How space allocation matters:  
The case of parking places  

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Abstract  

In densely built-up urban areas, road users compete to park their cars. Parking policies are at the heart of local urban plans. In the literature, they are dealt with from an urban economics perspective. The distinctive feature of this article is to consider parking places from the theory of non-renewable and open access resources. We define parking space as a scarce and homogeneous commodity. Through a static micro-based allocation model, we investigate the mechanisms underpinning space allocation. The model estimates the optimal space consumption rates to be set under the various cases identified. Although the model is original, the results are in line with the literature on resources. We show that overlooking the shadow cost associated with the spatial constraint when the resource is allocated sequentially induces pressures towards more parking places and drives down the social welfare.  

Key words: optimal resource allocation, public policies, competition for space, parking places  

JEL: D6, D61, Q28, Q30
1 Introduction

Common resources are prone to conflicts in their allocation. Competition among agents and uses arises when resource scarcity is real or felt. In densely built-up urban areas, road users compete to park their cars. Conflicts for park places are especially intense for on-street parking, considered as more convenient than off-street parking. Parking policies are at the heart of local urban plans. They take various forms, from paid-parking schemes throughout the city to the creation of large parking lots located next to public transportation facilities on the outskirts. Besides being a sensitive political issue challenging each local election, rivalry of use for park places generates various negative impacts. Road users looking for a place drive around longer and therefore increase road congestion and polluting emissions. Moreover, the time they loose in this search is a cost for them and for the society as a whole. Finally, some agents might be unable to find a park place in the city center and see their welfare decrease sharply.

Parking policies are generally treated from an urban economics perspective. In this sense, land is considered as heterogeneous in terms of natural endowments, climate or accessibility (Ricardo, 1817; Von Thünen, 1826). Relative scarcity of space is highlighted through the concept of land rents. Locations close to the central business district (CBD) display higher land rents since the CBD is assumed to be the attractive center where jobs concentrate. Issues related to workers and firms localization are core questions of urban economics. Distances, and especially commuting distances, constitute a key parameter in the analysis. Standard urban economic models in the tradition of Alonso, Muth and Mills assume relative scarcity of land, without taking into account the absolute scarcity of space. Their models imply infinite endowments in land.

Studies related to the various functions the Earth provides reveal the finite nature of the resource land. Worries arise from the finite nature of Earth combined with the expected growth in demand for these functions (Meyfroidt et al., 2010; Godfray et al., 2010; Wirsenius et al., 2010). Needs for food, housing or environment increase with the population. The accounting approach of Lambin and Meyfroidt (2011) summarizes estimations on the topic and provides an explicit insight of absolute land scarcity. Growing needs for agricultural uses (food, raw materials, energy) have become a public concern (World Bank, 2010). Projections for demands in urban uses (housing, transportation) are also on a rising trend, due to changes in demographics, revenues and consumption patterns (Döös, 2002; Alig et al., 2004; Seto et al., 2010). Rising urban land compete with agricultural functions and more importantly with natural functions (ecosystem services, biodiversity) which are often overlooked though their crucial role (Turner et al., 2007). Despite potential land use changes among functions, the scarcity of land is absolute.

In this work, we focus on artificial urban land, and especially public parking places. With intertemporal or long run perspectives, parking spaces can be considered as renewable resources. The stock of parking places is not fixed, and can be increased or decreased through land use changes. However, in the short run, land allocated to parking places is fixed. In this sense, we consider parking spaces as non-renewable resources. Renewable and non-renewable resources have similar features. Any resource is potentially renewable
on the (very) long run, and any renewable resource is exhaustible if the exploitation rate is higher than the regeneration rate. The time horizon and the exploitation pattern allow for a distinction between non-renewable and renewable resources. Moreover, a feature that distinguishes artificial space from other non-renewable resources is its propensity to be consumed without depletion or damage. For this reason, theory of resource allocation at time $t$ can be properly applied to space. This article deals with the spatial management of an exhaustible resource at time $t$. As far as we know, the spatial perspective of resource allocation is not directly treated in environmental economics. The literature on both non-renewable and renewable resources mainly deals with intertemporal issues. Space is usually not the subject of environmental economics, while it is a core concept of geographical and urban economics. However, urban economics rather focus on location issues without explicitly considering the scarcity of space.

Environmental economics provide an abundant literature on several issues related to natural resources. A major concern is the allocation over time of non-renewable resources (Dasgupta and Heal [chapter 6 and 12], 1979; Conrad and Clark [chapter 3], 1987; Solow, 1974). A seminal work from Harold Hotelling (1931) highlights the concept of scarcity rent, a positive rent that results from the scarce nature of the resource. The Hotelling rule states that the price of the resource, and thus the scarcity rent, must grow at the rate of interest on the optimal extraction path. This simple model has then been improved by introducing heterogeneity in the resource (Solow and Wan, 1976), uncertainty about the stock of the resource (Kemp and Long, 1979; Gilbert, 1979), or imperfect information in contracts (Gaudet et al., 1995; Osmundsen, 1998). Concerns about the market structure that best allows for optimal resource extraction are also discussed (Stiglitz, 1976; Stiglitz and Dasgupta, 1982). The notion of scarcity is thoroughly explored, and scarcity indices are debated in the literature (Pindyck, 1978; Halvorsen and Smith, 1984). In addition, many studies deal with issues related to the choice of a discounting rate (Arrow et al., 1995; Winkler, 2006). The above mentioned issues partly apply to renewable resources. Fish stock management is probably the most prolific topic (Dasgupta and Heal [chapter 5], 1979; Conrad and Clark [chapter 2], 1987; Clark, 1990; Hannesson, 1993). Gordon (1954) and Schaefer (1957) developed fisheries management models in which they estimate the maximum sustainable yield taking into account the regeneration rate of the resource. Extensive work has also been carried out on forestry (Samuelson, 1976; Bowes and Krutilia, 1985; Montgomery and Adams, 1995) and water resources (Conrad and Clark [chapter 3], 1987; Young and Haveman, 1985; Vorosmarty et al., 2000), and to a smaller extent to agricultural land and soil erosion, biodiversity and wildlife (McConnell, 1983; Barrett, 1997; Tisdell, 2002; Perrings and Pearce, 1994). A spatial dimension appears in the spatial management of resources, which is mainly addressed through fishing site choice (Wilen, 2000). Contrary to fossil fuels and minerals, renewable resources such as fish or forests are generally not privately owned and issues related to the management of common property resources or free access to the resource are raised. Garrett Hardin (1968) investigates the issue of free access in his famous “Tragedy of the Commons”. Contrary to a frequent confusion, conflicts do not arise from the nature of common goods, but from the rules (or absence of rules) governing their access. The different types of property regimes range from open access regimes (e.g. fishery resources in international waters).

\footnote{Also called Hotelling rent}
to private property regimes, and include State property and common property regimes (Seabright, 1993). Various policies can be implemented to overcome the overexploitation of free access resources. Besides taxes, quotas are frequently used in fisheries to limit the quantity of the resource extracted.

We define public park places as common resources, at the crossroads of private and public goods. Like private goods, park places are rivalrous; but like public goods, they are non-excludable. Any agent can park her car on public roads, but this prevents another agent to park her car at the same place. After the nature of the particular good studied, let us analyze the regime of access to public park places. Although fees must be required for the use of the good, park places can be considered as available on free access, also called open access. Indeed, fees are usually not financially binding and everyone can access the good. For this reason, use conflicts among agents occur, especially when population and activity density is high. By definition, these particular externalities are not taken into account in the cost born by agents, what leads consumption to stumble over the spatial constraint. The economic scarcity results from the gap between desired and available goods, *i.e.*, between offer and demand. Although scarcity has an absolute nature, this concept is generally considered as relative, since people’s desires may change, as well as the way they meet their needs (substitutes) (Baumgärtner *et al.*, 2006). Considering park places, people face local scarcity, not global scarcity. Indeed, parking needs are localized, and the resource (park lots) is local too.

This article investigates the mechanisms underpinning space allocation. In the context of parking places, it is not too strong an assumption to define space as a homogenous commodity. We particularly highlight the role the shadow cost associated with a scarce resource plays in allocation mechanisms. The shadow cost indicates how much it costs agents to be bounded in their parking space consumption. We show that overlooking the shadow cost of the spatial constraint when space is allocated sequentially induces pressures towards more parking places and drives down the social welfare.

The results confirm the standard conclusions of the literature. Park places are non-renewable resources and the allocation models evidence the scarcity rent resulting from the limited nature of the resource. Moreover, the whole resource is to be consumed by agents when there is a shadow cost stemming from the spatial constraint. Exploring intertemporal allocation of non-renewable resources, Hotelling (1931) concludes that at the end of the world, the resource has been exhausted. On the other hand, park places are available on free access. Following the literature on open access resources such as fish stock, the analysis we conduct shows that space consumption generates use conflicts and a lower social welfare. Although the model developed thereafter is original, the results are in line with the literature on non-renewable and open access resources. Our contribution consists in analyzing a particular resource, space, and to explore allocation mechanisms on a spatial perspective. While studies on fisheries exhibit overexploitation of the resource, we demonstrate mis-consumption of park places. The distinctive feature of this paper consists in considering space as a non-renewable resource, without considering distances and questions of urban economics.

Through a static micro-based allocation model, we investigate three situations (S): simultaneous allocation (S1), sequential allocation without public intervention (S2), and
sequential allocation with public intervention (S3). Under each situation, four possible cases (C) are identified: whether shadow costs are associated with the spatial constraint or not, and whether exogenous features constrain space consumption or not. Exploring space allocation process allows us to compare the utility and social welfare achieved under each situation. Simultaneous allocation of parking places (S1) leads to an equilibrium providing the highest social welfare possible. Nevertheless, commodities prone to externalities and conflicts of use experience market failures. When competition is high, parking places are indeed allocated sequentially (S2). In this case, the order of access to space is a key element. Results show that the social welfare is lower than in a perfectly functioning market. Therefore, public authority can intervene (S3) to guarantee optimal welfare. We then draw the multiple equilibria obtained after regulatory measures are introduced (S3). The model estimates the optimal space consumption rates to be set under the various cases identified. We compare specifically the low-competition case (C1) with the high-competition case (C2). When there is no shadow cost associated with the spatial constraint (C1), our findings reveal that the various situations (S1, S2, S3) are equivalent. However, when a shadow cost is associated with the spatial constraint, a sequential allocation without public intervention (S2) results in a lower social welfare than expected with other situations (S1, S3).

The paper is organized as follows. The next section introduces the model developed for the three situations (S1, S2, S3) presented above. Section 3 presents the main results in terms of space consumption, utility and social welfare. Section 4 draws the social welfare analysis and investigates the space consumption rates to be set by the public authority. Section 5 concludes.

2 The model

We consider an economy with two commodities: parking places, also called "space", \( s \) and a composite good \( x \). Space \( s \) is allocated among a continuum of \( N \) agents, simplified to 2 agents without loss of generality\(^2\). Any agent \( k = i, j \) aims to maximize her own utility expressed by the following utility function: \( u_k(s, x) = \alpha_k \ln s_k + \ln x_k \). Agents draw their utility from the consumption of both commodities \((s_k, x_k)\). Utility is supposed to increase with space consumption. Moreover, we assume that both commodities are necessary. Consequently, we use logarithmic utility functions, since they satisfy the Inada conditions \((s_k, x_k > 0)\). Any agent has a relative preference \( \alpha_k \) for parking places compared to composite good, with \( \alpha_k > 0 \). Preferences depend on the quality of space, not on prices.

We suppose that any agent faces two constraints: a budget constraint and a spatial constraint. The budget constraint is assumed to be saturated, so that \( R_k = p_s s_k + p_x x_k \). Let \( R_k \) be the income of agent \( k \). A price \( p_s \) applies to any agent for the consumption of a unit of space. Similarly, a price \( p_x \) is set for a unit of composite good. As in standard fisheries models, revenues and prices are set exogenously. Consumers are price-takers and compete with one another. The spatial constraint means that space consumption is limited by the total quantity of space available in the economy, noted \( \overline{s} \), so that \( s_k \leq \overline{s} \).

\(^2\) In a more realistic approach, the agents we are bringing into play in the model must be considered as categories of agents, with each category entailing several agents.
The sum of the quantities consumed by agents cannot exceed the total space available, so that \( \sum_{k=1}^{n} s_k \leq \bar{S} \) for \( n \) agents. The spatial constraint introduces interdependent relationships between agents. Indeed, space consumptions are interrelated and generate rivalry of use. Furthermore, it is assumed that agents need a minimum quantity of space, so that \( s_k \geq \bar{s} > 0 \). We suppose that anyone can afford this minimum quantity, \( R_k > p_s \bar{s} \), and that the remaining revenue left for composite good is positive, \( \bar{R}_k > 0 \). Finally, the model considers that the total space available is larger than the sum of minimal quantities for each agent, so that \( \bar{S} \geq \sum_{k=1}^{n} s_k \) in case of \( n \) agents. In addition, we assume a fixed share \( x \) of public parking space in the city. The urban area is assumed to be surrounded by natural space.

The allocation process is described first for homogeneous agents, taken as a benchmark, and then for heterogeneous agents. Homogeneous agents are defined with identical revenues \( (R_i = R_j) \) and preferences \( (u_i(s, x) = u_j(s, x) \) with \( \alpha_i = \alpha_j \). On the contrary, heterogeneous agents are defined with different revenues \( (R_i \neq R_j) \) and preferences \( (u_i(s, x) \neq u_j(s, x) \) with \( \alpha_i \neq \alpha_j \).

Agents can have either strong or low relative preferences for parking places. If both agents have low relative preferences for space, each agent consumes only a small share of the total space. As the resource is not fully consumed, there is no use competition and hence no need for regulation. On the other hand, if both agents have strong relative preferences for parking places, conflicts of use arise.

We develop simple theoretical utility maximization models for three situations (S): simultaneous space allocation (S1), sequential space allocation without public intervention (S2), and sequential space allocation with public intervention (S3). Under each situation, four possible cases (C) are identified\(^3\): whether a shadow cost is associated with the spatial constraint on quantities or not, and whether exogenous features constrain space consumption or not. We note \( \mu_k \) the shadow cost associated with the spatial constraint on quantities. Let \( \gamma_k \) be the marginal cost derived from the consumption of the minimum quantity. Table 1 below describes the cases. In Case 1 (C1), no shadow cost is associated with the spatial constraint on quantities and its consumption is not constrained by exogenous features. In Case 1' (C1'), no shadow cost is associated with the spatial constraint on quantities but its consumption is constrained by exogenous features. In Case 2 (C2), a shadow cost is associated with the spatial constraint on quantities and its consumption is not constrained by exogenous features. In Case 2' (C2'), a shadow cost is associated with the spatial constraint on quantities but its consumption is constrained by exogenous features. Thereafter, we are especially interested in the comparison between Case 1 and Case 2. In Case 1, competition for parking places is relatively low, while in Case 2, competition is relatively high. Competition arises when shadow costs are associated with the spatial constraint on quantities.

\(^3\)Under S1 and S3, the four cases appear (C1, C2, C1' and C2'), while under S2, only C1 and C2 appear since one of the two spatial constraints is dropped.
Table 1: Four cases emerge under each situation

<table>
<thead>
<tr>
<th>No shadow cost</th>
<th>Shadow cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_k = 0$</td>
<td>$\mu_k &gt; 0$</td>
</tr>
<tr>
<td>thus $s_i + s_j \leq S$</td>
<td>thus $s_i + s_j = S$</td>
</tr>
</tbody>
</table>

Unconstrained space consumption

<table>
<thead>
<tr>
<th>$\gamma_k = 0$</th>
<th>CASE 1</th>
<th>CASE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>thus $s_k \geq s$</td>
<td>Low competition</td>
<td>High competition</td>
</tr>
</tbody>
</table>

Constrained space consumption

<table>
<thead>
<tr>
<th>$\gamma_k &gt; 0$</th>
<th>CASE 1’</th>
<th>CASE 2’</th>
</tr>
</thead>
<tbody>
<tr>
<td>thus $s_k = s$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.1 Simultaneous space allocation (S1)

First, we build a utility maximization model for the situation of simultaneous space allocation (S1). Competing for parking places, agents adjust their consumption levels simultaneously to reach equilibrium, as in a perfectly functioning market economy. The utility maximization model of any agent $k = i, j$ is as follows:

$$\max_{s_k, x_k} u_k (s, x) = \alpha_k \ln s_k + \ln x_k$$  \hspace{1cm} (1)

with

$$R_k = p_s s_k + p_x x_k$$

$$s_i + s_j \leq S$$

$$s_k \geq s$$

We use the Karush-Kuhn-Tucker method to solve the above maximization model:

$$L (s_k, x_k, \lambda_k, \mu_k, \gamma_k) = \alpha_k \ln s_k + \ln x_k + \lambda_k (R_k - p_s s_k - p_x x_k) + \mu_k (S - s_i - s_j) + \gamma_k (s_k - s)$$  \hspace{1cm} (2)

with $\lambda_k$ the marginal value of a unit of space for any agent $k$. Any agent is constrained by (1) her budget, (2) the total quantity of space available in the economy, and (3) a minimum amount of space she has to consume.

For any agent $k$, the first-order conditions are:

$$\begin{cases}
\frac{\partial L}{\partial s_k} = \frac{\alpha_k}{s_k} - \lambda_k p_s - \mu_k + \gamma_k = 0 \\
\frac{\partial L}{\partial x_k} = \frac{1}{x_k} - \lambda_k p_x = 0 \\
\frac{\partial L}{\partial \lambda_k} = R_k - p_s s_k - p_x x_k = 0 \\
\frac{\partial L}{\partial \mu_k} = R_k - p_s s_k - p_x x_k = 0 \\
\mu_k (S - s_i - s_j) = 0 \\
\mu_k \geq 0 \\
S - s_i - s_j \geq 0 \\
\gamma_k (s_k - s) = 0 \\
\gamma_k \geq 0 \\
s_k - s \geq 0
\end{cases}$$  \hspace{1cm} (3)
2.2 Sequential space allocation without public intervention (S2)

Second, we build a utility maximization model for the situation of sequential space allocation without public intervention (S2). Agents access the market sequentially. Some categories of agents may access parking places first and therefore limit the amount left to others. We assume that agent $i$ maximizes her utility first. Her spatial constraint is defined by the whole space available minus the minimum quantity to be consumed by the other agent, so that $s_i \leq \overline{S} - \underline{s}$. As agent $i$ disposes of almost the whole space, we suppose that she is able to consume the minimal quantity required. Hence, we remove the second spatial constraint. $\overline{S}' = \overline{S} - s_i$ is then to be shared among the other agents. The utility maximization model of agent $i$ is as follows:

$$\max_{s_i, x_i} u_i(s, x) = \alpha_i \ln s_i + \ln x_i$$

$$\text{s.t.} \quad \begin{cases} R_i = p_s s_i + p_x x_i \\ s_i \leq \overline{S} - \underline{s} \end{cases}$$

given that $s_k \geq \underline{s}$, we get

$$L(s_i, x_i, \lambda_i, \mu_i) = \alpha_i \ln s_i + \ln x_i + \lambda_i (R_i - p_s s_i - p_x x_i) + \mu_i (\overline{S} - \underline{s} - s_i)$$

Agent $j$ maximizes her utility in second position. By analogy, we obtain the utility maximization model of agent $j$. The spatial constraint turns to $s_j \leq \overline{S}'$.

2.3 Sequential space allocation with public intervention (S3)

Third, we build a utility maximization model for the situation of sequential space allocation with public intervention (S3). Agents access the market sequentially, but the public authority intervenes to set space consumption rates. The aim is to protect agents accessing parking places in second position. The space consumption rate is noted $\tau$ for agent $i$, and $(1 - \tau)$ for agent $j$, with $0 < \tau < 1$. The space consumption of agent $i$ (agent $j$) cannot exceed a certain share of the total space, so that $s_i \leq \tau \overline{S}$ ($s_j \leq (1 - \tau) \overline{S}$). Space consumption rates are supposed to be set according to agents’ relative preferences for parking places and their income. Agent $i$ maximizes her utility first. The utility maximization model of agent $i$ is as follows:

$$\max_{s_i, x_i} u_i(s, x) = \alpha_i \ln s_i + \ln x_i$$

$$\text{s.t.} \quad \begin{cases} R_i = p_s s_i + p_x x_i \\ s_i \leq \tau \overline{S} \\ s_i \geq \underline{s} \end{cases}$$
we get

$$L(s_i, x_i, \lambda_i, \mu_i, \gamma_i) = \alpha_i \ln s_i + \ln x_i + \lambda_i (R_i - p_s s_i - p_x x_i) + \mu_i (\tau \bar{S} - s_i) + \gamma_i (s_i - \bar{s})$$ (7)

By analogy, we obtain the utility maximization model of agent $j$. The spatial constraints turn to $s_j \leq (1 - \tau) \bar{S}$ and $s_j \geq \bar{s}$.

3 Results

Results are presented situation by situation (S1, S2, S3), then case by case (C1, C2)\(^4\). Each situation allows for a comparison between the indirect utilities achieved by both agents. The indirect utility is derived from the optimal consumption of both goods, so that $V_k (R_k, p_s, p_x, \bar{S}, \bar{s}, \alpha_k) = u_k (s^*, x^*)$ for any agent $k$. Under each situation, different cases are possible. Within a situation, results may differ according to the case considered. Hence, within a given situation, comparison is made between cases. We then give a measure of the social welfare obtained under the situation analyzed. The utilitarian form of social welfare is expressed as $W = \eta V_i + \varphi V_j$. We define the social welfare as the aggregation of the indirect utilities of all agents constituting the society, so that

$$W = \sum_{k=1}^{n} V_n$$ for $n$ agents. In our model, we have two agents, agent $i$ and agent $j$.

Finally, each case is explored in terms of space consumption, indirect utilities and social welfare. Similarly, for each case several situations can occur. Hence, within a given case, comparison is made between situations. We discuss first results for homogeneous agents, then for heterogeneous agents.

3.1 Situation-by-situation results (S1, S2, S3)

Results found for any agent $k$ under simultaneous space allocation (S1) are summarized in Table 1.1 (homogeneous agents) and Table 1.2 (heterogeneous agents), Appendix A. Under simultaneous allocation, we note $V_{kSi}^{S_i m(l)}$ the indirect utilities whatever the case $l = 1, 2, 1', 2'$. They are detailed in Table 1.3 (homogeneous agents) and Table 1.4 (heterogeneous agents), Appendix A. When agents are homogeneous, we find that indirect utilities are equivalent whatever the case considered: $V_{iSi}^{S_i m(l)} = V_{jSi}^{S_j m(l)}$, with $s_i = s_j$. Agents have equal space consumption. When agents are heterogeneous, we get $V_{iSi}^{S_i m(l)} \neq V_{jSi}^{S_j m(l)}$, with $s_i \neq s_j$. Space consumption differs between agents\(^5\), according to their relative preferences for parking places and their income. No agent is clearly better off the other. We note $W_{Si}^{S_i m(l)}$ the social welfare under simultaneous space allocation, so that $W_{Si}^{S_i m(l)} = V_{iSi}^{S_i m(l)} + V_{jSi}^{S_j m(l)}$.

\(^4\)When discussing the results, we focus on C1 and C2 only. C1’ and C2’ do not lead to interesting interpretations in terms of space consumption. Indeed, in these cases space consumption is constrained and agents have no other choice but to consume the minimal quantity of space. Detailed results for C1’ and C2’ are yet provided in appendices.

\(^5\)A particular case would be that the difference in the relative preferences for parking places is exactly compensated by an inverse difference in income, leading to $s_i = s_j$ even for heterogeneous agents. Although quite unlikely, it is not to be excluded.
Results found for any agent $k$ under sequential space allocation without public intervention (S2) are summarized in Table 2.1 (homogeneous and heterogeneous agents), Appendix B. Under sequential allocation, we note $V_{i}^{Seq(l)}$ the indirect utilities whatever the case $l = 1, 2, 1', 2'$. They are detailed in Table 2.2 (homogeneous and heterogeneous agents), Appendix B. When agents are homogeneous, their indirect utilities are equivalent in Case 1: $V_{i}^{Seq(1)} = V_{j}^{Seq(1)}$ with $s_{i} = s_{j}$. However in Case 2, the agent maximizing her utility first (here, agent $i$) clearly gets a higher utility than the other agent (here, agent $j$): $V_{i}^{Seq(2)} > V_{j}^{Seq(2)}$ with $s_{i} > s_{j}$. When agents are heterogeneous, in Case 1 we get $V_{i}^{Seq(1)} \neq V_{j}^{Seq(1)}$ with $s_{i} \neq s_{j}$. Space consumption differs between agents, according to their relative preferences for parking places and their income. No agent is clearly better off the other. However in Case 2, all other things being equal, the agent maximizing her utility first (here, agent $i$) clearly gets a higher utility than the other agent (here, agent $j$): $V_{i}^{Seq(2)} > V_{j}^{Seq(2)}$ with $s_{i} > s_{j}$. This interpretation may not hold if agent $i$ has lower relative preferences for parking places and their income. No agent is clearly better off the other. We note $W^{Seq(l)}$ the social welfare under sequential space allocation without public intervention, so that $W^{Seq(l)} = V_{i}^{Seq(l)} + V_{j}^{Seq(l)}$.

Results found for any agent $k$ under sequential space allocation with public intervention (S3) are summarized in Table 3.1 (homogeneous and heterogeneous agents), Appendix C. Under sequential allocation with public intervention, we note $V_{k}^{SeqPI(l)}$ the indirect utilities whatever the case $l = 1, 2, 1', 2'$. They are detailed in Table 3.2 (homogeneous and heterogeneous agents), Appendix C. When agents are homogeneous, we expect the respective rates of space consumption $\tau$ and $(1 - \tau)$ to be equal, so that $\tau = (1 - \tau) = 0.5$. Consequently, their indirect utilities are equivalent whatever the case: $V_{i}^{SeqPI(l)} = V_{j}^{SeqPI(l)}$ with $s_{i} = s_{j}$. When agents are heterogeneous, we get $V_{i}^{SeqPI(l)} \neq V_{j}^{SeqPI(l)}$ with $s_{i} \neq s_{j}$. The government regulation limits the quantity of space agent $i$ can acquire. Space consumption differs between agents, according to the rates set. Since space consumption rates are supposed to be set according to agents’ relative preferences for parking places and their income, no agent is clearly better off the other. We note $W^{SeqPI(l)}$ the social welfare under sequential space allocation with public intervention, so that $W^{SeqPI(l)} = V_{i}^{SeqPI(l)} + V_{j}^{SeqPI(l)}$.

3.2 Case-by-case results (C1, C2)

In Case 1, there is no shadow cost associated with the spatial constraint on quantities ($\mu_{k} = 0$). Provided that the total quantity of space available, $\overline{S}$, is not constrained by exogenous features such as demography or sociology ($\gamma_{k} = 0$), agents are able, if desired, to obtain a quantity of space larger than the minimal quantity, so that $s_{k} \geq \underline{s}$. In this case, the space available may not be fully consumed, so that $s_{i} + s_{j} \leq \overline{S}$. Therefore, competition for parking places is relatively low. Case 1 is consistent with low relative preferences for parking places.

The optimal space consumption of agent $k$ is given by $s_{k} = \left( \frac{\alpha_{k}}{1 + \alpha_{k}} \right) \left( \frac{R_{k}}{p_{S}} \right)$ under all three situations (S1, S2, S3) and for homogeneous as well as heterogeneous agents. Space

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$^{6}$This finding holds only when $(\overline{S} - \underline{s}) > \underline{s}$, i.e., when $\overline{S} > 2\underline{s}$ for an economy with 2 agents.
consumption depends on agents’ relative preferences for parking places, as well as on their income and the price of the resource. All things being equal, the higher the relative preferences for parking places, the larger the quantity of space consumed, and the smaller the quantity of composite good consumed.

In Case 1, the indirect utilities present a similar structure whatever the situation analyzed:

$$V_k^{(1)} = V_k^{Sim(1)} = V_k^{Seq(1)} = V_k^{SeqPI(1)} = \alpha_k \left[ \ln \left( \frac{\alpha_k}{1+\alpha_k} \right) + \ln \left( \frac{R_i}{p_s} \right) \right] + \left[ \ln \left( \frac{1}{1+\alpha_k} \right) + \ln \left( \frac{R_i}{p_x} \right) \right].$$

Homogeneous agents achieve identical utilities, so that $V_i^{(1)} = V_j^{(1)}$ with $V_i^{Sim(1)} = V_i^{Seq(1)} = V_i^{SeqPI(1)}$ and $V_j^{Sim(1)} = V_j^{Seq(1)} = V_j^{SeqPI(1)}$. When agents are heterogeneous, they obtain different utilities depending on their relative preferences for parking places and respective incomes, so that $V_i^{(1)} \neq V_j^{(1)}$ with $V_i^{Sim(1)} = V_i^{Seq(1)} = V_i^{SeqPI(1)}$ and $V_j^{Sim(1)} = V_j^{Seq(1)} = V_j^{SeqPI(1)}$. No agent is clearly better off the other. Likewise, the social welfare is similar whatever the situation considered. It is expressed as follows:

$$W^{Sim(1)} = W^{Seq(1)} = W^{SeqPI(1)}$$

$$= V_i^{(1)} + V_j^{(1)}$$

$$= \left\{ \alpha_i \left[ \ln \left( \frac{\alpha_i}{1+\alpha_i} \right) + \ln \left( \frac{R_i}{p_s} \right) \right] + \left[ \ln \left( \frac{1}{1+\alpha_i} \right) + \ln \left( \frac{R_i}{p_x} \right) \right] \right\}$$

$$+ \left\{ \alpha_j \left[ \ln \left( \frac{\alpha_j}{1+\alpha_j} \right) + \ln \left( \frac{R_j}{p_s} \right) \right] + \left[ \ln \left( \frac{1}{1+\alpha_j} \right) + \ln \left( \frac{R_j}{p_x} \right) \right] \right\}.$$  

Consequently, when no shadow cost is associated with the spatial constraint on quantities (C1), the various situations (S1, S2, S3) are equivalent. Agents are therefore indifferent between the three situations.

**Proposition 1 - When no shadow cost is associated with the spatial constraint on quantities (C1), the various situations (S1, S2, S3) are equivalent.**

In Case 2, a shadow cost is associated with the spatial constraint on quantities ($\mu_k > 0$). Provided that the total quantity of space available, $\overline{S}$, is not constrained by exogenous features such as demography or sociology ($\gamma_k = 0$), agents are able, and desire, to obtain a quantity of space larger than the minimal quantity, so that $s_k \geq \overline{s}$. As a shadow cost is associated with the spatial constraint, Case 2 is consistent with high relative preferences for parking places. Agents compete against one another to get the largest quantity of space possible, within their budget and spatial constraints. Competition for parking places is relatively high. Therefore, the space available is fully consumed, so that $s_i + s_j = \overline{S}$.

The optimal space consumptions vary according to the situation considered. Under simultaneous space allocation (S1), the optimal space consumptions are given by $s_k = \left( \overline{S} \right)$ for homogeneous individuals, and by $s_i = \overline{S} - s_j$ and $s_j = \overline{S} - s_i$, for heterogeneous individuals. The total space available is shared between agents, either equally (homogeneous agents) or not (heterogeneous agents). Under sequential space allocation without public intervention (S2), the optimal space consumptions are $s_i = \overline{S} - \overline{s}$ and $s_j = \overline{s}$ for both homogeneous and heterogeneous individuals when agent $i$ accesses parking places first. Agent $i$ obtains the maximum quantity of the space available in the economy (i.e.,
the whole space minus the minimum quantity required for the other agent). Agent $j$ is then not able to get anything but the minimum quantity. Under sequential space allocation (S3), the optimal space consumptions are $s_i = \tau \bar{s}$ and $s_j = (1 - \tau) \bar{s}$ for both homogeneous and heterogeneous individuals when agent $i$ accesses parking places first. For homogeneous agents, the optimal rates of space consumption is $\tau = (1 - \tau) = 0.5$. For heterogeneous agents, the optimal rates of space consumption differ, with $0 < \tau < 1$. Since the rates set by the public authority are based on agents’ relative preferences, space consumption depends on agents’ preferences.

In Case 2, the indirect utilities vary according to the situation analyzed. Under simultaneous space allocation (S1), homogeneous agents achieve a level of utility given by

$$V_i^{Sim}(2) = \alpha_i \left[ \ln \bar{s} - \ln 2 \right] + \left[ \ln \bar{R}_i - \ln p_x \right],$$

with $V_i^{Sim}(2) = V_j^{Sim}(2)$. The indirect utility for heterogeneous agents is

$$V_i^{Sim}(2) = \alpha_i \left[ \ln (\bar{s} - s_j) \right] + \left[ \ln \bar{R}_i - \ln p_x \right]$$

and $V_j^{Sim}(2) = \alpha_j \left[ \ln (\bar{s} - s_i) \right] + \left[ \ln \bar{R}_j - \ln p_x \right]$, with $V_i^{Sim}(2) \neq V_j^{Sim}(2)$. No agent is clearly better off the other. Under sequential space allocation without public intervention (S2), for both homogeneous and heterogeneous agents we have $V_i^{Seq}(2) = \alpha_i \left[ \ln (\bar{s} - s_i) \right] + \left[ \ln \bar{R}_i - \ln p_x \right]$ and $V_j^{Seq}(2) = \alpha_j \left[ \ln s_j \right] + \left[ \ln \bar{R}_j - \ln p_x \right]$, with $V_i^{Seq}(2) \neq V_j^{Seq}(2)$. All other things being equal, agent $i$ is better off agent $j$, so $V_i^{Seq}(2) > V_j^{Seq}(2)$. Under sequential resource allocation with public intervention (S3), we obtain $V_i^{SeqPI}(2) = \alpha_i \left[ \ln (\bar{s} - s_i) \right] + \left[ \ln \bar{R}_i - \ln p_x \right]$ and $V_j^{SeqPI}(2) = \alpha_j \left[ \ln (1 - \tau) + \ln \bar{S} \right] + \left[ \ln \bar{R}_j - \ln p_x \right]$, with $V_i^{SeqPI}(2) = V_j^{SeqPI}(2)$ for homogeneous agents and $V_i^{SeqPI}(2) \neq V_j^{SeqPI}(2)$ for heterogeneous agents. No agent is clearly better off the other. Comparative statics related to the shadow cost associated with the spatial constraint on quantities are presented in Appendix D. The social welfare differs according to the situation considered. Under S2, agent $j$ has no other choice but to consume only the minimum quantity of space. The agent maximizing her utility in second position is clearly worse off. We prove that her low utility level cannot be fully compensated by the high utility level of agent $j$. Therefore and all other things being equal, the social welfare achieved under S2 is lower than that expected under the other situations (S1, S3), for both homogeneous and heterogeneous agents. We have $W^{Seq}(2) < W^{Sim}(2)$ and $W^{Seq}(2) < W^{SeqPI}(2)$, so that:

$$V_i^{Seq}(2) + V_j^{Seq}(2) < V_i^{Sim}(2) + V_j^{Sim}(2) \quad \iff \quad \alpha_i \left[ \ln (\bar{s} - s_i) \right] + \left[ \ln \bar{R}_i - \ln p_x \right] + \alpha_j \left[ \ln s_j \right] + \left[ \ln \bar{R}_j - \ln p_x \right]$$

and

$$V_i^{Seq}(2) + V_j^{Seq}(2) < V_i^{SeqPI}(2) + V_j^{SeqPI}(2) \quad \iff \quad \alpha_i \left[ \ln \bar{s} - \ln 2 \right] + \left[ \ln \bar{R}_i - \ln p_x \right] + \alpha_j \left[ \ln \bar{S} - \ln 2 \right] + \left[ \ln \bar{R}_j - \ln p_x \right]$$

with $W^{Sim}(2) = W^{SeqPI}(2)$ when $\tau$ is optimally set. Consequently, when there is a shadow cost associated with the spatial constraint, a sequential allocation without public intervention (S2) results in a lower social welfare. The various situations (S1, S2, S3) are not
equivalent in terms of social welfare achieved. Neither are they equivalent for the agents. Agent $i$ prefers $S_2$ ($V_{Si}^{Seq(2)} > V_{Si}^{Sim(2)}$ and $V_{Si}^{Seq(2)} > V_{i}^{SeqPI(2)}$), while agent $j$ goes for $S_1$ or $S_3$ ($V_{j}^{Sim(2)} > V_{j}^{Seq(2)}$ and $V_{j}^{SeqPI(2)} > V_{j}^{Seq(2)}$).

Proposition 2 - When a shadow cost is associated with the spatial constraint, a sequential allocation without public intervention ($S_2$) results in a lower social welfare.

4 Social welfare analysis

Given the limited but necessary nature of space, a shadow cost is likely to be associated with the spatial constraint. Case 2 is therefore the most plausible case. Moreover, we assume that the society wants to move towards the highest welfare achievable. Since open access resources are prone to market failures, simultaneous allocation of space is unlikely to happen. A sequential allocation would rather take place. Without any public intervention, a sequential allocation results in a lower social welfare than that expected under simultaneous allocation. A public authority can intervene to guarantee optimal welfare to society. Her role is to set a space consumption rate $\tau$ that is optimal to ensure optimal social welfare. The public authority sets a space consumption rate $\tau$ that maximizes the social welfare:

$$\max_\tau W_{SeqPI(2)} = \alpha_i \left[ \ln \tau + \ln \bar{S} \right] + \left[ \ln \bar{R}_i - \ln p_x \right] + \alpha_j \left[ \ln (1 - \tau) + \ln \bar{S} \right] + \left[ \ln \bar{R}_j - \ln p_x \right]$$  \hspace{1cm} (11)

The F.O.C. are:

$$\frac{\partial W_{SeqPI(2)}}{\partial \tau} = 0 \Leftrightarrow W_{SeqPI(2)}' = \left( \frac{\alpha_i}{\tau} \right) - \left( \frac{p_s \bar{S}}{R_i - p_s \tau \bar{S}} \right) - \left( \frac{\alpha_j}{(1 - \tau)} \right) + \left( \frac{p_s \bar{S}}{R_j - p_s (1 - \tau) \bar{S}} \right) = 0$$  \hspace{1cm} (12)

which can be split up into two subfunctions as follows:

$$\left( \frac{\alpha_i}{\tau} \right) - \left( \frac{\alpha_j}{(1 - \tau)} \right) = \left( \frac{p_s \bar{S}}{R_i - p_s \tau \bar{S}} \right) - \left( \frac{p_s \bar{S}}{R_j - p_s (1 - \tau) \bar{S}} \right)$$  \hspace{1cm} (13)

with

$$g(\tau) = \left( \frac{\alpha_i}{\tau} \right) - \left( \frac{\alpha_j}{(1 - \tau)} \right) \hspace{1cm} (14)$$

$$f(\tau) = \left( \frac{p_s \bar{S}}{R_i - p_s \tau \bar{S}} \right) - \left( \frac{p_s \bar{S}}{R_j - p_s (1 - \tau) \bar{S}} \right)$$

We find:

$$g'(\tau) = -\frac{\alpha_i}{\tau^2} - \frac{\alpha_j}{(1 - \tau)^2} < 0 \hspace{1cm} (15)$$

$$f'(\tau) = \left( \frac{p_s \bar{S}}{R_i - p_s \tau \bar{S}} \right)^2 + \left( \frac{p_s \bar{S}}{R_j - p_s (1 - \tau) \bar{S}} \right)^2 > 0$$
$g(\tau)$ is a decreasing function, with $\lim_{\tau \to 0}(\tau) = +\infty$, $\lim_{\tau \to 1}(\tau) = -\infty$ and $g(\tau) = 0$ when $\tau = \left(\frac{\alpha_i - \alpha_j}{\alpha_i + \alpha_j}\right)$. Since $g''(\tau) = 0$ when $\tau = (1 - \tau) = 0.5$, there exists an inflection point so that $g(0.5) = 0$ when $\alpha_i = \alpha_j$ (a), $g(0.5) > 0$ when $\alpha_i > \alpha_j$ (b), and $g(0.5) < 0$ when $\alpha_i < \alpha_j$ (c).

$f(\tau)$ is an increasing function, whose limits vary according to various cases:

- **Case A**: $R_i$ is slightly larger than $R_j$
  \[R_j + p_s\bar{S} > R_i > R_j, \text{ with } \lim_{\tau \to 0}(\tau) < 0 \text{ and } \lim_{\tau \to 1}(\tau) > 0\]

- **Case B**: $R_i$ is larger than $R_j$
  \[R_j + p_s\bar{S} = R_i > R_j, \text{ with } \lim_{\tau \to 0}(\tau) < 0 \text{ and } \lim_{\tau \to 1}(\tau) = 0\]

- **Case C**: $R_i$ is strongly larger than $R_j$
  \[R_i > R_j + p_s\bar{S}, \text{ with } \lim_{\tau \to 0}(\tau) < 0 \text{ and } \lim_{\tau \to 1}(\tau) < 0\]

- **Case D**: $R_i$ is slightly smaller than $R_j$
  \[R_j > R_i > R_j - p_s\bar{S}, \text{ with } \lim_{\tau \to 0}(\tau) < 0 \text{ and } \lim_{\tau \to 1}(\tau) > 0\]

- **Case E**: $R_i$ is smaller than $R_j$
  \[R_j > R_i = R_j - p_s\bar{S}, \text{ with } \lim_{\tau \to 0}(\tau) = 0 \text{ and } \lim_{\tau \to 1}(\tau) > 0\]

- **Case F**: $R_i$ is strongly smaller than $R_j$
  \[R_i < R_j - p_s\bar{S}, \text{ with } \lim_{\tau \to 0}(\tau) > 0 \text{ and } \lim_{\tau \to 1}(\tau) > 0\]

Case A is close to Case D. For clarity purposes, they are merged in the following graph.

The optimal rates $\tau$ obtained under the various cases considered above are illustrated in Figure 1. We expect $\tau = (1 - \tau) = 0.5$ for homogeneous agents, and $\tau \neq (1 - \tau)$ for heterogeneous agents.

The optimal $\tau^*$ obtained under the various cases considered above are illustrated in Figure 1. We expect $\tau = (1 - \tau) = 0.5$ for homogeneous agents, and $\tau \neq (1 - \tau)$ for heterogeneous agents. The lower income inequalities (as in Case A or D), the closer to 0.5 the rate is. All other things being equal, the higher the income or the relative preferences for parking places, the larger the space consumption rate. When the higher-incomer displays higher relative preferences for parking places than the lower-incomer, then $\tau^*$ moves away from 0.5. However, if the higher-incomer displays lower relative preferences for parking places then the lower-incomer, than $\tau^*$ moves closer to 0.5.

Furthermore, the relation between the social welfare and the price of space ($p_s$), or the total quantity of space available ($\bar{S}$), is consistent with the intuition. Indeed, it is equivalent to reduce the price of space along with an increase in the total quantity, and to increase the price of space while reducing the total quantity available. Increasing $p_s$ or $\bar{S}$ moves $f(\tau)$ upwards when $R_i > R_j$ (the slope of the function increases), which means that $\tau^*$ decreases, or downwards when $R_i < R_j$ (the slope of the function increases too), which means that $\tau^*$ increases. Therefore, all other things being equal, increasing the price of space or the quantity available brings $\tau^*$ closer to 0.5. The resource is then shared more equally in order to maintain the highest social welfare possible.
Figure 1: Optimal space allocation rates
5 Concluding remarks

This article explores the basic mechanisms underpinning space allocation. We especially highlight the role the shadow cost plays in allocation process. We demonstrate that space consumption and utility obtained by homogeneous and heterogeneous agents vary according to the allocation process. The social welfare differs too. We show that under sequential resource allocation and when a shadow cost is associated with the spatial constraint, the social welfare is lower than expected under other situations. Market failures arise and lead to a pressure towards the creation of additional parking places. New parking places will be created through land use changes, thus limiting other uses. Moreover, when natural uses are turned into urban uses (so called phenomenon of "land artificialization"), environmental externalities are generated, in particular irreversibility of land use changes, loss of biodiversity, ecosystem fragmentation and lower resilience.

We evidence that the creation of additional parking places can be overcome with appropriate public intervention. It is a question of first recognizing and then internalizing the shadow cost associated with the spatial constraint. The model aims to investigate the optimal space consumption rates to be set in order to prevent pressures towards more parking places. The optimal space consumption rate proves straightforward for homogeneous agents, with space shared equally. When agents are heterogeneous, the optimal rate depends on the relative size of the preferences for parking places and of the respective incomes of agents.

One of the main limits of this article relates to the structure of the model. Indeed, a one-period model does not allow us to observe the effective creation of additional parking places. However, we can assume that the public authority regulates the creation of artificial surfaces. Even when it does not directly intervene in the allocation process, he can set a maximum quantity $\rho$ to be turned into parking places in case of pressure towards more parking uses. In this case, in period 2, the total parking space to be allocated among agents would be $S + \rho$. Moreover, two mechanisms can alter the absolute scarcity over time and are not accounted for in the present analysis: the extensive margin and the intensive margin. The former mechanism, the extensive margin, means that land use changes allow for the creation of additional park places (urban sprawl). The latter mechanism, the intensive margin, means that more needs would be met with a finite stock of park places (urban densification).

Another limit relates to the choice of the resource: parking space. Optimization regards parking space as a homogeneous resource. Yet though the apparent homogeneity of parking places, space can also be considered as heterogeneous. Urban lands may face different topographical constraints, infrastructure endowments or accessibility (Saiz, 2010). Agents’ preferences would vary according to the type of space (horizontal vs. vertical$^7$), its localization, existing amenities (urban or environmental) on the space or nearby, etc. A research perspective is to take into account the heterogeneity of space through a multi-amenity approach.

$^7$Vertical space, such as multi-floor parking facilities, and horizontal space, such as on-street parking.
Finally, following the emission trading scheme conceived by Ronald Coase (1960) and elaborated by John Dales (1968) for water resources, our model could be enriched with the creation of a market scheme of space consumption rights for parking. The trading scheme would substitute for the introduction of a space consumption rate set exogenously by the public authority. It would allow agents with low preferences for parking places to exchange part of the space they could acquire with agents having higher preferences for parking places but who were not able to get as much space as desired. According to Hannesson (2004), individual transferable quotas are a first best regulatory regime for open access resources.
References


Appendices

**Appendix A: Simultaneous space allocation (S1)**

Results and indirect utilities found in Case 1 and Case 1’ under simultaneous space allocation are identical for homogeneous and heterogeneous agents. The only difference to keep in mind is that $\alpha_i = \alpha_j$ and $R_i = R_j$ for homogeneous agents, while $\alpha_i \neq \alpha_j$ and $R_i \neq R_j$ for heterogeneous agents. Table 1.2 and Table 1.4 only include the results and indirect utilities that differ.

<table>
<thead>
<tr>
<th>Table 1.1: Results for homogeneous agents under simultaneous space allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
</tr>
<tr>
<td>Case 1’ with $R_k = R_k - p_s \bar{s}$</td>
</tr>
<tr>
<td>Case 2 with $R_k = R_k - \left( \frac{1}{2} \right)p_s \bar{S}$</td>
</tr>
<tr>
<td>Case 2’ with $R_k = R_k - \left( \frac{1}{2} \right)p_s \bar{S}$</td>
</tr>
</tbody>
</table>

Let us check the conditions under which $\mu_k$ is strictly positive.

\[
\mu_k = \left( \frac{\alpha_k}{\bar{s}} \right) - \left( \frac{p_s}{R_k} \right) > 0
\]

\[
\mu_k = \left( \frac{\alpha_k}{\bar{s}} \right) - \left( \frac{p_s}{R_k - \left( \frac{1}{2} \right)p_s \bar{S}} \right) > 0
\]

\[R_k > \left( \frac{1+\alpha_k}{\alpha_k} \right) \left( \frac{\bar{S}}{2p_s} \right)
\]

Given that $\left( \frac{1+\alpha_k}{\alpha_k} \right) > 0$ since $\alpha_k > 0$, $\mu_k > 0$ if and only if $R_k > \frac{1}{2}p_s \bar{S}$, which is true when $\tilde{R}_k > 0$. This result can easily be generalized to the other situations studied thereafter.

<table>
<thead>
<tr>
<th>Table 1.2: Results for heterogeneous agents under simultaneous space allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2 with $R_i = R_i - p_s (S - s_i)$ and $\tilde{R}_i = R_i - p_s (\bar{S} - s_i)$</td>
</tr>
<tr>
<td>Case 2’ with $R_i = R_i - p_s (S - s_i)$, $\tilde{R}_i = R_i - p_s (\bar{S} - s_i)$ and $\tilde{R}_k = R_k - p_s \bar{s}$</td>
</tr>
</tbody>
</table>

\[
\gamma_i = \left( \frac{p_s}{R_i} \right) - \left( \frac{\alpha_i}{S - s_i} \right) + \mu_i \quad \text{or} \quad \gamma_k = 0
\]

\[
\gamma_j = \left( \frac{p_s}{R_j} \right) - \left( \frac{\alpha_j}{S - s_j} \right) + \mu_j \quad \text{or} \quad \gamma_k = 0
\]

\[
\mu_i = \left( \frac{\alpha_i}{S - s_i} \right) - \left( \frac{p_s}{R_i} \right) \quad \text{and} \quad \gamma_i = \left( \frac{p_s}{R_i} \right) - \left( \frac{\alpha_i}{S - s_i} \right) + \mu_i
\]

\[
\mu_j = \left( \frac{\alpha_j}{S - s_j} \right) - \left( \frac{p_s}{R_j} \right) \quad \text{and} \quad \gamma_j = \left( \frac{p_s}{R_j} \right) - \left( \frac{\alpha_j}{S - s_j} \right) + \mu_j
\]

\[
\gamma_k = 0
\]
Table 1.3: Indirect utilities for homogeneous agents under simultaneous space allocation

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Indirect utility $V_{k}^{m(1)}(R_k, p_s, p_x, S, \alpha_k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1’ with $R_k = R_k - p_s \bar{S}$</td>
<td>$V_{k}^{m(1)} = \alpha_k \ln \left( \frac{\alpha_k}{\pi \alpha_k} \right) + \ln \left( \frac{2k}{p_x} \right) + \ln \left( \frac{2k}{\bar{p}_x} \right)$</td>
</tr>
<tr>
<td>Case 2’ with $R_k = R_k - \left( \frac{1}{2} \right) p_s \bar{S}$</td>
<td>$V_{k}^{m(2)} = \alpha_k \ln \left( \frac{\alpha_k}{\pi \alpha_k} \right) + \ln \left( R_k - \ln p_x \right)$</td>
</tr>
</tbody>
</table>

NB: The terms $\ln \left( \frac{\alpha_k}{\pi \alpha_k} \right)$ and $\ln \left( \frac{1}{\pi \alpha_k} \right)$ might be negative. However, the comparison between indirect utilities would not be impacted.

Table 1.4: Indirect utilities for heterogeneous agents under simultaneous space allocation

<table>
<thead>
<tr>
<th>Case 2 with $R_i = R_i - p_s \left( \bar{S} - s_i \right)$ and $R_j = R_j - p_s \left( \bar{S} - s_j \right)$</th>
<th>Indirect utilities $V_{k}^{m(1)}(R_i, p_s, p_x, S, \alpha_k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_i = \bar{S} - \frac{\bar{S}}{2}$; $s_j = \frac{\bar{S}}{2}$ Given that $\bar{S}' = \frac{\bar{S}}{2}$</td>
<td>$V_{i}^{m(3)} = \alpha_i \ln \left( \bar{S} - s_i \right) + \ln \left( R_i - \ln p_x \right)$ and $V_{j}^{m(3)} = \alpha_j \ln \left( \bar{S} - s_j \right) + \ln \left( R_j - \ln p_x \right)$</td>
</tr>
<tr>
<td>Case 2’ with $R_i = R_i - p_s \left( \bar{S} - s_i \right)$, $R_j = R_j - p_s \left( \bar{S} - s_i \right)$ and $R_k = R_k - p_s \bar{S}$</td>
<td>$V_{k}^{m(4)} = V_{k}^{m(2)}$</td>
</tr>
</tbody>
</table>

Appendix B: Sequential space allocation without public intervention (S2)

The results found in Case 1 under sequential space allocation are identical to those found in Case 1 under simultaneous allocation.

Table 2.1: Results for homogeneous and heterogeneous agents under sequential space allocation without public intervention

<table>
<thead>
<tr>
<th>Case 2 with $R_i = R_i - p_s \left( \bar{S} - \frac{\bar{S}}{2} \right)$ and $R_j = R_j - p_s \left( \bar{S} - s_j \right)$</th>
<th>$s_i = \bar{S} - \frac{\bar{S}}{2}$; $s_j = \frac{\bar{S}}{2}$ Given that $\bar{S}' = \frac{\bar{S}}{2}$</th>
<th>$x_k$</th>
<th>$\lambda_k$</th>
<th>$\mu_k$</th>
<th>$\eta_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_k = \left( \frac{R_i}{p_x} \right)$</td>
<td>$\lambda_k = \left( \frac{1}{R_k} \right)$</td>
<td>$\mu_i = \left( \frac{\alpha_i}{\pi \alpha_i} \right) - \left( \frac{p_x}{R_i} \right)$</td>
<td>$\mu_j = \left( \frac{\alpha_j}{\bar{p}_x} \right) - \left( \frac{p_x}{R_j} \right)$</td>
<td>$\eta_k = 0$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2: Indirect utilities for homogeneous and heterogeneous agents under sequential space allocation without public intervention

<table>
<thead>
<tr>
<th>Indirect utility $V_{i}^{m(1)}(R_i, p_s, p_x, S, \alpha_k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{i}^{m(1)} = V_{k}^{m(1)}$</td>
</tr>
<tr>
<td>$V_{i}^{m(3)} = \alpha_i \ln \left( \bar{S} - s_i \right) + \ln \left( R_i - \ln p_x \right)$ and $V_{j}^{m(3)} = \alpha_j \ln \left( \bar{S} - s_j \right) + \ln \left( R_j - \ln p_x \right)$</td>
</tr>
</tbody>
</table>

Appendix C: Sequential space allocation with public intervention (S3)

The results found in Case 1 under sequential space allocation with public intervention are identical to those found in Case 1 under the two previous situations. The results found in Case 1’ are identical to those found in Case 1’ under simultaneous allocation.
Table 3.1: Results for homogeneous and heterogeneous agents under sequential space allocation with public intervention

<table>
<thead>
<tr>
<th>Case</th>
<th>$s_k$ with $\tilde{R}_i$</th>
<th>$x_k$ with $\tilde{R}_j$</th>
<th>$\lambda_k$</th>
<th>$\mu_k$</th>
<th>$\gamma_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>$R_i - p_x (\tau S)$ and $R_j = R_k - p_x (1 - \tau) S$</td>
<td>$x_k = \left( \frac{R_k}{p_x} \right)$</td>
<td>$\lambda_k = \left( \frac{1}{R_k} \right)$</td>
<td>$\mu_i = \left( \frac{\alpha_i}{\tau S} \right)$ and $\mu_j = \left( \frac{\alpha_j}{1 - \tau)S} \right)$</td>
<td>$\gamma_i = 0$</td>
</tr>
<tr>
<td>Case 2</td>
<td>$R_i - p_x (\tau S)$ and $R_j = R_k - p_x (1 - \tau) S$</td>
<td>$x_k = \left( \frac{R_k}{p_x} \right)$</td>
<td>$\lambda_k = \left( \frac{1}{R_k} \right)$</td>
<td>$\mu_i = \left( \frac{\alpha_i}{\tau S} \right)$ and $\mu_j = \left( \frac{\alpha_j}{1 - \tau)S} \right)$</td>
<td>$\gamma_i = 0$</td>
</tr>
</tbody>
</table>

Table 3.2: Indirect utilities for homogeneous and heterogeneous agents under sequential space allocation with public intervention

<table>
<thead>
<tr>
<th>Case</th>
<th>Indirect utility $V_k^{SeqPI(1)}(R_k, p_x, p_s, S, \tilde{\tau}, \tilde{\alpha})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 with $R_k = R_k - p_x S$</td>
<td>$V_k^{SeqPI(1)} = V_k^{SeqPI(1)} = V_k^{SeqPI(1)}$</td>
</tr>
<tr>
<td>Case 2 with $R_i = R_i - p_x \tau S$ and $R_j = R_j - p_x (1 - \tau) S$</td>
<td>$V_k^{SeqPI(3)} = \alpha_i \ln \tau + \ln S + \ln R_i - \ln p_x S$ and $V_k^{SeqPI(3)} = \alpha_j \ln \tau + \ln S + \ln R_j - \ln p_x S$</td>
</tr>
<tr>
<td>Case 2' with $R_i = R_i - p_x \tau S$, $S, \tilde{\tau}$, and $R_k = R_k - p_x S, S$</td>
<td>$V_k^{SeqPI(4)} = V_k^{SeqPI(3)} = V_k^{SeqPI(3)}$</td>
</tr>
</tbody>
</table>

Appendix D: Analyzing the shadow cost

We characterize the evolution of the shadow cost associated with the spatial constraint, $\mu_k$, towards other variables (relative preferences for the resource $\alpha_k$, and incomes $R_k$). Let us consider $\mu_i = \left( \frac{\alpha_i}{\tau S} \right) - \left( \frac{p_x}{R_i - p_x \tau S} \right)$ and $\mu_j = \left( \frac{\alpha_j}{1 - \tau)S} \right) - \left( \frac{p_x}{R_j - p_x (1 - \tau)S} \right)$. Four different situations are identified.

- If $\alpha_i < \alpha_j$ and $R_i < R_j$, then $\left( \frac{\alpha_i}{\tau S} \right) < \left( \frac{\alpha_j}{1 - \tau)S} \right)$ and $\left( \frac{p_x}{R_i - p_x \tau S} \right) > \left( \frac{p_x}{R_j - p_x (1 - \tau)S} \right)$, so that $\mu_i < \mu_j$.

- If $\alpha_i < \alpha_j$ and $R_i > R_j$, then $\left( \frac{\alpha_i}{\tau S} \right) < \left( \frac{\alpha_j}{1 - \tau)S} \right)$ and $\left( \frac{p_x}{R_i - p_x \tau S} \right) < \left( \frac{p_x}{R_j - p_x (1 - \tau)S} \right)$, so that the situation is undefined.

- If $\alpha_i > \alpha_j$ and $R_i < R_j$, then $\left( \frac{\alpha_i}{\tau S} \right) > \left( \frac{\alpha_j}{1 - \tau)S} \right)$ and $\left( \frac{p_x}{R_i - p_x \tau S} \right) > \left( \frac{p_x}{R_j - p_x (1 - \tau)S} \right)$, so that the situation is undefined.

- If $\alpha_i > \alpha_j$ and $R_i > R_j$, then $\left( \frac{\alpha_i}{\tau S} \right) > \left( \frac{\alpha_j}{1 - \tau)S} \right)$ and $\left( \frac{p_x}{R_i - p_x \tau S} \right) < \left( \frac{p_x}{R_j - p_x (1 - \tau)S} \right)$, so that $\mu_i > \mu_j$. 

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Consequently, if agent $i$ has relatively higher preferences for parking places $\alpha_i$ than her budget is low compared to agent $j$, then $\mu_i > \mu_j$. High preferences for parking places, as well as a high income, make the shadow value grow. The relation between $\mu_i$ and $\mu_j$ depends on the relative size of preferences and incomes between agents.