \(A\). Recall that we have obtained the coefficient of the "normalized" contract from the original ones using the following transformations:

\[
a_0 = W(1 - a_\hat{p}), \quad a_\hat{p} = \frac{S}{1 + S}, \quad a_y = A
\]

While the accounting based bonus \(A\) simply equals \(a_y\) in (10), the term \(S\) representing long-term incentive plans based on stock performance is equal to

\[
S = \frac{a_\hat{p}}{1 - a_\hat{p}} = \frac{\Sigma \tau}{1 + kr\tau_\hat{g}^{-1} + \Sigma (kr - \tau)}
\]

whose comparative statics correspond to the ones of \(a_\hat{p}\).

### 4 Empirical Implications

Most of existing results are based on the relation between PPS and stock liquidity (e.g. Chen and Swan). In Section 3 we show that this relation is spurious since liquidity is an endogenous measure resulting from the ratio of informed vs. uninformed trade. There can be henceforth ambiguous theoretical relations between these two dimensions.

**Implication 1** more liquidity leads to a better aggregation of dispersed and heterogeneous information and more market based pay ceteris paribus
(Chordia, Roll and Subrahmanyam (2008), Chen and Swan (2009)). This result is confirmed empirically by Jayaraman and Milbourn (2009), who find that the proportion of cash-based compensation to total annual compensation is lower in firms with higher stock liquidity.

**Implication 2** *more informative stock prices lead to more market based pay ceteris paribus*

Jayaraman and Milbourn (2009) show some exploratory evidence that less informative prices are associated with a lower performance sensitivity with respect to the stock price.

**Implication 3** *(NEW) highly volatile (=more precise) prices, more stock-based compensation*

**Implication 4** *(NEW) more noise trade, less stock-based compensation*

## 5 Concluding remarks

We provide a model in which market-based pay addresses the problem of compensating a CEO when the impact of his actions extends well beyond his tenure at the firm. Using a standard noisy rational expectations model, we examine the optimal design of market-based pay when the firm’s stock price is the outcome of self-interested, asymmetrically informed trading in presence of risk-neutral market makers. We show that even if the stock price is efficient for valuation, it is not efficient for providing incentives. The reason is that the stock price does not react one-to-one to the actual effort decision of the CEO. This property alters the comparative statics of the textbook trade-off between risk and incentives. Our results yield a number of novel empirical predictions on the impact of market conditions, and in particular stock price volatility and noise trading volume, on optimal pay structures.
References


Appendix

We make use of the following two results (see Vives, 2009, Technical Appendix). The first result is the standard conditional distribution for normally distributed variables:

Result 1 (Conditional expectation) Let $Y_i$ be a $(n_i \times 1)$ a multi-variate normally distributed vector with mean $\mu_i$, $i=1,2$, and variance-covariance matrices $\Sigma_{ij}$, then

$$Y_2|Y_1 = y_1 \sim N([\mu_2 + \Sigma_2 \Sigma_{11}^{-1}(y_1 - \mu_1)], [\Sigma_{22} - \Sigma_2 \Sigma_{11}^{-1}\Sigma_{12}])$$

The second result gives the certainty equivalent for CARA utility when wealth is a linear combination of a normally distributed variable and its chi-square distributed square:

Result 2 (Certainty equivalent) Let $z \sim N(0, \sigma^2)$ and $W = az^2 + bz + c$ then

$$E[-\exp(-rW)] = -\frac{1}{\sigma \sqrt{1/\sigma^2 + 2ra}} \exp\left(-r\left(c - \frac{rb^2}{2(1/\sigma^2 + 2ra)}\right)\right)$$

Proof of Proposition 1

Applying Result 1, we obtain

$$E[v|z] = e^* \left(1 - \frac{\beta^2 \tau_u}{\beta^2 \tau_u + \tau_\theta}\right) + \frac{\beta \tau_u}{\beta^2 \tau_u + \tau_\theta} z$$

Letting $\lambda = \frac{\beta \tau_u}{\beta^2 \tau_u + \tau_\theta}$, substituting $z = \beta(e + \theta) + u$ and denoting $\tau = \beta^2 \tau_u + \tau_\theta$ gives the result $\hat{p} = E[v|z]$. ■

Proof of Proposition 2

The mean and variance of the manager’s income are, respectively:

$$E[I] = a_0 + a_\beta [(1 - \lambda \beta)e^* + \lambda \beta e] + a_y e$$

$$Var[I] = a_\beta^2 [(\lambda \beta)^2 \tau_\theta^{-1} + \lambda^2 \tau_u^{-1}] + a_y^2 (\tau_\theta^{-1} + \tau_\eta^{-1}) + 2a_\beta a_y (\lambda \beta \tau_\eta^{-1})$$

The incentive constraint therefore becomes

$$a_\beta \lambda \beta + a_y - ke = 0 \quad (A.1)$$

The participation constraint will bind since the manager’s utility is increasing in $a_0$ (through expected income alone) while the objective function of problem (8) is decreasing in $a_0$ and the incentive constraint (??) is independent of $a_0$. 

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After substituting the binding participation constraint into the objective function and using (A.1) to substitute for $e$, problem (8) becomes:

$$\max_{a_p, a_y} \frac{1}{k} (a_p \lambda \beta + a_y) - \frac{r}{2} \left[ a_p^2 (\lambda \beta)^2 \tau_-^{-1} + \lambda^2 \tau_u^{-1} \right] + a_y^2 (\tau_-^{-1} + \tau_y^{-1}) + 2 a_p a_y (\lambda \beta \tau_-^{-1}) - \frac{1}{2k} (a_p \lambda \beta + a_y)^2$$

The first order conditions with respect to $a_p$ and $a_y$ are:

$$\lambda \beta = a_p \left[ k r (\lambda \beta)^2 \tau_-^{-1} + \lambda^2 \tau_u^{-1} \right] + a_y \left[ k r \lambda \beta \tau_-^{-1} + \lambda \beta \right]$$

$$1 = a_y \left[ k r \left( \tau_-^{-1} + \tau_y^{-1} \right) + 1 \right] + a_p \left[ k r \lambda \beta \tau_-^{-1} + \lambda \beta \right]$$

Solving for $a_p$ and $a_y$ yields:

$$a_p = \frac{\beta \lambda \tau_-}{\lambda \tau_- \tau_y + \beta^2 \tau_u \tau_\theta + k r \tau \tau_\theta + k r \tau + k r \beta^2 \tau_u}$$

$$a_y = \frac{\tau_\theta}{\tau_\theta \tau_\eta + \beta^2 \tau_u \tau_\theta + k r \tau_\theta + k r \tau + k r \beta^2 \tau_u}$$

which rearranged and using $\Sigma^{-1} \equiv \beta^2 \tau_u + \tau_\eta$ become:

$$a_p = \frac{1}{1 + k r \left( \tau_\theta^{-1} + \Sigma \right)} \Sigma \tau$$

$$a_y = \frac{1}{1 + k r \left( \tau_\theta^{-1} + \Sigma \right)} \Sigma \tau_\eta.$$

The optimal effort is recovered from the incentive constraint (A.1):

$$k e = \frac{1}{1 + k r \left( \tau_\theta^{-1} + \Sigma \right)} \Sigma \left( \lambda \beta \frac{\tau_-}{\lambda \tau_u + \tau_\eta} \right)$$

$$\Rightarrow e = \frac{1}{k} \frac{1}{1 + k r \left( \tau_\theta^{-1} + \Sigma \right)}$$

**Result 1:** $\Sigma = Var(\hat{v} - v) = E[(\hat{v} - v)^2]$ where $\hat{v}$ is the best linear unbiased estimator of $v = e + \theta$ i.e.

$$\hat{v} = \Sigma (\tau \hat{p} + \tau_\eta y).$$

**Proof:** Recall the definition of $\Sigma = (\tau - \tau_\theta + \tau_\eta)^{-1} = (\beta^2 \tau_u + \tau_\eta)^{-1}$. From the definition
of \( \hat{v} \) one obtains

\[
\text{Var}(\hat{v}) = \text{Var}(\Sigma \tau \hat{p} + \Sigma \tau _\eta y) = \text{Var}(\Sigma \tau \hat{p}) + \text{Var}(\Sigma \tau _\eta y) + 2Cov(\Sigma \tau \hat{p}, \Sigma \tau _\eta y) = (\Sigma \tau )^2 \text{Var}(\hat{p}) + (\Sigma \tau _\eta )^2 \text{Var}(y) + 2\Sigma^2 \tau \tau _\eta Cov(\hat{p}, y).
\]

Given risk-neutral pricing, \( \hat{p} = E[v | \hat{p}] \), so we can decompose the variance

\[
\text{Var}(\hat{p}) = \text{Var}(v) - \text{Var}(v | \hat{p}) = \tau^-1 - \tau^{-1}
\]

Substituting in \( \text{Var}(\hat{v}) \) for the various terms:

\[
\text{Var}(\hat{v}) = (\Sigma \tau )^2 \text{Var}(\hat{p}) + (\Sigma \tau _\eta )^2 \text{Var}(y) + 2\Sigma^2 \tau \tau _\eta Cov(\hat{p}, y) = \Sigma^2 \left( \tau^2 \left( \frac{1}{\tau \theta} - \frac{1}{\tau} \right) + \tau _\eta^2 \left( \frac{1}{\tau \theta} + \frac{1}{\tau _\eta} \right) + 2\Sigma \tau \tau _\eta \lambda \beta \frac{1}{\tau \theta} \right)
\]

which after some manipulations and using \( \lambda \beta = \frac{\beta^2 \tau _\eta \tau}{\tau} = 1 - \frac{\tau _\eta}{\tau} \), can be written as

\[
\text{Var}(\hat{v}) = \frac{\Sigma}{\tau \theta} (\tau + \tau _\eta)
\]

Given that \( \hat{v} \) is the BLUEstimator of \( v \), we have that \( \hat{v} = v + \epsilon \) where the error term \( \epsilon \) represents the estimation error and it is orthogonal to \( \hat{v} \) (hence to \( \hat{p} \) and \( y \)). We can then write

\[
\text{Var}(\hat{v}) = \text{Var}(v) + \text{Var}(\epsilon)
\]

and substituting for \( \text{Var}(\hat{v}) \) and \( \text{Var}(v) \):

\[
\frac{\Sigma}{\tau \theta} (\tau + \tau _\eta) = \frac{1}{\tau \theta} + \text{Var}(\epsilon)
\]

\[
\text{Var}(\epsilon) = \frac{\Sigma}{\tau \theta} (\tau + \tau _\eta) - \frac{1}{\tau \theta} = \Sigma
\]

using \( \Sigma = (\tau - \tau \theta + \tau _\eta)^{-1} \).

Notice finally that an analogous result holds if the only contractible variable is the (normal-}

\[\text{Var}(v) = \text{Var}(\hat{p}) + \text{Var}(v | \hat{p}) = \text{Var}(E[v | \hat{p}]) + \text{Var}(\text{orthogonal error})\]

so the more volatile (ex-ante) is the price \( \hat{p} \), the more information it contains about \( v \). Henceforth volatility is not necessarily bad for contracting.
ized) price $\hat{p}$, where formulas must be corrected all through eliminating $y = 0$ and its precision $\tau_q$. ■