Trade Volume and Economic Growth

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Abstract
Do open economies earn better economic performance than closed economies? The most basic measure of openness is the simple trade volume, which is exports plus imports divided by GDP. A large number of empirical studies found a positive relationship, but few theoretical studies used this definition in endogenous growth models. We develop a two-country (Home and Foreign) by two-good (consumption good and investment good) by one factor (capital) endogenous growth model with international knowledge spillover to study the relationship between trade volume and economic growth. We find that trade volume are positively related to economic growth rate when a variation from the production coefficients in both countries.

Keywords: two-country endogenous growth model, trade shares, economic growth

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1. Introduction

Is trade openness good for economic performance? Although there are many empirical studies which provide an affirmative answer to this question, the relationship is still called in question. For instance, Rodriguez and Rodrik (2001) and Winters (2004) consider the results for trade liberalization from existing empirical studies are fragile. The results are crucially dependent on the variables selection for trade openness and independent variables in the regressions. Moreover, Yanikkaya (2003) and DeJong and Ripoll (2006) indicate that if we use trade restriction to represent trade openness, it is easy to get a reverse result in developing countries. Lee et al. (2004) also find that openness has a small, positive, but not particularly robust effect on growth. They have to rely, however, on the nontestable assumption that the structural shocks in the system of simultaneous equations are uncorrelated. This causality criticism is echoed by Billmeier and Nannicini (2009) and they find that the result is uncertain if we use trade restriction to measure openness.

So far, the relationship between trade openness and growth is a highly debated topic in the growth and development literature, at least from the perspective of trade restriction. However, if we use trade volume to measure openness, all empirical studies support the positive relationship, such as Frankel and Romer (1999), Irwin and Terviö (2002), Yanikkaya (2003), Alcala and Ciccone (2004), Noguer and Siscart (2005) and Chang et al. (2009).

In theoretical literatures, most of them focus on trade restriction and have a controversial result. For example, in R&D-based endogenous growth models, they find that trade restriction lowers the world economic growth rate but a reverse result under some conditions, such as Romer (1990), Grossman and Helpman (1990), Rivera-Batiz and Romer (1991a), Dinopouloua and Segerstrom (1999), Baldwin and Robert-Nicoud (2008) and Gustafsson and Segerstrom (2010). Similarly, in capital-accumulating endogenous growth models, Naito (2006b) find that the relationship between tariff and growth is correlated with the factor intensity of production technology in a small-open economy. Lee (2011) also finds that this relationship depends on the absolute advantage in the
production of investment goods in a two-country endogenous growth model.

Most empirical studies use the ratio of exports plus imports to GDP to measure trade volume. However, there are few papers to discuss the relationship between trade volume and economic growth in theoretical literatures. For example, Osang and Pereira (1997) introduce a composite consumption good, as in Bovenberg (1989), into a human capital accumulating growth model to analyze this issue. They find that the relationship is decided by the agent’s attitude toward saving. Moreover, Doi et al. (2007) also find that the relationship is related to factor intensity in a human capital accumulating endogenous growth model with psychic cost and externality. These results are inconsistent with the empirical studies. This is just the main motivation of our paper.

Our paper uses trade volume to represent the trade openness to examine the relationship between openness and economic growth. We introduce production externality which includes international knowledge spillover into a two-country capital-accumulation endogenous growth model. The international knowledge spillover is verified by Coe and Helpman (1995) and this set-up is also cited by Ghosh and Mourmouras (2002) and Lee (2011) to solve the knife-edge problem in a two-country endogenous growth model.

In this model, there are two countries (Home and Foreign) and two goods in each country: a pure consumption good and a pure investment good. The two goods are respectively produced by two sectors with capital as the only private input. The capital may be thought of as a composite of various types of physical and human capital as outlined in Rebelo (1991). We find that trade volume are positively related to long-run economic growth rate when a variation from the production coefficients in both countries. This result is consistent with empirical studies.

The intuition can be explained by a higher Home’s productivity for the investment goods. The higher Home’s productivity for the investment goods means that Home has more comparative advantage in the investment goods, and hence exports more investment goods and imports more consumption goods. It follows a higher volume of trade. On the other hand, a higher this productivity also furthers marginal product of capital and the corresponding economic growth rate. Therefore, trade volume and economic growth rate
are positive correlated.

The structure of the paper is as follows. Section 2 sets up the basic model. Section 3 analyzes the long-run equilibrium. In Section 4 we examine the relationship between trade volume and growth. Finally, some concluding remarks are made in Section 5.

2. The Model

There are two countries (Home and Foreign) and two goods in each country: a pure consumption good and a pure private investment good. The two goods are respectively produced by two sectors with capital as the only private input. The capital may be thought of as a composite of various types of physical and human capital as outlined in Rebelo (1991). International knowledge spillovers are introduced in this economy, which furthers each country’s productivity. The markets are competitive. Firms produce goods and make rental payments for capital input and distribute their profits to the households that own capital and firms. The households use the income to purchase the two goods. We also assume that the numbers of the households and firms are normalized to one, and thus each variable defined below expresses its aggregated value as well. Following tradition of Oniki and Uzawa, we assume that while the two goods are tradable, capital stock is not internationally mobile.

2.1. Firms in the two countries

There are two sectors to produce each good with capital, $K_i$, as the only private input in each country. There is a positive production externality in each production technology, such as the setting of Romer (1986). Different from Romer (1986), the production externality also originates from the activities of foreign country through international trade. That is, we assume that the new knowledge also instantaneously spills over across different countries.\(^1\) Thus, the externality may be represented by total capital in both countries.

For simplicity, abstracting from domestic knowledge spillover, the Home’s productive externality is just represented by the Foreign’s total capital stock, $K^*$, which is in line with

\(^1\) Although there is a lot of international technology diffusion whose marginal cost of use is not zero, we only focus on the public-good type for simplicity. The required proportion for this kind of international knowledge spillover may be arbitrarily small, and then this set-up is quite reasonable.
Lee (2010). Thus, the production function of good \( i \) in the Home country is:

\[
Y_i = A_i K_i^{1-\alpha} \bar{K}^\alpha, \quad i = 1, 2, \tag{1}
\]

where \( i = 1 \) (resp. 2) corresponds to the consumption (resp. investment) good, \( \bar{K} \) is the Foreign’s total capital which stands for the productive externality, and \( \bar{K}^* = K^* \) in the equilibrium. Parameter \( A_i \) is the productivity coefficient in sector \( i \), and \( \alpha \) measures the productivity of international knowledge spillover which is identical between the two sectors.

The full employment conditions are:

\[
K = K_1 + K_2, \tag{2}
\]

where \( K \) is the total capital stock.

Let \( p \) be the international price of consumption good in terms of the investment good. Given \( p \) and the rental rate of capital, \( r \), if the Home country produces both goods simultaneously, then the first-order conditions for the representative competitive firm in each sector are respectively:

\[
(1 - \alpha) p \frac{Y_1}{K_1} = r, \tag{3a}
\]

\[
(1 - \alpha) \frac{Y_2}{K_2} = r, \tag{3b}
\]

where \( r \) is also the interest rate if we assume that there is no depreciation of capital.

Suppose Foreign has a similar production structure. Thus, the production function of good \( i \) in the Foreign country is:

\[
Y_i^* = A_i^* K_i^{1-\alpha} \bar{K}^\alpha, \quad i = 1, 2, \tag{4}
\]

Similarly the first-order conditions for the Foreign’s representative firm are:

\[
(1 - \alpha) p \frac{Y_1^*}{K_1^*} = r^*, \tag{5a}
\]

\[
(1 - \alpha) \frac{Y_2^*}{K_2^*} = r^*. \tag{5b}
\]

### 2.2 Households in the two countries
Home’s representative household earns factor income with \( k(0) \) units of capital endowed initially and profit from the two firms. The household’s budget constraint is thus:

\[
rK + \pi = pC + I,
\]

where \( \pi = \pi_1 + \pi_2 = [pY_1 - rK_1] + (Y_2 - rK_2) \) is the profit from the two firms, \( C \) is consumption, and \( I \) is investment. The law of motion for capital accumulation is:

\[
\dot{K} = I.
\]

For simplicity, no depreciation in capital is assumed.

Given \( r, p, \) and \( K(0) \), the representative agent’s problem is to choose \( c, I, \) and \( k \) to maximize the following discounted lifetime utility:

\[
U = \int_0^\infty u(C)e^{-\rho t} dt,
\]

subject to (6) and (7). Parameter \( \rho > 0 \) is the rate of time preference. For simplicity, following Ventura (1997), we assume the felicity has logarithmic form:

\[
u(C) = \ln C.
\]

To solve the household’s optimization problem, we set up the current-value Hamiltonian:

\[
H = \ln C + \lambda [rK + \pi - pC],
\]

where \( \lambda \) is the co-state variable of capital. The necessary conditions for optimality are then:

\[
\frac{1}{C} = \lambda p, \quad (9a)
\]

\[
\lambda r = \rho \lambda - \dot{\lambda}, \quad (9b)
\]

with the transversality conditions \( \lim_{t \to \infty} \lambda ke^{-\rho t} = 0 \). While condition (9a) represents marginal benefit is equal to marginal cost for consumption condition, (9b) presents Euler equations for capital.

The behavior of Foreign’s household is the same. Thus, the first-order condition for the household is parallel to (9a) and (9b) with asterisked variables. Moreover, for simplicity, we assume the rate of time preferences between these two countries are the same as follows.
\[ \rho = \rho^*. \]
Thus, the necessary conditions for the Foreign’s household are:

\[ \frac{1}{C^*} = \lambda^* p, \quad (10a) \]
\[ \lambda^* r^* = \rho \lambda^* - \dot{\lambda}^*, \quad (10b) \]
with the transversality conditions \[ \lim_{t \to +\infty} \lambda^* K^* e^{-rt} = 0. \]

3. **Two-country World Market Equilibrium**

For free trade, the world commodity market-clearing condition for the consumption goods is:

\[ C + C^* = Y_1 + Y_1^*. \quad (11) \]

Once (11) is satisfied, the world market for the investment goods is automatically cleared.

We are now ready to analyze the equilibrium. The equilibrium is a path \{ \( Y_1, Y_2, Y_1^*, Y_2^*, C, C^*, K, K^*, K_1, K_2, K_1^*, K_2^*, I, I^*, r, r^*, p, \lambda, \lambda^* \) \} and is determined by (1)-(6), (9)-(11).

3.1. **Transformation of the economic system**

In order to analyze the equilibrium, it is necessary to transform the equilibrium conditions with perpetual growth into a system with stationary variables. Denote \( z = \frac{K^*}{K}, x = \frac{pC}{K}, x^* = \frac{pC^*}{K^*}, v = \frac{K_1}{K}, \) and \( v^* = \frac{K_1^*}{K^*}. \) In what follows we briefly explain the transformation.

First, multiplying (11) by \( p/K \) and utilizing \((3a)-(5b),\) the world market-clearing condition may be rewritten as:

\[ x + x^* z = A_2 \left( \frac{z}{1-v} \right)^\alpha v + A_2^* \left[ \frac{1}{(1-v^*) z} \right]^\alpha v^* z. \quad (12) \]

Next, \((3a)-(3b)\) and \((5a)-(5b)\) can be solved to yield the relationship between \( v \) and \( v^*. \)

\[ \left( \frac{v}{1-v} \right)^\alpha = \frac{A_1 A_2^*}{A_1^* A_2} \left( \frac{v^*}{1-v^*} \right)^\alpha. \quad (13) \]

This implies that the Home’s fraction of capital allocated to the consumption sector and the Foreign’s fraction are positively related. We find that \( v > v^* \) when Home has a
comparative advantage in the pure consumption good \(A_1/A_2>A_1^*/A_2^*\) and \(v<v^*\) when Home has a comparative advantage in the pure investment good \(A_1/A_2<A_1^*/A_2^*\).

Differentiating (9a) and using (3b) and (9b) lead to the following growth rate of consumption.

\[
\frac{\dot{C}}{C} = (1-\alpha)A_2\left(\frac{z}{1-v}\right)^\alpha - \rho - \frac{\dot{p}}{p}.
\]  

(14a)

Based on (6) and using (3a)-(3b), we get the growth rate of capital:

\[
\frac{\dot{K}}{K} = A_2\left(\frac{z}{1-v}\right)^\alpha - x.
\]  

(14b)

The growth rate of \(x\) can be derived by (14a) and (14b) as follows.

\[
\frac{\dot{x}}{x} = \frac{\dot{p}}{p} + \frac{\dot{C}}{C} - \frac{\dot{K}}{K} = x - \alpha A_2\left(\frac{z}{1-v}\right)^\alpha - \rho.
\]  

(15)

Moreover, the counterpart in the foreign country is:

\[
\frac{\dot{x}^*}{x^*} = x^* - \alpha A_2^*\left[\frac{1}{(1-v^*)z}\right]^\alpha - \rho.
\]  

(16)

Finally, with the help of (14b), the growth rate of \(z\) is:

\[
\frac{\dot{z}}{z} = \frac{\dot{K}^*}{K^*} - \frac{\dot{K}}{K} = A_2\left[\frac{1}{(1-v^*)z}\right]^\alpha - A_2\left(\frac{z}{1-v}\right)^\alpha + x - x^*.
\]  

(17)

The dynamics of the system can therefore be described by (12)-(13) and (15)-(17), which determine the equilibrium paths of five variables: \(x, x^*, z, v, \) and \(v^*\). Other variables are solved by other equations.

### 3.2. Long-Run Equilibrium

We now analyze the equilibrium in a steady state. A steady state is a perfect foresight equilibrium with a BGP under which \(x, x^*, z, v,\) and \(v^*\) are constant. Denote \(\bar{z}, \bar{x}, \bar{x}^*, \bar{v},\) and \(\bar{v}^*\) as the values in a BGP. Based on (15) and (16), we then obtain:

\[
\bar{x} = \alpha A_2\left(\frac{\bar{z}}{1-\bar{v}}\right)^\alpha + \rho,
\]  

(18a)
\[
\tilde{x}^* = A_2^* \left( \frac{1}{(1 - \tilde{v}^*) \tilde{z}} \right)^{\alpha} + \rho. \tag{18b}
\]

Utilizing (18a) and (18b), Equation (17) implies that the rates of economic growth between these two countries are identical.

\[
(1 - \alpha) A_2 \left( \frac{\tilde{z}}{1 - \tilde{v}} \right)^{\alpha} - \rho = (1 - \alpha) A_2^* \left( \frac{1}{(1 - \tilde{v}^*) \tilde{z}} \right)^{\alpha} - \rho. \tag{19}
\]

This equation indicates \( \tilde{x} = \tilde{x}^* \). Moreover, this equation is also the main difference from the existing literature. After \( \tilde{v} \) and \( \tilde{v}^* \) are determined, the growth rates between countries may be different. Thanks to international knowledge spillover, \( \tilde{z} \) appears in both Home’s and Foreign’s growth rates. Now, the two growth rates may converge to be identical through the adjustment of \( \tilde{z} \) - that is, (19) can determine the value of \( \tilde{z} \). The past literature, such as Bond, Trask and Wang (2003) and Doi, Nishimura and Shimomura (2007), always assume no international difference in production technology and preference, such that this equation is met automatically.

Substituting (18a), (18b) and (19) into (12), the world market-clearing condition can be rewritten by:

\[
\frac{A_2}{(1 + \tilde{z})(1 - \tilde{v})} \left( \frac{\tilde{z}}{1 - \tilde{v}} \right)^{\alpha} \left[ \tilde{v} - \alpha + \tilde{z} (\tilde{v}^* - \alpha) \right] = \rho \tag{20}
\]

Therefore, the three endogenous variables, \( \tilde{z}, \tilde{v}, \) and \( \tilde{v}^* \), are determined by (13), (19), and (20).

Now we assess the existence of \( \tilde{z}, \tilde{v}, \) and \( \tilde{v}^* \). Based on (13), we find that \( \tilde{v}^* = \tilde{v}^* (\tilde{v}) \).

Moreover, given \( \tilde{v} \), (19) can determine a unique \( \tilde{z} = \tilde{z} (\tilde{v}) = \left[ \frac{A_2^*}{A_2} \left( \frac{1 - \tilde{v}}{(1 - \tilde{v}^*) \tilde{z}} \right)^{\alpha} \right]^{\frac{1}{\alpha}} \). It is easy to find that the left-hand side (LHS) of (20) is increasing in \( \tilde{v} \) and \( LHS \to \infty \) as \( \tilde{v} \to 1 \) and \( LHS \to -\alpha A_2 \tilde{z}^{\alpha} < 0 \) as \( \tilde{v} \to 0 \). Consequently, \( \tilde{v} \) can be determined uniquely by (20) as depicted in Figure 1.

[Insert Figure 1 here]

This figure indicates there is an unique solution for \( \tilde{v} \). Given \( \tilde{v} \), (13) and (19) can determined \( \tilde{z} \) and \( \tilde{v}^* \). In sum, we get the following proposition.
**Proposition 1** Suppose the only difference between Home and Foreign is the productivity coefficient in the firms’ technology. The long-run equilibrium is uniquely determined with incomplete specialization in each country.

4. Trade Volume and Growth

This section examines the long-run relationship between trade volume and economic growth rate. We follow the empirical literatures to define the trade volume as the ratio of exports plus imports to GDP.

Now we examine the trade volume. Substituting (3a) and (3b) into (6), we find that Home’s resource constraint becomes:

\[ p(\tilde{C} - \tilde{Y}_1) = \tilde{Y}_2 - \tilde{I} \]

If Home imports consumption goods and export investment goods, then this equation implies that Home’s import volume equals export volume. With the help of (11), Home’s total trade volume is

\[ p(\tilde{C} - \tilde{Y}_1) + \tilde{Y}_2 - \tilde{I} = 2p(\tilde{C} - \tilde{Y}_1) = 2p((\tilde{C} - \tilde{Y}_1) - (\tilde{C}^* - \tilde{Y}_1^*)) \]

Thus, utilizing (3a)-(3b), (5a)-(5b) and (19), the ratio of exports plus imports to GDP is:

\[
T = \frac{p(\tilde{C} - \tilde{Y}_1) + \tilde{Y}_2 - \tilde{I}}{pY_1 + Y_2} = \left[ \frac{\rho}{A_2} \left( \frac{1 - \tilde{v}}{\tilde{z}} \right)^{\alpha} - (\tilde{v} - \alpha) \right] - \tilde{z} \left[ \frac{\rho}{A_2} \left( \frac{1 - \tilde{v}}{\tilde{z}} \right) - (\tilde{v}^* - \alpha) \right]
\]

Moreover, the long-run economic growth rate can be derived by (14a) as follows.

\[
g = (1 - \alpha)A_2 \left( \frac{\tilde{v}}{1 - \tilde{v}} \right)^{\alpha} - \rho.
\]

For simplicity, we assume \(A_1/A_2 = A_1^*/A_2^*\) at the beginning. Note that under \(A_1/A_2 = A_1^*/A_2^*\), (13) induces \(\tilde{v} = \tilde{v}^*\), and (19) and (20) indicates \(\tilde{C} = \tilde{Y}_1\) and \(\tilde{C}^* = \tilde{Y}_1^*\) which means this economy is like an autarkic economy. Under \(A_1/A_2 = A_1^*/A_2^*\), (19) and (20) implies both values of the two square brackets in (21) are zero and hence this ratio is also equal to zero. In addition, with (19) and (20), the economic growth rate in (22) can be rewritten as
\[ g = \rho \left( \frac{1 - \alpha}{\bar{v} - \alpha} - 1 \right). \]  

(23)

Now we examine the relationship between \( T \) and \( g \). These two endogenous variables are unchanged as long as the exogenous variables are constant. Thus, we need to assume some variations of the exogenous variables to investigate the relationship between these two endogenous variables. In this section, we assume the variation in exogenous variables only originates from the production coefficients of the two countries, such as \( A_1 \), \( A_2 \), \( A_1^* \), and \( A_2^* \). Without loss of generality, we change the value of \( A_2 \) and hold the other parameter values constant. In what follows, we briefly show the derivation.

First, we examine the effect of \( A_2 \) on \( T \). We assume \( A_2 \) increases a little such that \( A_1/A_2 < A_1^*/A_2^* \). It means that Home has comparative advantage in the pure investment good. Equation (11) also implies:

\[
\left[ \frac{\rho}{A_2} \left( \frac{1 - \bar{v}}{\bar{z}} \right)^\alpha \right] - (\bar{v} - \alpha) + \frac{\rho}{A_2^*} \left( \frac{1 - \bar{v}^*}{\bar{z}} \right)^\alpha - (\bar{v}^* - \alpha) = 0.
\]

The values of the two brackets are the same except \( \bar{v} \) and \( \bar{v}^* \). Under \( A_1/A_2 < A_1^*/A_2^* \), (13) also implies \( \bar{v} < \bar{v}^* \) and then the value of the first brackets is greater than second brackets. It follows that the value of first brackets is positive and the value of second brackets is negative. In other words, Home imports the pure consumption good and exports the pure investment good. Thus, the ratio of exports plus imports to GDP in (21) becomes greater than zero. The effect of \( A_2 \) on \( T \) is positive, i.e. \( dT/dA_2 > 0 \).

Second, we examine the effect of \( A_2 \) on \( g \). Under \( A_1/A_2 = A_1^*/A_2^* \), (19) and (20) imply:

\[(A_2 A_2^*)^{1/2} (1 - \bar{v})^\alpha (\bar{v} - \alpha) = \rho.\]

This expression also indicates

\[ \frac{d\bar{v}}{dA_2} = \frac{-(1 - \bar{v})(\bar{v} - \alpha)}{2A_2[\alpha(\bar{v} - \alpha) + (1 - \bar{v})]} < 0.\]

Thus, it is easy to find that the effect of \( A_2 \) on \( g \) is positive by (23) as follows.

\[ \frac{dg}{dA_2} = -\rho(1 - \alpha)(\bar{v} - \alpha)^{-2} \frac{d\bar{v}}{dA_2} > 0. \]

As a result, the relationship between trade volume and economic growth rate is
positive as follows.

\[
\frac{dT}{dg} = \frac{dT / dA_1}{dg / dA_2} > 0.
\]

That is, if we only vary one of the values of \(A_1, A_2, A_1^*,\) and \(A_2^*\), we find that there is a positive relationship between trade volume and economic growth rate under \(A_1/A_2 = A_1^*/A_2^*\) or small enough difference of these two values of the relative productivities. In sum, we get the following proposition.

**Proposition 2** Assume the only variation in the economy is from the production coefficients in both countries and the difference of relative productivities is small enough. Trade volume and economic growth rate are positive correlated.

The intuition can be explained as follows. A higher \(A_2\) means that Home has more comparative advantage in the investment good, and hence exports more investment goods and imports more consumption goods. It follows a higher volume of trade. On the other hand, a higher \(A_2\) also furthers marginal product of capital and so does the economic growth rate. Therefore, trade volume and economic growth rate are positive correlated in this case.

**5. Concluding Remarks**

We have presented a basic two-country endogenous growth model, which is regarded as an integration of a two-country economy and endogenous growth. We find that trade volume are positively related to economic growth rate when a variation from the production coefficients in both countries. This result is consistent with empirical studies.

Since the model is a basic one, there are many directions to extend this model to discuss trade and growth issues. One direction is to extend this model to include two factors, such as physical capital and labor or physical capital and human capital. This may re-examine the hypothesis of the Heckscher-Ohlin model. Moreover, international knowledge is a public good in this model, but in the real world most of these are a private good. If the firms absorbing international knowledge should pay the cost, then we can
extend this model to a R&D structure such as Romer (1990).
Reference


real exchange rate, Oxford Economic Papers 54, 72-90.


Figure 1: long-run equilibrium

\[ -\alpha A_2 z^\alpha \]

\[ \rho \]

\[ 0 \]

\[ \alpha \]

\[ \bar{v} \]

\[ 1 \]

\[ LH \]