Cross-Licensing and Competition*

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Abstract

We study bilateral cross-licensing agreements among \( N(> 2) \) firms that engage in competition after the licensing phase. It is shown that the most collusive cross-licensing royalty, i.e. the one that allows the industry to achieve the monopoly profit, is sustainable as the outcome of bilaterally efficient agreements. In a symmetric setting, if the terms of the cross-licensing agreements are not observable to third parties, the monopoly royalty is the unique symmetric bilaterally efficient royalty. However, when the terms of the agreements are public, the most competitive royalty (i.e. zero) can also be bilaterally efficient. Policy implications regarding the antitrust treatment of cross-licensing agreements are derived from these results.

Keywords: Cross-Licensing, Collusion, Antitrust and Intellectual Property.

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1 Introduction

A cross-license is an agreement between two firms that grants each the right to practice the other’s patents (Shapiro, 2001, and Régibeau and Rockett, 2011). Cross-licensing is not a new phenomenon and has been a widespread practice in many industries. For instance, Taylor and Silberston (1973) report that a significant share of licensing is done by cross-licensing in many sectors: 50% in the telecommunications and broadcasting industry, 25% in the electronic components sector, 23 percent in the pharmaceutical industry, etc. In particular, cross-licensing in the semiconductor industry has been intensively studied (Grindley and Teece, 1997, Hall and Ziedonis, 2001 and Galasso, 2012). Moreover, cross-licensing has been extensively used as a way to settle patent disputes (Shapiro, 2001).

Cross-licensing can be especially useful in Information Technology (IT) industries, such as the semiconductor and computer industries, where the intellectual property rights necessary to market a product are held by a large number of parties, a situation known as a patent thicket (U.S. DOJ and FTC, 2007). According to FTC (2011, pp.55-56), "The IT patent landscape involves products containing a multitude of components, each covered by numerous patents. ... This contrasts with the relationship between products and patents in the pharmaceutical and biotech industries where innovation is generally directed at producing a discrete product covered by a small number of patents." Patent thickets raise many concerns and are considered as one of the most crucial intellectual property issues of the day (Shapiro, 2007, and Régibeau and Rockett, 2011).

However, like other solutions to the patent thicket problem such as patent pools and cooperative standard setting, cross-licensing may negatively affect competition. According to the U.S. antitrust guidelines for the licensing of intellectual property (U.S. DOJ and FTC, 1995, p.18), "when a licensing arrangement affects parties in a horizontal relationship, a restraint in that arrangement may increase the risk of coordinated pricing, output restrictions, or the acquisition or maintenance of market power". More precisely, cross-licensing can reduce competition through high royalties. In particular, firms in a duopoly can sign a cross-licensing agreement with royalties high enough to replicate the monopoly profit (Fershtman and Kamien, 1992). For this reason, competition authorities

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1Patent thickets arise for multiple reasons: firms’ desire to strengthen and broaden their patent rights (Hall and Ziedonis, 2001, and Galasso and Schankerman, 2010), the cumulative nature of technology (von Graevenitz et al., 2012), the lack of resources and misaligned incentives at patent offices (Jaffe and Lerner, 2004, and Bessen and Meurer, 2008), etc. Patent thickets are particularly prevalent in areas related to information technology such as semiconductors, audio-visual technology, telecommunications and optics (von Graevenitz et al., 2011).
prohibit the use of royalties that are disproportionate with respect to the market value of the license.\textsuperscript{2}

However, as long as bilateral cross-licensing agreements are signed in an industry with more than two competing firms, it is unclear whether two firms would agree on high royalties since this might weaken their competitive positions with respect to their rivals. Also, if the terms of licensing are publicly observable, firms may want to agree on low royalties to be perceived by their rivals as efficient and therefore aggressive. Actually, both American and European competition authorities grant antitrust safety zone to (cross-) licensing agreements signed between firms whose combined market share is below a certain threshold. For instance, Article 3 of EC Technology Transfer Block Exemption Regulation provides antitrust exemption to bilateral licensing agreements between competitors if their combined market share does not exceed 20\%\textsuperscript{3}. However, to the best of our knowledge, the existing theoretical economic literature on cross-licensing does not provide any rationale for such exemptions since it studies bilateral cross-licensing only in a duopoly setting.

In this paper, we investigate the formation of bilateral cross-licensing agreements among $N$ ($> 2$) competing firms and assess their effects on competition. Each firm is initially endowed with a patent portfolio and can get access to its rivals’ patents through cross-licensing agreements involving the payment of fixed fees and royalties. After the cross-licensing phase, firms engage in quantity competition. As a first step, this paper considers symmetric firms with symmetric patent portfolios and focuses on symmetric equilibria where all cross-licensing agreements specify the same royalty. Our analysis allows us to identify the conditions under which such granting of antitrust safety zone can be justified.

We disentangle different strategic effects that drive the choice of royalties in a bilateral cross-licensing agreement. In the benchmark case of a multilateral licensing agreement signed by all $N$ firms, the firms would agree on the royalty that allows them to achieve the monopoly outcome. When we consider a bilateral cross-licensing, two firms in an agreement will internalize the externalities exerted on each other and hence have an incentive to raise the price by reducing joint production through high royalty. Is this

\textsuperscript{2}For instance, according to the Guidelines on the application of Article 81 of the EC Treaty to technology transfer agreements (European Commission, 2004), “... Article 81(1) may be applicable where competitors cross license and impose running royalties that are clearly disproportionate compared to the market value of the licence and where such royalties have a significant impact on market prices.”

\textsuperscript{3}Similarly, according to the US guideline (U.S. DOJ and FTC, 1995, p.22), “... the Agencies will not challenge a restraint in an intellectual property licensing arrangement if (1) the restraint is not facially anticompetitive and (2) the licensor and its licensees collectively account for no more than twenty percent of each relevant market significantly affected by the restraint.”
coordination effect the only one that matters when there are $N(>2)$ competing firms? If not, what are other effects and under which conditions is the anticompetitive coordination effect dominated by other (potentially pro-competitive) effects? The answers to these questions can generate interesting policy implications regarding the antitrust treatment of cross-licensing.

We assume that the larger the set of patents to which a firm has access, the lower its marginal cost. Alternatively, we can assume that the larger the set of patents to which a firm has access, the higher the value of its product. We actually show that our model of cost-reducing patents can be equivalently interpreted as a model of value-increasing patents. We distinguish public cross-licensing agreements from private agreements. The terms of a private agreement are observable only to the parties who sign the agreement while in the case of a public agreement, the terms are observed by all the firms.

We focus on bilaterally efficient agreements. A set of cross-licensing agreements is said to be bilaterally efficient if each agreement maximizes the joint profit of the pair of firms who signed the agreement given all other cross-licensing agreements. In the case of public agreements, we find that when two firms decide bilateral royalties to maximize their joint profit, they take into account two opposite effects: the coordination effect and the Stackelberg effect. The Stackelberg effect captures the fact that firms can influence the output levels to be chosen by their rivals through their choice of royalties. This effect disappears when agreements are private.

Our main results are as follows. If agreements are private, the unique bilaterally efficient set of symmetric cross-licensing agreements is the one involving the monopoly royalty $r^m$, i.e. the one that induces the firms to achieve the monopoly outcome. If agreements are public, the outcome depends on which between the Stackelberg effect and the coordination effect is stronger: if the coordination effect is stronger, the unique symmetric bilaterally efficient set of agreements is the one involving the monopoly royalty; if the Stackelberg effect is stronger, there are two symmetric bilaterally efficient set of agreements: the one involving the monopoly royalty and the one involving zero royalty. In both cases, the Pareto-dominant bilaterally efficient royalty is the monopoly royalty. This implies that the most collusive outcome can be sustained even if cross-licensing agreements are decided bilaterally. We show that this result still holds in the case of cross-licensing agreements decided by coalitions of any size comprising more than two firms. However, for a given number $N$ of competing firms, the Stackelberg effect is more likely to dominate the coordination effect as the number of firms involved in an agreement decreases.
A given symmetric royalty $r$ is bilaterally efficient if no pair of firms has an incentive to deviate from $r$. We show that a coalition has no incentive to deviate from $r$ if and only if it has no incentive to change its joint output from the level it would produce if it maintains $r$. In addition, we can decompose the joint profit of two firms as the sum of the downstream profit (from sales of their products) and the upstream profit (from licensing their patents). Therefore, when we study a coalition’s deviation in terms of joint output, in the case of a public agreement, we need to distinguish four cases depending on whether the coordination effect dominates the Stackelberg effect and whether the downstream profit effect dominates the upstream profit effect. For instance, if the Stackelberg effect dominates the coordination effect and the downstream profit effect dominates the upstream profit effect, the coalition has an incentive to boost its downstream profit, which requires it to increase the joint output (i.e. to deviate to a lower royalty) in order to induce the rivals to produce less. In contrast, if the Stackelberg effect dominates the coordination effect but the upstream profit effect dominates the downstream profit effect, the coalition has an incentive to boost its upstream licensing revenue, which requires it to reduce the joint output in order to induce the rivals to produce more. As $r$ increases, the upstream profit effect becomes more important: obviously, when $r = 0$, there is no upstream profit. It turns out that at $r = r^m$, the downstream profit effect is equal to the upstream profit effect and the coalition has no incentive to deviate. For $r$ in $[0, r^m)$, the coalition has an incentive to deviate to a higher royalty (a lower royalty) if the coordination effect dominates (is dominated by) the Stackelberg effect. In a symmetric way, for $r > r^m$, the coalition has an incentive to deviate to a lower royalty (a higher royalty) if the coordination effect dominates (is dominated by) the Stackelberg effect. Therefore, when the coordination effect dominates the Stackelberg effect, $r = r^m$ is a unique bilaterally efficient royalty; when the Stackelberg effect dominates the coordination effect, there are two bilaterally efficient royalties: $r = 0$ and $r = r^m$. The case of private agreement is a special case of the public agreement with no Stackelberg effect and therefore $r = r^m$ is a unique bilaterally efficient royalty.

Finally, our analysis generates the following policy implications. The antitrust exemptions granted to bilateral licensing agreements signed by competitors whose combined market share is smaller than a certain threshold can actually allow the firms in a given industry to implement the fully cooperative outcome (i.e. $r = r^m$) regardless of whether the licensing agreements are public or private. However, when the agreements are public and the Stackelberg effect dominates the coordination effect, bilateral cross-licensing may lead to the most competitive outcome (i.e. zero royalty) as well.
If an antitrust authority prefers to implement a lower retail price than the one in the fully cooperative outcome, they should enforce some upper bound on royalty such that firms cannot use royalties that are disproportionate with respect to the market value of the license. As long as this upper bound is smaller than \( r^m \), this will be the bilaterally efficient royalty when licensing agreements are private. If licensing agreements are public, the upper bound is bilaterally efficient only if the coordination effect dominates the Stackelberg effect; otherwise, the enforcement of such an upper bound makes zero royalty the unique bilaterally efficient outcome. In addition, we show that for given number of competing firms in an industry, the Stackelberg effect is more likely to dominate the coordination effect as the size of coalition (i.e. the number of firms signing together a cross-licensing agreement) is smaller. Therefore, granting exemptions to bilateral agreements makes some sense; actually, we show that in Cournot competition, if there are at least four competing firms and the coalition size is limited to two, the Stackelberg effect always dominates the coordination effect. Therefore, the antitrust exemptions granted to bilateral licensing agreements below a certain threshold of combined market share can be justified only if the following conditions are met (i) zero royalty is socially preferred to \( r^m \) (ii) licensing agreements are public (iii) firms are NOT exempted from using royalties disproportionate with respect to the market value of the license and \( r^m \) is one of such disproportionate royalties.

2 Related literature

Our paper contributes to the literature on the competitive effects of cross-licensing agreements and patent pools. In a pioneering paper, Priest (1977) shows how these two practices can be used as a disguise for cartel arrangements. Fershtman and Kamien (1992) develop a model in which two firms engage in a patent race for two complementary patents and use it to shed light on the social trade-off underlying cross-licensing agreements: One the one hand, cross-licensing improves the efficiency of the R&D investments by eliminating the duplication of efforts. But on the other hand, it favors price collusion between the firms. Eswaran (1994) shows that cross-licensing can enhance the degree of collusion achieved in a repeated game by credibly introducing the threat of increased rivalry in the market for each firm’s product. Shapiro (2001) argues that patent pools tend to raise (lower) welfare when patents are perfect complements (substitutes), an idea which is generalized to intermediate levels of substitutability/complementarity by Lerner and Tirole.
(2004) and further extended to the case of uncertain patents by Choi (2010). To the best of our knowledge, the present paper is the first formalized study of the competitive effects of bilateral cross-licensing agreements in an industry comprised of more than two firms.

Our paper is also related to the literature on strategic formation of networks surveyed in Goyal (2007) and Jackson (2008). In particular, the concept of bilateral efficiency we use is similar to the widespread refinement of pairwise stability in the network literature (Jackson and Wolinsky, 1996). Goyal and Moraga-Gonzales (2001) and Goyal, Moraga-Gonzales, and Konovalov (2008) also study two-stage games where a network formation stage is followed by a competition stage. However, they examine ex ante R&D cooperation, while we study ex post licensing agreements, and, contrary to the present paper, they do not allow for transfers among firms. Bloch and Jackson (2007) develop a general framework to examine the issue of network formation with transfers among players. A crucial difference between their framework and our setting is that we allow the agreements among firms to involve the payment of per-unit royalties on top of fixed fees while they only consider the case of lump-sum transfers.

More generally, our paper is related to the literature on negotiations and cooperative arrangements in industrial organization (e.g. Gans and Inderst, 2001, 2003, and, for a detailed survey, Gans and Inderst, 2007). Of particular interest to us is the paper by De Fontenay and Gans (2012) who provide an analysis of a non-cooperative pairwise bargaining game between agents in a network and show that the equilibria of their bargaining game are those that result in bilaterally efficient agreements. This finding supports our focus on bilaterally efficient agreements.

3 The Model

3.1 The model of cost-reducing patents

Consider an industry consisting of $N \geq 3$ symmetric firms producing a homogeneous good. Each firm owns one patent covering a cost-reducing technology and can get access to its rivals’ patents through cross-licensing agreements. We assume that the patents are symmetric in the sense that the marginal cost of a firm only depends on the number of patents it has access to. Let $c(n)$ be a firm’s marginal cost when it has access to a

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4Our concept of bilateral efficiency is also similar to the bilateral contracting principle used in the vertical relations literature (Hart and Tirole, 1990; Whinston, 2006).

5Given that we consider that firms have symmetric patent portfolios, we can assume, without loss of generality, that each firm has one patent.
number \( n \in \{1, \ldots, N\} \) of patents with \( c(N)(\equiv \bar{c}) \leq c(N-1) \leq \ldots \leq c(1)(\equiv \bar{c}) \) and let \( \mathbf{c} \equiv (c_1, \ldots, c_N) \) be the vector representing the marginal costs of all firms in the industry. Let \( \Delta c \equiv \bar{c} - \underline{c} \).

We consider a two-stage game in which prior to engaging in Cournot competition, each pair of firms can sign a cross-licensing agreement whereby each party gets access to the patented technology of the other one. More precisely, the two-stage game is described as follows:

- **Stage 1: Cross-licensing.**

Each pair of firms \((i, j)\) decides whether to sign a cross-licensing agreement and determine the terms of the agreement if any. We assume that a bilateral cross-licensing agreement between firm \(i\) and firm \(j\) specifies a pair of royalties \((r_{i\rightarrow j}, r_{j\rightarrow i}) \in \mathbb{R}^2\) and a lump-sum transfer \(F_{i\rightarrow j} \in \mathbb{R}\), where \(r_{i\rightarrow j}\) (resp. \(r_{j\rightarrow i}\)) is the per-unit royalty paid by firm \(i\) (resp. firm \(j\)) to firm \(j\) (resp. firm \(i\)) and \(F_{i\rightarrow j}\) is a transfer made by firm \(i\) to firm \(j\) (which is negative if firm \(i\) receives a transfer from firm \(j\)). Assume that all bilateral negotiations occur simultaneously. For the purpose of this paper we do not need to specify the underlying pairwise bargaining game: we will focus on the negotiation outcomes (and whether these satisfy a given property) and set aside the way such outcomes are achieved.

- **Stage 2: Cournot competition.**

Firms compete à la Cournot with the cost structure inherited from Stage 1.

Depending on the observability of royalties signed by \(i\) and \(j\) to other firms, we distinguish between public cross-licensing and private cross-licensing. In the case of public cross-licensing, all firms observe all the cross-licensing agreements signed at stage 1 before they engage in Cournot competition. In contrast, in the case of private cross-licensing, the terms of cross-licensing agreement signed between \(i\) and \(j\) remain known only to \(i\) and \(j\) and therefore each firm \(k\) should form a conjecture about the licensing terms signed between \(i\) and \(j\).

We assume that the firms face an inverse demand function \(P(\cdot)\) satisfying the following standard conditions (Novshek, 1985):

- **A1** \(P(\cdot)\) is twice continuously differentiable and \(P'(\cdot) < 0\) whenever \(P(\cdot) > 0\).
- **A2** \(P(0) > \bar{c} > \underline{c} > P(Q)\) for \(Q\) sufficiently high.
- **A3** \(P'(Q) + QP''(Q) < 0\) for all \(Q \geq 0\) with \(P(Q) > 0\).
These mild assumptions ensure the existence and uniqueness of a Cournot equilibrium \((q^*_i)_{i=1,...,n}\) satisfying the following (intuitive) comparative statics properties, where \(c_i\) denote firm \(i\)'s marginal cost:

\[
\frac{\partial q^*_i}{\partial c_i} < 0 \quad \text{and} \quad \frac{\partial q^*_j}{\partial c_i} > 0 \quad \text{for any} \ j \neq i; \quad \frac{\partial Q^*}{\partial c_i} < 0 \quad \text{for any} \ i,
\]

where \(Q^* = \sum_i q^*_i\) is the total equilibrium output;

\[
\frac{\partial \pi^*_i}{\partial c_i} < 0 \quad \text{and} \quad \frac{\partial \pi^*_j}{\partial c_i} > 0 \quad \text{for any} \ j \neq i,
\]

where \(\pi^*_i\) is firm \(i\)'s equilibrium profit (see e.g. Amir et al., 2012).

### 3.2 Alternative equivalent formulation: value-increasing patents

Instead of assuming that access to more patents reduces a firm’s marginal cost, we can assume that access to more patents increases the value of the product produced by the firm. We below show that our model of cost-reducing patents can be equivalently interpreted as a model of value-increasing patents.

We consider a constant symmetric marginal cost \(c\) for all firms. Each firm has one patent. Let \(v(n)\) represent the value of the product produced by a firm when the firm has access to \(n \in \{1, ..., N\}\) number of distinct patents with \(v(N) \geq v(N-1) \geq ... \geq v(1) (\equiv v)\). Let \(\mathbf{v} \equiv (v_1, ..., v_N)\) be the vector representing the value of each firm’s product after the licensing stage.

We define Cournot competition for given \(\mathbf{v} \equiv (v_1, ..., v_N)\) as follows. Each firm \(i\) simultaneously chooses its quantity \(q_i\). Given \(\mathbf{v} \equiv (v_1, ..., v_N)\), \(\mathbf{q} \equiv (q_1, ..., q_N)\) and \(Q = q_1 + ... + q_N\), the quality-adjusted equilibrium prices are determined by the following two conditions:

- **indifference condition:**
  \[
  v_i - p_i = v_j - p_j \quad \text{for all} \ (i, j) \in \{1, ..., N\}^2;
  \]

- **market-clearing condition:**
  \[
  Q = D(p) \quad \text{where} \quad p_i = p + v_i - v.
  \]

In other words, \(p\) is the price for the product of a firm which has access to its own patent only. The market clearing condition means that this price is adjusted to make the total supply equal to the demand. The indifference condition implies that the price
each firm charges is adjusted such that all consumers who buy any product are indifferent among all products. A micro-foundation of this setup can be provided as follows. There is a mass one of consumers. Each consumer has a unit demand and hence buys at most one unit among all products. A consumer’s gross utility from having a unit of product of firm $i$ is given by $u + v_i$: $u$ is specific to the consumer while $v_i$ is common to all consumers. Let $F(u)$ represent the cumulative distribution function of $u$. Then, by construction of quality-adjusted prices, any consumer is indifferent among all products and the marginal consumer indifferent between buying any product and not buying is characterized by $u + v - p = 0$, implying

$$D(p) = 1 - F(p - v).$$

In equilibrium, $p$ is adjusted such that $1 - F(p - v) = Q$.

Let $P(Q)$ be the inverse demand curve. In the equilibrium, a firm’s profit is given by

$$\pi_i = \left( P(Q) + v_i - v - c - \sum_{j \neq i} r_{i \rightarrow j} \right) q_i + \sum_{j \neq i} r_{j \rightarrow i} q_j.$$

After making the following change of variables

$$c - (v_i - v) = c_i,$$

the profit can be equivalently written as

$$\pi_i = \left( P(Q) - c_i - \sum_{j \neq i} r_{i \rightarrow j} \right) q_i + \sum_{j \neq i} r_{j \rightarrow i} q_j,$$

which is the profit expression in our original model of cost-reducing patents. In particular, the change of variables implies

$$c - (v(n) - v) = c(n),$$

which in turn implies

$$v(n + 1) - v(n) = c(n) - c(n + 1).$$

Therefore, our model of cost-reducing patents can be equivalently interpreted as a model of value-increasing patents. In what follows, for our formal analysis, we stick to the model of cost-reducing patents.
4 Benchmark: multilateral licensing agreement

We here consider the case of a multilateral licensing agreement signed by all \(N\) firms. We focus on a symmetric outcome where all firms pay the same royalty \(r\) to each other.

Let \(P^m(c)\) be the monopoly price when each firm’s marginal cost is \(c\). It is characterized by
\[
\frac{P^m(c) - c}{P^m(c)} = \frac{1}{\varepsilon(P^m(c))}.
\]
where \(\varepsilon(.)\) is the elasticity of demand.

Given a symmetric \(r\), each firm’s marginal cost is \(c + (N - 1)r\). The firms will agree on a royalty to achieve the monopoly price. Given a symmetric \(r\), at stage 2, firm \(i\) chooses \(q_i\) to maximize
\[
\frac{P(Q_i + q_i) - c}{P(Q_i + q_i)} = \frac{1}{\varepsilon(P(Q_i + q_i))}.
\]
where \(Q_i = Q - q_i\) is the quantity chosen by all other firms. Note that this is the same as maximizing \([P(Q_i + q_i) - c] q_i + rQ_i\) when \(Q_i\) is considered by firm \(i\) as given. Let \(r^m\) be the royalty that allows to achieve the monopoly price \(P^m(c)\). Then, from the first-order condition associated to firm \(i\)’s maximization program, we have
\[
N \left[ \frac{P^m(c) - c - (N - 1)r^m}{P^m(c)} \right] = \frac{1}{\varepsilon(P^m(c))}. \quad (2)
\]
From (1) and (2), \(r^m\) is determined by
\[
N \left[ P^m(c) - c - (N - 1)r^m \right] = P^m(c) - c,
\]
which is equivalent to
\[
\frac{P^m(c) - c}{N} = r^m. \quad (3)
\]

Now we examine under which condition the multilateral licensing agreement will lead to a higher downstream price compared to the situation before cross-licensing. Let \(Q^*(N, \bar{c})\) represent the industry output when \(N\) firms with the marginal cost \(\bar{c}\) compete in quantity. Therefore, the equilibrium price prior to licensing is \(P(Q^*(N, \bar{c}))\). At \(P(Q^*(N, \bar{c}))\) we have
\[
\frac{P(Q^*(N, \bar{c}) - c)}{P(Q^*(N, \bar{c}))} = \frac{1}{\varepsilon(P(Q^*(N, \bar{c})))} \quad (4)
\]
From (2) and (4), \(P^m(c) = P(Q^*(N, \bar{c}))\) if
\[
N \left[ P(Q^*(N, \bar{c})) - \bar{c} \right] = P(Q^*(N, \bar{c})) - c,
\]

which is equivalent to
\[
\Delta c = (N - 1) [P(Q^*(N, \bar{c})) - \bar{c}].
\] (5)

Note that the right hand side of (5) does not depend on \(\Delta c\). As the cost reduction from licensing increases, \(\bar{c}\) is smaller and the monopoly price is smaller. Therefore, we must have \(P^m(\bar{c}) \geq P(Q^*(N, \bar{c}))\) if and only if \(\Delta c \leq (N - 1) [P(Q^*(N, \bar{c})) - \bar{c}]\).

**Proposition 1** (Multilateral licensing). Assume that A1-A3 hold and that all firms in the industry jointly agree on a symmetric royalty. Then they agree on \(r^m = (P^m(\bar{c}) - \bar{c}) / N\) to achieve the monopoly price \(P^m(\bar{c})\). Moreover:

(i) If \(\Delta c < (N - 1) (P(Q^*(N, \bar{c})) - \bar{c})\), the post-licensing monopoly price is higher than the pre-licensing equilibrium price.

(ii) If \(\Delta c = (N - 1) (P(Q^*(N, \bar{c})) - \bar{c})\), the post-licensing monopoly price is the same as the pre-licensing equilibrium price.

(iii) If \(\Delta c > (N - 1) (P(Q^*(N, \bar{c})) - \bar{c})\), the post-licensing monopoly price is lower than the pre-licensing equilibrium price.

Note that when \(\Delta c = 0\), i.e. when patents are perfect substitutes, the multilateral licensing agreement described above serves as a purely collusive device.

5 Characterization of the bilaterally efficient public agreements

We now characterize the bilaterally efficient cross-licensing outcomes, i.e. the outcomes such that each pair of firms \((i, j)\) does not find it jointly optimal to modify the terms of the agreement they signed. In other words, each cross-licensing agreement is required to maximize the joint payoff of the two parties involved in it, given all other cross-licensing agreements. In this section we focus on cross-licensing agreements that are observable to all firms in the industry.

5.1 Preliminaries

Assume that all firms are always active (i.e. produce a positive output) whatever the cross-licensing agreements that are signed.\(^6\) Under this assumption, any given pair of

\(^6\)Without this assumption, some firms may decide not to be active depending on the cross-licensing agreements previously signed. We plan to study this situation in another paper which focuses on the relationship between cross-licensing and entry barrier.
firms find it (jointly) optimal to license one’s patent to the other. To see why, assume that firm $i$ does not license its patent to $j$. These two firms can (weakly) increase their joint profits if $i$ licenses its patent to $j$ by specifying the payment of a per-unit royalty $r_{j-i}$ equal to the reduction in marginal cost allowed by $j$’s use of $i$’s patent. Such licensing agreement would not affect the level of joint output but will (weakly) decrease the cost of firm $j$. It will therefore (weakly) increase their joint profits.

Consider now a symmetric situation where all firms cross-license each other and let $r$ denote the (common) per-unit royalty paid by any firm $i$ to have access to the patent of any firm $j \neq i$ with $i, j = 1, ..., N$. Let $S(r, N)$ denote such a set of cross-licensing agreements. We below study the joint incentives of a coalition of two firms, say 1 and 2, to deviate from the symmetric royalty $r$. The next lemma shows that it is sufficient to focus on deviations specifying identical royalties between the two firms in the coalition. Indeed the joint payoff of any asymmetric deviation can be replicated by a symmetric one as the joint payoff depends on the royalties paid by each firm of the coalition to the other only through their sum.

**Lemma 1** Consider a symmetric set of cross-licensing agreements $S(r, N)$. The joint payoff a coalition {i, j} gets from a deviation to a cross-licensing agreement in which firm $i$ (resp. firm $j$) pays a royalty $r_{i-j}$ (resp. $r_{j-i}$) to firm $j$ (resp. firm $i$) depends on $(r_{i-j}, r_{j-i})$ only through the sum $r_{i-j} + r_{j-i}$.

**Proof.** See Appendix. ■

This lemma shows that to determine whether the set $S(r, N)$ of cross-licensing agreements is bilaterally efficient, it is sufficient to investigate the incentives of the coalition to deviate by changing the royalty they charge each other from $r$ to $\hat{r} \neq r$.

For given $(r, \hat{r})$, let $Q^*(r, \hat{r})$ denote the total industry output and $Q^*_{12}(r, \hat{r})$ denote the sum of the outputs of firms 1 and 2 in the (second-stage) equilibrium of Cournot competition. Let $Q^*_{-12}(r, \hat{r}) \equiv Q^*(r, \hat{r}) - Q^*_{12}(r, \hat{r})$ denote the total equilibrium output of their rivals. Then, the considered set of symmetric agreements is bilaterally efficient if and only if:

$$r \in \text{Arg} \max_{\hat{r} \geq 0} (\pi^*_1 + \pi^*_2)(r, \hat{r})$$

where:
\[ (\pi_1^* + \pi_2^*) (r, \hat{r}) = [P (Q^*(r, \hat{r})) - (c + (N - 2) r)] Q^*_{12}(r, \hat{r}) + 2r Q^*_{12}(r, \hat{r}) \]
\[ = [P (Q^*_{12}(r, \hat{r}) + BR_{-1,2}(Q^*_{12}(r, \hat{r}))) - (c + (N - 2) r)] Q^*_{12}(r, \hat{r}) + 2r BR_{-12}(Q^*_{12}(r, \hat{r})) \]

where \( BR_{-12}(.) \) is defined as follows. If \( N = 3 \), then \( BR_{-12}(.) \) is the best-response function of firm 3. If \( N \geq 4 \), then \( BR_{-12}(.) \) is the aggregate response of the coalition’s rivals: for any joint output \( Q_{12} = q_1 + q_2 \) of firms 1 and 2, \( BR_{-12}(Q_{12}) \) is the (unique) real number such that \( \frac{BR_{-12}(Q_{12})}{N-2} \) is the best-response of any firm \( i \) \( (i = 3, ..., N) \) to each firm \( i = 1, 2 \) producing \( q_i \) and each firm \( i \) \( (i = 3, ..., N) \) producing \( \frac{BR_{-12}(Q_{12})}{N-2} \).

After observing the coalition’s deviation to \( \hat{r} \neq r \), the rivals of the coalition expect the coalition to produce \( Q^*_{12}(r, \hat{r}) \) and will best respond to this quantity by producing \( BR_{-12}(Q^*_{12}(r, \hat{r})) \) in aggregate. In other words, from a strategic point of view, the coalition’s deviation to \( \hat{r} \neq r \) is equivalent to its commitment to produce \( Q^*_{12}(r, \hat{r}) \) as a Stackelberg leader. We therefore define the following two-stage Stackelberg game.

**Definition:** For any \( r \geq 0 \) and \( N \geq 3 \), the Stackelberg game \( G(r, N) \) is defined by the following elements:

**Players:** There are \( N - 1 \) players: coalition \( \{1,2\} \) and each firm \( i \) \( (i = 3, ..., N) \). Each player has a marginal production cost \( c \) (excluding royalties) and pays a per-unit royalty \( r \) to each of the other players; each firm \( i \) \( (i = 3, ..., N) \) pays \( r \) to each member of the coalition.

**Actions:** The coalition \( \{1,2\} \) chooses its (total) output \( Q_{12} \) within the interval \( I(r, N) \equiv [Q^*_{12}(r, \infty), Q^*_{12}(r, 0)] \) and each firm \( i \) \( (i = 3, ..., N) \) can choose any quantity \( q_i \geq 0 \).

**Timing:** There are two stages.
- Stage 1: The coalition \( \{1,2\} \) acts as a Stackelberg leader and chooses \( Q_{12} \) within the interval \( [Q^*_{12}(r, \infty), Q^*_{12}(r, 0)] \).
- Stage 2: If \( N = 3 \), then firm 3 chooses its output \( q_3 \) (given \( Q_{12} \)). If \( N \geq 4 \), then each firm \( i \) \( (i = 3, ..., N) \) simultaneously chooses its quantity \( q_i \geq 0 \) (given \( Q_{12} \)).

The following lemma shows that the set of cross-licensing agreements \( S(r, N) \) is bilaterally efficient if and only if choosing \( Q^*_{12}(r, r) \) is optimal for the coalition in the Stackelberg game \( G(r, N) \).

**Lemma 2** The symmetric set of cross-licensing agreements \( S(r, N) \) is bilaterally efficient if and only if choosing to produce \( Q^*_{12}(r, r) \) is a subgame-perfect equilibrium strategy of the coalition \( \{1,2\} \) in the Stackelberg game \( G(r, N) \).
5.2 Incentives to deviate: downstream and upstream profits

In what follows we identify two opposite effects on the incentives of the coalition \{1, 2\} to marginally expand or contract its output with respect to \(Q_{12}(r, r)\). Note that the coalition’s marginal cost is \(c + (N - 2) r\) whereas each member’s marginal cost at the Cournot competition is \(c + (N - 1) r\). The difference between the two has to do with the royalty payment between 1 and 2. In what follows, we call \(rQ_{12}\) the royalty saving of the coalition (compared to a single firm producing the same quantity \(Q_{12}\)).

Then, the coalition’s profit can be rewritten as

\[
\pi_{12}(Q_{12}, r) = \left[ P(Q_{12} + BR_{12}(Q_{12})) - (c + (N - 1) r) \right] Q_{12} + r \left[ Q_{12} + 2BR_{-12}(Q_{12}) \right].
\]

The term \(\pi_{12}^D(Q_{12}, r)\) represents the coalition’s downstream market profit from selling its products, which is defined with respect to each member’s marginal cost at the Cournot competition stage.\(^7\) The term \(\pi_{12}^U(Q_{12}, r)\) represents the coalition’s profit from the upstream market of patent licensing. This profit is composed of the royalty saving and the licensing revenue from all non-member firms. We below study the effect of a (local) variation of \(Q_{12}\) on each of the two sources of profit.

- Effect on the downstream market profit

Starting with the effect on the coalition’s downstream market profit \(\pi_{12}^D(Q_{12}, r)\), we have

\[
\frac{\partial \pi_{12}^D}{\partial Q_{12}}(Q_{12}, r) = P'(Q_{12} + BR_{-12}(Q_{12})) Q_{12} BR_{-12}(Q_{12}) + P'(Q_{12} + BR_{-12}(Q_{12})) Q_{12}(r) \left[ P(Q_{12} + BR_{-12}(Q_{12})) - (c + (N - 1) r) \right]
\]

which, when evaluated at \(Q_{12}^*(r, r)\), is given by

\[
\frac{\partial \pi_{12}^D}{\partial Q_{12}}(Q_{12}^*(r, r), r) = P'(Q^*(r, r)) Q_{12}^*(r, r) BR_{-12}^*(Q_{12}^*(r, r)) + P'(Q^*(r, r)) Q_{12}^*(r, r) \left[ P(Q^*(r, r)) - (c + (N - 1) r) \right].
\]

---

\(^7\)Defining the downstream market profit with respect to the individual marginal cost greatly facilitates our analysis since we can use each firm’s first order condition at the Cournot competition stage (see (9)).
The term $P' (Q^*(r, r)) Q_{12}^*(r, r) BR'_{12}(Q_{12}^*(r, r)) > 0$ in (8) captures the (usual) *Stackelberg effect*: the leader has an incentive to increase its output $Q_{12}$ above the Cournot level $Q_{12}^*(r, r)$ because such an increase will be met with a decrease in the aggregate output of the followers (one can easily check that $BR'_{12}(Q_{12}^*(r, r)) < 0$). It will turn out to be useful to rewrite this term as $-2 [P (Q^*(r, r)) - (\zeta + (N - 1) r)] BR'_{12}(Q_{12})$, which is obtained from the F.O.C. of firm $i$ (with $i = 1, 2$) in the Cournot game:

$$P' (Q^*(r, r)) \frac{Q_{12}^*(r, r)}{2} + P (Q^*(r, r)) - (\zeta + (N - 1) r) = 0.$$  

(9)

The term $P' (Q^*(r, r)) Q_{12}^*(r, r) + [P (Q^*(r, r)) - (\zeta + (N - 1) r)]$ in (8) represents the marginal downstream profit of the coalition in a setting where it would play a simultaneous quantity-setting game. This term captures a *coordination effect*: the coalition has an incentive to reduce output below the Cournot level $Q_{12}^*(r, r)$ since the joint output of the coalition when each member chooses its quantity in a non-cooperative way is too high with respect to what maximizes its joint downstream profit (in a simultaneous quantity-setting game). Indeed, using again (9), we have:

$$P' (Q^*(r, r)) Q_{12}^*(r, r) + [P (Q^*(r, r)) - (\zeta + (N - 1) r)] = - [P (Q^*(r, r)) - (\zeta + (N - 1) r)] < 0.$$  

Therefore, the overall marginal effect of a local increase of $Q_{12}$ (above the Cournot level $Q_{12}^*(r, r)$) on the coalition’s downstream profit is given by:

$$\frac{\partial \pi^D}{\partial Q_{12}} (Q_{12}(r, r), r) = - [P (Q^*(r, r)) - (\zeta + (N - 1) r)] [1 + 2BR'_{12}(Q_{12}^*(r, r))] .$$  

(10)

The first term between brackets in (10) is positive while the sign of the second term between brackets in (10) is ambiguous. If the aggregate response of the coalition’s rival is relatively strong such that $1 + 2BR'_{12}(Q_{12}^*(r, r)) < 0$, then the Stackelberg effect dominates the coordination effect and the coalition has an incentive to increase its quantity above $Q_{12}^*(r, r)$, i.e. $\frac{\partial \pi^D}{\partial Q_{12}} (Q_{12}^*(r, r), r) > 0$. In contrast, if the aggregate response of the coalition’s rival is relatively weak such that $1 + 2BR'_{12}(Q_{12}^*(r, r)) < 0$, then the coordination effect dominates the Stackelberg effect and the coalition has an incentive to reduce its quantity below $Q_{12}^*(r, r)$, i.e. $\frac{\partial \pi^D}{\partial Q_{12}} (Q_{12}^*(r, r), r) < 0$.

- Effect on the upstream market profit

Let us now turn to the effect of a local variation in $Q_{12}$ on the coalition’s upstream
market profit $\pi'_{12}(Q_{12})$. We have:

$$
\frac{\partial \pi'_{12}}{\partial Q_{12}}(Q_{12}(r, r), r) = r \left( 1 + 2BR'_{-12}(Q_{12}(r, r)) \right).
$$

(11)

One the one hand, a marginal increase in $Q_{12}$ results in a larger royalty saving. On the other hand, a marginal increase in $Q_{12}$ induces the rivals of the coalition to reduce their output by $|BR'_{-12}(Q_{12})|$ and hence results in a reduction of $r |BR'_{-12}(Q_{12})|$ in the licensing revenues that each member of the coalition gets from the rivals. If the Stackelberg effect dominates the coordination effect such that $1 + 2BR'_{-12}(Q_{12}) < 0$ holds, then the reduction in the licensing revenue from rivals is larger than the increase in the royalty saving such that an increase in $Q_{12}$ reduces the upstream profit. The reverse holds if $1 + 2BR'_{-12}(Q_{12}) > 0$.

Even if there is ambiguity about the sign of the effect of an increase in $Q_{12}$ (above the Cournot level $Q_{12}(r, r)$) on each of the coalition’s downstream and upstream profits, what is very interesting to notice in (10) and (11) is that the sign of the effect on the downstream profit is always opposite to the sign of the effect on the upstream profit (whenever $1 + 2BR'_{-12}(Q_{12}(r, r)) \neq 0$). For instance, when the Stackelberg effect dominates the coordination effect (i.e., $1 + 2BR'_{-12}(Q_{12}(r, r)) < 0$), the marginal effect of $Q_{12}$ on the downstream profit gives incentives to increase output, i.e. $\frac{\partial \pi'_{12}}{\partial Q_{12}}(Q_{12}(r, r), r) > 0$, while the marginal effect on the upstream profit yields incentives to reduce output, i.e. $\frac{\partial \pi'_{12}}{\partial Q_{12}}(Q_{12}(r, r), r) < 0$. The reverse holds when the coordination effect dominates the Stackelberg effect.

### 5.3 Incentives to deviate: four cases

From the analysis of the previous subsection, by summing up (10) and (11), the total effect of a marginal increase in $Q_{12}$ on the coalition’s profit can be described in a simple way as:

$$
\frac{\partial \pi_{12}}{\partial Q_{12}}(Q_{12}^*(r, r), r) = [c + Nr - P(Q^*(r, r))] [1 + 2BR'_{-12}(Q_{12}^*(r, r))].
$$

(12)

We can distinguish four cases depending on whether or not the Stackelberg effect dominates the coordination effect and which is more important between the effect of a local deviation on the downstream profit and the one on the upstream profit.

- Stackelberg effect vs. coordination effect
Let us first examine the term $1 + 2BR'_{-12}(Q_{12}^*)(r, r)$ which determines whether or not the Stackelberg effect dominates the coordination effect. The F.O.C. for the maximization program of any firm $i$ ($i = 3, ..., N$), when the coalition produces a given quantity $Q_{12}$, can be written as:

$$P'(Q_{12} + BR_{-12}(Q_{12})) \frac{BR_{-12}(Q_{12})}{N - 2} + P(Q_{12} + BR_{-12}(Q_{12}) - (\zeta + (N - 1)r) = 0.$$ 

Differentiating the latter with respect to $Q_{12}$ (and dropping the argument $(r, r)$) yields

$$BR'_{-12}(Q_{12}) = -\frac{P''(Q)BR_{-12}(Q_{12}) + (N - 2)P'(Q)}{P''(Q)BR_{-12}(Q_{12}) + (N - 1)P'(Q)}.$$ 

This, combined with $P'(Q) < 0$, proves that $1 < BR'_{-12}(Q_{12}) < 0$ (a result that will be useful later), and, when evaluated at $Q_{12} = Q_{12}^*$, yields

$$1 + 2BR'_{-12}(Q_{12}^*) = -\frac{P''(Q^*)BR_{-12}(Q_{12}^*) + (N - 3)P'(Q^*)}{P''(Q^*)BR_{-12}(Q_{12}^*) + (N - 1)P'(Q^*)} = -\frac{N-2}{N} P''(Q^*)Q^* + (N-3)P'(Q^*)$$ 

because $BR_{-12}(Q_{12}^*) = \frac{N-2}{N} Q^*$ as the corresponding equilibrium is symmetric. Distinguishing between the two scenarios $P''(Q^*) \leq 0$ and $P''(Q^*) > 0$ and using the fact that $BR_{-12}(Q_{12}^*) \leq Q^*$, one can easily show that A3 implies that the denominator is always negative. Therefore, from $P'(Q^*) < 0$, it follows that

$$1 + 2BR'_{-12}(Q_{12}^*) \leq 0 \iff \frac{Q^*P''(Q^*)}{P'(Q^*)} \geq -\frac{N(N - 3)}{N - 2}. \quad (13)$$

This shows that the coordination effect dominates the Stackelberg effect ($1 + 2BR'_{-12}(Q_{12}^*) > 0$) if and only if the slope of the inverse demand is positive ($P''(Q^*) > 0$) and sufficiently elastic ($\frac{Q^*P''(Q^*)}{P'(Q^*)} < -\frac{N(N - 3)}{N - 2}$). This result follows from the fact that a local increase in $Q_{12}$ affects the marginal revenue $P'(Q) q_i + P(Q)$ of a firm $i$ ($i = 3, ..., N$) outside the coalition by affecting the inverse demand $P(Q)$ and its slope $P'(Q)$. If the magnitude of the effect on $P'(Q)$, which is proportional to $P''(Q)$, is high relative to the magnitude of the effect on $P'(Q)$, which is proportional to $P''(Q)$, then the magnitude of the adjustment that each rival of the coalition has to make to equalize its marginal revenue to its marginal cost will be relatively low. In particular, for $N \geq 4$, under A3, the Stackelberg effect always dominates the coordination effect.
• Downstream profit vs. upstream profit

Let us now examine the term $f(r, N) = c + Nr - P(Q^*(r, r))$. The effect of an increase in $Q_{12}$ on the downstream profit is more important than the effect on the upstream profit if $f(r, N) < 0$. For instance, when $r = 0$, there is no upstream profit and we have $f(0, N) = c - P(Q^*(0, 0)) < 0$. Intuitively, we expect that the upstream profit becomes more important as $r$ increases, which turns out to be true as we below show that $\frac{\partial f}{\partial r}(r, N) = N - \frac{\partial Q^*}{\partial r} P'(Q^*) > 0$. Adding the F.O.C.s for all firms’ maximization program, when each of them produces at the effective marginal cost $c + (N - 1)r$, yields

$$P'(Q^*) Q^* + NP(Q^*) - N(c + (N - 1)r) = 0.$$ 

Differentiating the latter with respect to $r$ leads to

$$\frac{\partial Q^*}{\partial r} [P'(Q^*) + P''(Q^*) Q^*] + N \left[ P'(Q^*) \frac{\partial Q^*}{\partial r} - (N - 1) \right] = 0.$$ 

From $P'(Q^*) + P''(Q^*) Q^* < 0$ (by A3) and $\frac{\partial Q^*}{\partial r} < 0$, it follows that $P'(Q^*) \frac{\partial Q^*}{\partial r} - (N - 1) < 0$, which implies that $\frac{\partial f}{\partial r}(r; N) > 0$ for any $N \geq 3$. Since $f(r, N)$ strictly increases with $r$, we expect that there exists a unique $r > 0$ at which $c + Nr - P(Q^*(r, r)) = 0$.

Surprisingly, it turns out that the unique $r$ at which $f(r, N) = 0$ is $r_m$ in (3). At $r = r_m$, we have that $P(Q^*(r_m, r_m)) = P_m$ and, therefore, $c + Nr_m - P(Q^*(r_m, r_m)) = 0$. Thus, for $r < r_m$, the downstream profit is more important than the upstream one and the reverse holds for $r > r_m$.

5.4 Bilaterally efficient royalties

From the previous analysis of local deviations, we know that there are four possible cases depending on which between the Stackelberg effect and the coordination effect is stronger and which between the downstream profit effect and the upstream profit effect is stronger. Consider first the case in which the Stackelberg effect dominates the coordination effect. Then, if the downstream profit effect is more important than the upstream profit effect (i.e. $r$ belongs to $[0, r_m)$), the coalition has an incentive to decrease its royalty in order to reduce the rivals’ outputs, which generates $r = 0$ as the unique potential bilateral efficient royalty among royalties $r$ in $[0, r_m)$. If the upstream profit effect is more important than the downstream profit effect (i.e. $r > r_m$), the coalition has an incentive to increase its royalty to boost the rivals’ production and thereby the licensing revenue from the
rivals. At \( r = r_m \), the downstream profit effect is equal to the upstream profit effect and the coalition has no incentive to deviate at least locally. In summary, when the Stackelberg effect dominates the coordination effect, there are two potential bilaterally efficient royalties: \( r = 0 \) and \( r = r_m \).

Now let us consider the case in which the coordination effect dominates the Stackelberg effect. Then, if the downstream profit effect is more important than the upstream profit effect (i.e. \( r \) belongs to \((0, r_m)\)), the coalition has an incentive to increase its royalty to make the retail price as close as possible to the monopoly price. In a symmetric way, if the upstream profit effect is more important than the downstream profit effect (i.e. \( r > r_m \)), the coalition has an incentive to decrease its royalty to make the retail price as close as possible to the monopoly price. In summary, when the coordination effect dominates the Stackelberg effect, we have a unique potential bilaterally efficient royalty: \( r = r_m \). Therefore, we have:

**Lemma 3** Under A1-A3, from the local analysis of the Stackelberg game \( G(r, N) \), we obtain the following candidates for the bilaterally efficient symmetric royalties:

(i) If the Stackelberg effect dominates the coordination effect \( \left( \frac{QP''(Q)}{P'(Q)} > -\frac{N(N-3)}{N-2} \right) \), then there are two candidate royalties, \( r = 0 \) and \( r = r_m \), for which \( S(r, N) \) can be bilaterally efficient.

(ii) If the coordination effect dominates the Stackelberg effect \( \left( \frac{QP''(Q)}{P'(Q)} < -\frac{N(N-3)}{N-2} \right) \), then \( r = r_m \) is the only possible value of \( r \) for which \( S(r, N) \) can be bilaterally efficient.

In order to determine the values of \( r \) for which \( S(r, N) \) is indeed bilaterally efficient, we need to switch from a local analysis (which provides necessary conditions for \( S(r, N) \) to be bilaterally efficient) to a global analysis (which confirms or denies that the potential candidates are bilaterally efficient). The following proposition characterizes the bilaterally efficient symmetric agreements depending on whether or not the Stackelberg effect dominates the coordination effect.

**Proposition 2** (public cross-licensing) Under A1-A3, the following holds:

(i) If the Stackelberg effect dominates the coordination effect \( \left( \frac{QP''(Q)}{P'(Q)} > -\frac{N(N-3)}{N-2} \right) \), then \( S(r, N) \) is bilaterally efficient if and only if \( r \in \{0, r_m\} \).

(ii) If the coordination effect dominates the Stackelberg effect \( \left( \frac{QP''(Q)}{P'(Q)} < -\frac{N(N-3)}{N-2} \right) \), then \( S(r, N) \) is bilaterally efficient if and only if \( r = r_m \).

**Proof.** See Appendix.
This proposition shows that the monopoly royalty is the Pareto-dominant bilaterally efficient royalty regardless of whether the Stackelberg effect dominates or is dominated by the coordination effect.

6 Private cross-licensing

In the previous section, we considered public cross-licensing agreements. In this section, we consider the alternative scenario of private cross-licensing: each bilateral cross-licensing agreement is only observable to the two firms involved in it. Hence, at the beginning of stage 2, firm $i$ is aware of only the royalties of the licensing contracts that it itself signed. Regarding the royalties agreed on between firm $j(\neq i)$ and $k(\neq i)$, $i$ should form an expectation. In a symmetric equilibrium in which every pair of firms agree on the same royalty $r$, every firm should believe that every pair of firms agreed on $r$ on the equilibrium path. We here make an assumption of passive beliefs: regardless of the terms of cross-licensing signed between $i$ and $j$ in the licensing stage, $i$ maintains the same belief about the terms signed by any pair of rivals and believes that every pair of rivals agreed on $r$.

Then, we can show that Lemma 1 continues to hold in the case of private cross-licensing and hence, without loss of generality, we can restrict a coalition to deviate with a symmetric royalty. Then, a result similar to Lemma 2 holds in that the symmetric set of cross-licensing agreements $S(r,N)$ is bilaterally efficient if and only if choosing to produce $Q_{12}^*(r,r)$ is optimal to the coalition $\{1,2\}$. The major difference between public cross-licensing and private cross-licensing is that in the latter case, there is no Stackelberg effect: formally speaking, the analysis under private cross-licensing can be derived from that of public cross-licensing by setting $BR'_{-12}(Q_{12})$ to zero. This implies that $S(r^m,N)$ is the unique bilaterally efficient set of symmetric agreements.

**Proposition 3 (private cross-licensing)** Under $A1$-$A3$, in the two-stage game of private cross-licensing followed by Cournot competition, $S(r^m,N)$ is the unique bilaterally efficient set of symmetric cross-licensing agreements.

Therefore, private cross-licensing strengthens the finding under public cross-licensing in Proposition 2 since the monopoly outcome is the unique symmetric outcome under private cross-licensing.
7 Robustness to coalitions of any size

In this section, we show that the main results previously obtained by considering a coalition of size 2 extend to a coalition of any given size \( k \) (with \( 3 \leq k \leq N \)). We will say that a set of cross-licensing agreements is \( k \)-efficient if no coalition of size \( k \) finds it optimal to change the terms of the cross-licensing agreements between the members of the coalition.

Since the case of \( k = N \) was studied in the benchmark of multilateral licensing by all firms in the industry, we study a coalition of size \( k \) (with \( 3 \leq k \leq N - 1 \)), first in the case of public cross-licensing and then in the case of private cross-licensing.

Suppose public cross-licensing. Consider the deviation of a coalition composed of \( \{1, \ldots, k\} \) in the licensing stage. Lemma 1 continues to hold in the case of coalition of size \( k \) and hence, without loss of generality, we can restrict attention to deviations involving a symmetric royalty \( \hat{r} \). For given \( (r, \hat{r}) \), let \( Q^*(r, \hat{r}) \) denote the total industry output and \( Q^*_k(r, \hat{r}) \) denote the sum of the outputs of the firms in the coalition in the (second-stage) equilibrium of Cournot competition. Let \( Q^*_{-k}(r, \hat{r}) \equiv Q^*(r, \hat{r}) - Q^*_k(r, \hat{r}) \) denote the total equilibrium output of the coalition’s rivals.

Denoting \( Q_k \) the total quantity produced by the considered coalition and \( r \) the common royalty paid to the firms outside the coalition, the coalition’s profit can be rewritten as

\[
\pi_k(Q_k, r) = \frac{P(Q_k + BR_{-k}(Q_k)) - (c + (N - 1) r)}{\pi^D_k(Q_k, r)} Q_k + r \frac{(k - 1)Q_k + kBR_{-k}(Q_k)}{\pi^U_k(Q_k, r)}.
\]

Equation (14) generalizes (6). Suppose that the coalition marginally expands or contracts its output \( Q_k \) with respect to \( Q^*_k(r, r) \). Then, we have

\[
\frac{\partial \pi_k^D}{\partial Q_k}(Q^*_k(r, r), r) = -\left[ P\left(Q^*(r, r)\right) - \left(c + (N - 1) r\right) \right] \frac{\partial Q_k}{\partial (Q_k, r)} Q_k + r \left( k - 1 + kBR'_{-k}(Q^*_k(r, r)) \right).
\]

\[
\frac{\partial \pi_k^U}{\partial Q_k}(Q^*_k(r, r), r) = r \left( k - 1 + kBR'_{-k}(Q^*_k(r, r)) \right).
\]

Summing up the two terms leads to

\[
\frac{\partial \pi_k}{\partial Q_k}(Q^*_k(r, r), r) = \left[c + Nr - P\left(Q^*(r, r)\right)\right] \frac{\partial Q_k}{\partial (Q_k, r)} Q_k + r \left( k - 1 + kBR'_{-k}(Q^*_k(r, r)) \right).
\]

Equation (15) generalizes (12). In particular, the first bracket term is the same in both equations and does not depend on \( k \) while the second bracket term in (15) depends
The Stackelberg effect dominates the coordination effect if and only if \( k - 1 + kBR_{-k}(Q^*_k(r,r)) < 0 \). We have

\[
    k - 1 + kBR_{-k}(Q^*_k(r,r)) \leq 0 \text{ iff } \frac{QP''(Q)}{P'(Q)} \leq -\frac{N(N-2k+1)}{N-k}.
\]

The important point is that at \( r = r^m \), the first bracket term is zero regardless of the coalition size: \( c + Nr^m - P(Q^*(r^m, r^m)) = 0 \). Therefore, we have the following result.

**Theorem 1** Assume that A1-A3 hold. In the two-stage game of public cross-licensing followed by Cournot competition;

(i) If the Stackelberg effect dominates the coordination effect \( \frac{QP''(Q)}{P'(Q)} > -\frac{N(N-2k+1)}{N-k} \), then \( S(r^m, N) \) is \( k \)-efficient if and only if \( r \in \{0, r^m\} \).

(ii) If the coordination effect dominates the Stackelberg effect \( \frac{QP''(Q)}{P'(Q)} < -\frac{N(N-2k+1)}{N-k} \), then \( S(r^m, N) \) is \( k \)-efficient if and only if \( r = r^m \).

Theorem 1 generalizes Proposition 2 to any given size of coalition. In addition, this theorem implies that even if we allow for coalitions of different sizes (for instance, firm 1 signs a licensing agreement with firm 2 and at the same time signs an agreement with firms \{3, 4\}), the monopoly outcome survives.

The theorem also implies that Proposition 3 of private cross-licensing generalizes to coalitions of any size since private cross-licensing is a particular case of public cross-licensing in which the Stackelberg effect is absent (i.e. \( BR_{-k} = 0 \)).

**Theorem 2** Assume that A1-A3 hold. In the two-stage game of private cross-licensing followed by Cournot competition, \( S(r^m, N) \) is the unique \( k \)-efficient set of symmetric agreements.

We below provide an intuition of our results by considering private cross-licensing. Given \( r = r^m \), the F.O.C. with respect to \( q_i \) for a single firm \( i \) is given by:

\[
P'(Q^m)\frac{Q^m}{N} + P(Q^m) = c + (N-1)r^m.
\]

The equality (16) shows that \( r^m \) allows to achieve the monopoly outcome when no firm can build a coalition with any other firms to make joint deviation. Suppose now that two firms \((i, j)\) can make a joint deviation: by agreeing on some royalties \((r_{i-j}, r_{j-i})\), they
can choose a joint output $q_i + q_j$ different from $2Q^m/N$. However, it turns out that the F.O.C. of the coalition is satisfied exactly at $q_i + q_j = 2Q^m/N$:

$$P'(Q^m) \frac{2Q^m}{N} + P(Q^m) = c + (N - 2)r^m. \quad (17)$$

The equality (17) can be equivalently written as

$$P'(Q^m) \frac{Q^m}{N} + P'(Q^m) \frac{Q^m}{N} + P(Q^m) = [c + (N - 1)r^m] - r^m. \quad (18)$$

Comparing (16) with (18) shows that the coalition has a smaller gain from output expansion than a single firm since it internalizes the effect on price reduction by $P'(Q^m) \frac{Q^m}{N}$ more than a single firm. Comparing (16) with (18) also shows that the coalition enjoys a reduction in marginal cost of $r^m$ with respect to a single firm. Substracting (16) from (18) reveals that the two opposite effects exactly cancel out:

$$P'(Q^m) \frac{Q^m}{N} = -r^m.$$

For this reason, coalition of any $k$ size has no incentive to deviate from producing $kQ^m/N$.

In the case of public cross-licensing, there is the Stackelberg effect which affects both the downstream profit and the upstream licensing revenue. For instance, if a coalition increases its production by reducing its royalty, its rivals reduce their output, which increases the coalition’s downstream profit but decreases its licensing revenue. It turns out that at $r^m$, the two opposite effects cancel out for any size $k$ of coalition.

8 Policy implications

The previous analysis reveals that market forces induce firms to sign cross-licensing agreements allowing them to achieve the monopoly outcome regardless of the size of coalitions and the information structure (i.e. whether the licensing terms are public or private). In this section, we derive two different policy implications. First, we interpret our results in terms of desirability (or undesirability) of cross-licensing of complementary patents (substitutable patents). Second, we discuss the policy of granting antitrust exemptions to bilateral cross-licensing agreements.

In our model, $\Delta c \equiv \tau - \zeta$ has some close connection with whether patents are close substitutes or complements. As patents become more complementary (respectively, more
substitutable), \( \Delta c \) would be larger (respectively, smaller). For instance, let \( \delta \equiv c(1) - c(2) \) be the cost reduction from the first extra patent. If the patents are neither substitutes nor complements, we have \( c(N) = \bar{c} - \delta(N - 1) \), which is equivalent to \( \Delta c = \delta(N - 1) \). If they are substitutes, the marginal cost reduction from extra patent should decrease such that we have \( c(N) > \bar{c} - \delta(N - 1) \) (i.e. \( \Delta c < \delta(N - 1) \)); if they are complements, the marginal cost reduction from extra patent should increase that we have \( c(N) < \bar{c} - \delta(N - 1) \) (i.e. \( \Delta c > \delta(N - 1) \)). Applying this interpretation to the results of Proposition 1, Theorem 1 and Theorem 2 leads to:

**Proposition 4** (i) When patents are relatively substitutes (i.e. \( \Delta c < (N-1) (P(Q^*(N, \bar{c})) - \bar{c}) \)), cross-licensing leads to a decrease in consumer surplus.

(ii) When patents are relatively complements (i.e. \( \Delta c > (N-1) (P(Q^*(N, \bar{c})) - \bar{c}) \)), cross-licensing leads to an increase in consumer surplus.

Therefore, absent any other policy remedies, a consumer-surplus-maximizing competition authority should allow cross-licensing only if patents are complementary enough.

We now discuss the antitrust exemptions granted to bilateral licensing agreements signed by competitors whose combined market share is smaller than a certain threshold. Our results show that such policy can actually allow the firms in a given industry to implement the fully cooperative outcome (i.e. \( r = r^m \)) regardless of whether the licensing agreements are public or private. The exception can occur when licensing agreements are public. In this case, bilateral cross-licensing may lead to the most competitive outcome (i.e. zero royalty) only if the Stackelberg effect dominates the coordination effect.

If the antitrust authority prefers to implement a lower retail price than the one in the fully cooperative outcome, they should enforce some upper bound on royalty such that firms cannot use royalties that are disproportionate with respect to the market value of the license. As long as this upper bound is smaller than \( r^m \), this will be the bilaterally efficient royalty when licensing agreements are private. If licensing agreements are public, the upper bound is bilaterally efficient only if the coordination effect dominates the Stackelberg effect; otherwise, the enforcement of such an upper bound makes zero royalty the unique bilaterally efficient outcome. In addition, Theorem 1 shows that for given number of competing firms in an industry, the Stackelberg effect is more likely to dominate the coordination effect as the size of coalition (i.e. the number of firms signing together a cross-licensing agreement) is smaller. Therefore, granting exemptions to bilateral agreements makes some sense; actually, in our model of Cournot competition, if there are at least four competing firms and the coalition size is limited to two, the Stackelberg effect
always dominates the coordination effect. For instance, if there are ten symmetric firms, any two firms’ combined market share is equal to the twenty percent threshold specified in Article 3 of EC Technology Transfer Block Exemption Regulation. Hence, we expect that the Stackelberg effect is likely to dominate the coordination effect in an industry in which firms can benefit from such exemptions.

Therefore, the EC’s granting of antitrust exemptions to bilateral licensing agreements below a certain threshold of combined market share can be justified only if the following conditions are met: (i) zero royalty is socially preferred to \( r^m \) (ii) licensing agreements are public (iii) firms are NOT exempted from using royalties disproportionate with respect to the market value of the license and \( r^m \) is one of such disproportionate royalties.

9 Concluding remarks

This paper is a first step toward understanding bilateral cross-licensing agreements among \( N \geq 3 \) competing firms. We plan to address several extensions in future research. There are two natural extensions to check the robustness of our results. One is to consider Bertrand competition instead of Cournot competition. The other is to consider asymmetric firms (asymmetric costs, asymmetric patent portfolios and asymmetric benefit from having access to rivals’ patents).

After checking the robustness, we can extend our setting to study some other related policy issues. First, we can introduce, in addition to incumbent firms, entrants with no (or weak) patent portfolios. This would allow us to study whether cross-licensing can be used to raise barriers to entry (Hall et al., 2012, U.S. DOJ and FTC, 2007). Second, we can include in the set of players non-practicing entities (NPEs) which do not compete in the downstream market. This would allow us to study the conditions under which NPEs weaken competition and (when these conditions are met) to isolate the anticompetitive effects generated by NPEs from the effects resulting from cross-licensing in the absence of NPEs. The issue of how NPEs affect competition and innovation is of substantial current interest to policy makers (Morton, 2012). Note that NPEs and entrants involve completely opposite asymmetries. The former are present in the upstream market of patent licensing and are absent in the downstream (product) market while the second are absent (or have very weak presence) in the upstream market and are present in the downstream market.
10 Appendix

Proof of Lemma 1

Assume, without loss of generality, that \((i, j) = (1, 2)\). The joint payoff firms 1 and 2 derive from a deviation to a licensing agreement involving the payment of \((r_{1-2}, r_{2-1})\) is:

\[
\begin{align*}
\pi_1^* + \pi_2^* &= \left[ P(Q^*) - \zeta - r_{1-2} - (N - 2) r \right] q_1^* + r_{2-1} q_2^* + r Q_{-12}^* \\
&\quad + \left[ P(Q^*) - \zeta - r_{2-1} - (N - 2) r \right] q_2^* + r_{1-2} q_1^* + r Q_{-12}^* \\
&= \left[ P(Q^*) - \zeta - r \right] (q_1^* + q_2^*) + 2 r Q_{-12}^*
\end{align*}
\]

which can be rewritten as

\[
\pi_1^* + \pi_2^* = \left[ P(Q^*) - \zeta - (N - 2) r \right] (Q^* - Q_{-12}^*) + 2 r Q_{-12}^* \tag{19}
\]

Denoting \(c_i\) the marginal cost of firm \(i\) (including the royalties paid to the other firms), the F.O.C. for firm \(i\)'s maximization program is:

\[ P(Q^*) - c_i + q_i^* P'(Q^*) = 0 \]

Summing the F.O.C.s for \(i = 1, 2, \ldots, N\) yields:

\[ NP(Q^*) - \sum_{i \geq 1} c_i + Q^* P'(Q^*) = 0 \]

which shows that \(Q^*\) depends on \((c_1, c_2, \ldots, c_N)\) only through \(\sum_{i \geq 1} c_i\). Moreover, summing the F.O.C.s for \(i = 3, \ldots, N\) yields

\[ (N - 2) P(Q^*) - \sum_{i \geq 3} c_i + Q_{-12}^* P'(Q^*) = 0 \]

which implies that \(Q_{-12}^*\) depends on \((c_1, c_2, c_3, \ldots, c_N)\) only through \(\sum_{i \geq 1} c_i\) and \(\sum_{i \geq 3} c_i\). From \((c_1, c_2, c_3, \ldots, c_N) = (\zeta + r_{1-2} + (N - 2) r, \zeta + r_{2-1} + (N - 2) r, \zeta + (N - 1) r, \ldots, \zeta + (N - 1) r)\) it then follows that both \(Q^*\) and \(Q_{-12}^*\) depend on \((r_{1-2}, r_{2-1})\) only through \(r_{1-2} + r_{2-1}\), which, combined with (19), implies that \(\pi_1^* + \pi_2^*\) depends on \((r_{1-2}, r_{2-1})\) only through \(r_{1-2} + r_{2-1}\).

Proof of Lemma 2
Since $Q_{12}^*(r, \hat{r})$ is strictly decreasing and continuous in $\hat{r}$ then $r \in \text{Arg}\max_{r \in [0, \delta]} \pi_1^* + \pi_2^*$ if and only if:

$$Q_{12}^*(r, r) \in \text{Arg}\max_{Q_{12} \in [Q_{12}(r, r), Q_{12}(r, 0)]} \pi_2 (Q_{12}) \equiv [P (Q_{12} + BR_{-12} (Q_{12})) - (\zeta + (N - 2) r)] Q_{12} + 2r BR_{-12} (Q_{12})$$

which means that $Q_{12}^*(r, r)$ is a subgame-perfect equilibrium strategy of the coalition \{1, 2\} in the game $G(r, N)$.

**Proof of Proposition 2**

Let us first show some general preliminary results which will be useful for the subsequent specific analysis of the four considered scenarios. We have:

$$\frac{\partial \pi_{12}}{\partial Q_{12}} (Q_{12}, r) = P' (Q_{12} + BR_{-12} (Q_{12})) Q_{12} (1 + BR_{-12} (Q_{12})) + P (Q_{12} + BR_{-12} (Q_{12})) Q_{12} + (\zeta + (N - 1) r) + (1 + 2 BR'_{-12} (Q_{12}))$$

$$= [\zeta + N r - P (Q_{12} + BR_{-12} (Q_{12}))] [1 + 2 BR'_{-12} (Q_{12})] + 2 \left[ P' (Q_{12} + BR_{-12} (Q_{12})) \frac{Q_{12}}{2} + P (Q_{12} + BR_{-12} (Q_{12})) - (\zeta + (N - 1) r) \right] [1 + BR'_{-12} (Q_{12})]$$

Let us show that $J(Q_{12}, r)$ is decreasing in $Q_{12}$. We have

$$\frac{\partial J(Q_{12}, r)}{\partial Q_{12}} = \left[ P'' (Q) \frac{Q_{12}}{2} + P' (Q) \right] \frac{(1 + BR'_{-12} (Q_{12}))}{>0} + \frac{P' (Q)}{<0}$$

Since $P'' (Q) \frac{Q_{12}}{2} + P' (Q) < \max \left[ P'' (Q) Q + P' (Q) , P' (Q) \right] < 0$ then $J(Q_{12}, r)$ is decreasing in $Q_{12}$. This, combined with the fact that the F.O.C. for each firm $i = 1, 2$, satisfied at the symmetric Cournot equilibrium equilibrium, is given by $J(Q_{12}^* (r, r), r) = 0$, yields that

$$J(Q_{12}, r) \leq 0 \iff Q_{12} \geq Q_{12}^* (r, r)$$

(20)

Since $1 + BR'_{-12} (Q_{12}) > 0$, it follows that

$$D(Q_{12}, r) \leq 0 \iff Q_{12} \geq Q_{12}^* (r, r)$$

(21)
Moreover, from
\[ BR_{-12}'(Q_{12}) = -\frac{P''(Q)BR_{-12}(Q_{12}) + (N-2)P'(Q)}{P''(Q)BR_{-12}(Q_{12}) + (N-1)P'(Q)} \]
it follows that
\[ 1 + 2BR_{-12}'(Q_{12}) = -\frac{P''(Q)BR_{-12}(Q_{12}) + (N-3)P'(Q)}{P''(Q)BR_{-12}(Q_{12}) + (N-1)P'(Q)} \]  \( (22) \)
which can be rewritten as
\[ 1 + 2BR_{-12}'(Q_{12}) = -\frac{\frac{N-2}{N}P'(Q)}{P''(Q)BR_{-12}(Q_{12}) + (N-1)P'(Q)} \left[ QP''(Q) \left( \frac{N}{N-2} \frac{BR_{-12}(Q_{12})}{Q} \right) + \frac{N(N-3)}{N-2} \right] \]  \( (23) \)

Since \( P''(Q)BR_{-12}(Q_{12}) + (N-1)P'(Q) \leq \max (\{(N-1)P'(Q), P''(Q)Q + (N-1)P'(Q)\}) < 0 \), it follows that
\[ 1 + 2BR_{-12}'(Q_{12}) \geq 0 \iff \frac{QP''(Q)}{P'(Q)} \geq -\frac{N(N-3)}{N-2} \left[ \frac{N-2}{N} \frac{Q}{BR_{-12}(Q_{12})} \right] \]  \( (23) \)
(for any \( Q_{12} \) such that \( BR_{-12}(Q_{12}) \neq 0 \)). From the fact that \( BR_{-12}(Q_{12}) - \frac{N-2}{N}Q = BR_{-12}(Q_{12}) - \frac{N-2}{N}(Q_{12} + BR_{-12}(Q_{12})) \) is decreasing in \( Q_{12} \) and \( BR_{-12}(Q_{12}^* (r, r)) = \frac{N-2}{N}Q^* (r, r) \) (by symmetry of the considered Cournot equilibrium), it follows that
\[ \frac{N-2}{N} \frac{Q}{BR_{-12}(Q_{12})} \geq 1 \iff Q_{12} \geq Q_{12}^* (r, r) \]

In particular we obtain the following result which will be useful for the next steps of the proof: If \( \frac{QP''(Q)}{P'(Q)} > -\frac{N(N-3)}{N-2} \) (for any \( Q \) such that \( P'(Q) \neq 0 \)) then \( 1 + 2BR_{-12}'(Q_{12}) < 0 \) for any \( Q_{12} \geq Q_{12}^* (r, r) \).

- Let us now show that \( r = r^m \) is bilaterally efficient regardless of whether the Stackelberg effect dominates or is dominated by the coordination effect.

\[
\frac{\partial \pi_{12}}{\partial Q_{12}}(Q_{12}, r^m) = P'(Q_{12} + BR_{-12}(Q_{12})) Q_{12} (1 + BR_{-12}'(Q_{12})) + \]
\[ [P(Q_{12} + BR_{-12}(Q_{12})) - (c + (N-1)r^m)] + r^m (1 + 2BR_{-12}'(Q_{12})) \]
\[ = 2 \left( P'(Q_{12} + BR_{-12}(Q_{12})) \frac{Q_{12}}{2} + r^m \right) (1 + BR_{-12}'(Q_{12})) + \]
\[ [P(Q_{12} + BR_{-12}(Q_{12})) - (c + Nr^m)] \]
Since \( \zeta + N r^m - P (Q^*(r^m, r^m)) = 0 \) then

\[
P (Q_{12} + BR_{-12}(Q_{12})) - (\zeta + N r^m) = P (Q_{12} + BR_{-12}(Q_{12})) - P (Q_{12}^*(r^m, r^m) + BR_{-12}(Q_{12}^*(r^m, r^m)))
\]

Moreover, combining \( \zeta + N r^m - P (Q^*(r^m, r^m)) = 0 \) with the F.O.C.

\[
P' (Q_{12}^*(r^m, r^m) + BR_{-12}(Q_{12}^*(r^m, r^m))) \frac{Q_{12}^*(r^m, r^m)}{2} + P (Q_{12}^*(r^m, r^m) + BR_{-12}(Q_{12}^*(r^m, r^m))) - \zeta - (N - 1) r^m = 0
\]
yields

\[
P' (Q_{12} + BR_{-12}(Q_{12})) \frac{Q_{12}}{2} + r^m = P' (Q_{12} + BR_{-12}(Q_{12})) \frac{Q_{12}}{2} - P' (Q_{12}^*(r^m, r^m) + BR_{-12}(Q_{12}^*(r^m, r^m))) \frac{Q_{12}^*(r^m, r^m)}{2}
\]

Therefore,

\[
\frac{\partial \pi_{12}}{\partial Q_{12}} (Q_{12}, r^m) = 2 \left[ P' (Q_{12} + BR_{-12}(Q_{12})) \frac{Q_{12}}{2} - P' (Q_{12}^*(r^m, r^m) + BR_{-12}(Q_{12}^*(r^m, r^m))) \frac{Q_{12}^*(r^m, r^m)}{2} \right]
\]

Using the fact that \( P'(Q) < 0 \) and \( 1 + BR'_{-12}(Q_{12}) > 0 \), it is straightforward to show that both functions \( P' (Q_{12} + BR_{-12}(Q_{12})) \frac{Q_{12}}{2} \) and \( P (Q_{12} + BR_{-12}(Q_{12})) \) are decreasing in \( Q_{12} \), which implies that

\[
\frac{\partial \pi_{12}}{\partial Q_{12}} (Q_{12}, r^m) \geq 0 \iff Q_{12} \leq Q_{12}^*(r^m, r^m)
\]

Therefore, \( Q_{12}^*(r^m, r^m) \) maximizes \( \pi_{12} (Q_{12}, r^m) \) over \([Q_{12}^*(r^m, r^m), Q_{12}^*(r^m, 0)]\), which is equivalent to the fact that \( S(r^m, N) \) is bilaterally efficient.

- Consider now the case \( \frac{P''(Q)}{P'(Q)} > -\frac{N(N-3)}{N-2} \) and let us show that \( r = 0 \) is bilaterally efficient, that is \( Q'_{12}^*(0, 0) \) maximizes \( \pi_{12} (Q_{12}, 0) \) over \([Q_{12}^*(0, \infty), Q_{12}^*(0, 0)]\). We have

\[
\frac{\partial \pi_{12}}{\partial Q_{12}} (Q_{12}, 0) = (\zeta - P (Q_{12} + BR_{-12}(Q_{12}))) (1 + 2BR'_{-12}(Q_{12})) + D(Q_{12}, 0)
\]

Let us show that \( P''(Q_{12} + BR_{-12}(Q_{12})) BR_{-12} (Q_{12}) + (N - 3)P'(Q_{12} + BR_{-12}(Q_{12})) < 0 \) for any \( Q_{12} \in [Q_{12}^*(0, \infty), Q_{12}^*(0, 0)] \), which, by (22), is sufficient to state that \( 1 + 2BR'_{-12}(Q_{12}) < 0 \) for any \( Q_{12} \in [Q_{12}^*(0, \infty), Q_{12}^*(0, 0)] \). On the one hand, if \( P''(Q_{12} + \ldots \right)
therefore, that two inequalities BR\_{-1,2}(Q_{12}) < 0 then it follows from BR\_{-1,2}(Q_{12}) ≥ 0 and P'(Q) < 0 that 1 + 2BR'_{-1,2}(Q_{12}) < 0. On the other hand, if P''(Q_{12} + BR\_{-1,2}(Q_{12})) ≥ 0 then from BR\_{-1,2}(Q_{12}) ≤ Q, it follows that P''(Q_{12} + BR\_{-1,2}(Q_{12}))BR\_{-1,2}(Q_{12}) + (N - 3)P'(Q_{12} + BR\_{-1,2}(Q_{12})) ≤ P''(Q)Q + (N - 3)P'(Q).

Note first that if P''(Q_{12} + BR\_{-1,2}(Q_{12})) ≥ 0, it must hold that N ≥ 4; otherwise the condition \( \frac{QP''(Q)}{P'(Q)} > -\frac{N(N-3)}{N-2} \) (which is one of the two conditions defining the present scenario) would be violated. This implies that P''(Q)Q + (N - 3)P'(Q) ≤ P''(Q)Q + P'(Q), which combined with A3 yields P''(Q)Q + (N - 3)P'(Q) < 0 and, therefore, that 1 + 2BR'_{-1,2}(Q_{12}) < 0. We are now in position to state that, for any Q_{12} ∈ [Q_{12}'(0, ∞), Q_{12}'(0, 0)], the latter inequality holds independently of whether P''(Q_{12} + BR\_{-1,2}(Q_{12})) < 0 or P''(Q_{12} + BR\_{-1,2}(Q_{12})) ≥ 0. Combining that with the fact that the two inequalities $\zeta - P(Q_{12} + BR_{-1,2}(Q_{12})) ≤ \zeta - P(Q_{12}'(0,0) + BR_{-1,2}(Q_{12}'(0,0))) < 0$ and $D(Q_{12}, 0) ≥ 0$ hold for any $Q_{12} ∈ [Q_{12}'(0, \bar{r}), Q_{12}'(0, 0)]$, we get that $\frac{\partial \pi_{12}}{\partial Q_{12}}(Q_{12}, 0) ≥ 0$ for any $Q_{12} ∈ [Q_{12}'(0, \bar{r}), Q_{12}'(0, 0)]$. This implies that $Q_{12}'(0, 0)$ maximizes $\pi_{12}(Q_{12}, 0)$ over $[Q_{12}'(0, ∞), Q_{12}'(0, 0)]$. Therefore, S(0, N) is bilaterally efficient.

11 References


and Remedies with Competition. Report of the FTC.