Sovereign debt spread and default in a simple model with self-fulfilling prophecies and asymmetric information

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Abstract:

The outbreak of the Greek crisis has revived the literature on the sovereign debt spreads. Recent evidence has shed new lights on the main determinants of interest rates spreads. It has been notably shown that the sharp increase of government bond yields cannot be entirely attributed to changes in macroeconomic fundamentals. Contagion effects can occur and self-fulfilling speculation may arise. Yet, this literature has been mainly empirical and needs sound theoretical foundations. The aim of this paper is to fill in this gap. We develop a simple model in the spirit of second generation currency crises models developed by (Obstfled, 1996) or (Sachs, Tornell, & Velasco, 1996). The model describes a strategic game between governments and financial markets. Eurozone countries face indeed a trade-off as governments may either commit and implement restrictive fiscal policies or default on debt. The cost of the commitment strategy increases when interest rates increase or when the fiscal multipliers are high. This leaves the opportunity for speculators to drive the economy towards a bad equilibrium in which the sovereign debt risk premium increases forcing the government to renege its commitment. Compared to Sachs, Tornell, & Velasco, (1996), we introduce a source of uncertainty about the cost of default in the model. The cost is not known to financial markets and can only be expected. By this way, we may introduce the possibility that governments do not default although risk premiums on bond yield is high.

JEL Codes: H63, E44, E61.

Key Words: Sovereign default, risk premium, self-fulfilling prophecies.

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Introduction

From 2011, EMU countries have engaged a strategy of frontloaded fiscal consolidation. Despite evidence that such a strategy could be self-defeating (Holland & Portes, 2012) and detrimental to growth, fiscal stance has remained restrictive in 2012, in 2013 and it will still be in 2014. This strategy is clearly sub-optimal (see OFCE, IMK, & ECLM, 2012) regarding new evidence on the size of fiscal multipliers. Recent literature has showed that fiscal multipliers depend on the macro-financial environment. They are indeed higher when the unemployment rate is high and when a banking crisis occurs, which is precisely the situation of some (if not most) EZ countries from 2009 and onwards.

From there, the strategy of frontloaded fiscal consolidation may be justified either by a misperception of the size of fiscal multipliers or by a credibility argument. Some countries (mainly from the South of Europe: Greece, Spain, Italy and Portugal) faced a rapid surge in the risk premium on sovereign bond yields. Whereas spreads did not exceed 60 basis points in the pre-crisis period, they had started to rise moderately with the outbreak of the financial crisis. Then, a break occurred at the autumn 2009 after the newly elected Greek government revised strongly upward the deficit figures for 2009 and after the Gulf emirate of Dubai asked for moratorium on the debt of a public conglomerate. These news triggered a regime-switch as financial markets suddenly realized that a default on public debt might not be excluded (Gibson, Hall, & Tavlas, 2012). Therefore, they started to cast some doubt on the credibility of the Greek government to deal with the need to restore public finances sustainability. Contagion rapidly gained other EMU countries which then felt urged to reduce public deficits to show their commitment to fiscal sustainability. Fiscal consolidation was perceived as the only solution to enforce credibility and to step down the risk premium. Conversely, it was thought that delaying the adjustment would have led to an explosion of the spreads and eventually to the split up of the Eurozone.

Credibility has yet not improved despite the measures taken ex-ante by governments to improve their fiscal position. Spreads have still increased in 2011 and 2012. Even if it is almost impossible to assess what would have been the development of spreads if countries had considered an alternative fiscal strategy, it remains that consolidation has not been a sufficient condition to ensure credibility. The deterioration of other macroeconomic fundamentals (notably the output gap) may also matter as less future growth resulting from harsh consolidation reduces the ability to repay debt in the future. Credibility and risks of default result from complex interactions where self-fulfilling prophecies or contagion effects also matter.

Actually, empirical literature on sovereign debt spreads has gained new momentum since 2009 and the outbreak of the Greek crisis. It has shed new lights on the main determinants of interest rates spreads. It has been suggested that a deterioration of the fiscal position (through changes in fiscal variables flows or stocks) have a positive impact on government bond yields, imposing by this way discipline on governments (Schucknecht, Von Hagen, & Wolswijk, 2009). But other fundamentals may also matter. External imbalances (Alessandrini, Fratianni, Hughes Hallet, & Presbitero, 2012) or business cycles

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4 The IMF (see Blanchard & Leigh, 2013) has indeed recognised that fiscal multipliers had been underestimated.

5 See (Haugh, Ollivaud, & Turner, 2009) for a synthesis on the impact of increases of debt or deficit on the interest rates.
(Grandes, 2007) have been shown to have an impact of bond yields spreads. However this recent evidence also confirms that the sharp increase of government bond yields cannot be entirely attributed to changes in macroeconomic fundamentals. Members of monetary union are notably more prone to liquidity squeeze as they are indebted in a money over which they do not have complete control (De Grauwe, 2011). As financial markets are aware that these countries cannot force the central bank to act as a lender of last resort for public debt, they may be more inclined to test the ability of members of monetary union to redeem their debt. These countries face the same situation as emerging countries issuing debt in foreign currencies: the “original sin” problem as emphasized by (Eichengreen, Hausmann, & Panizza, 2005). Hence, the relation between sovereign debt spreads and fundamentals may be nonlinear reflecting changes in the financial environment and notably the general risk pricing (Bernoth, Von Hagen, & Schuknecht, 2006). (Argyrou & Kontonikas, 2011) illustrate for example the contagion effect by suggesting that the rise in the spread on Greek bonds has been passed through the sovereign debt spread for most EMU countries during the crisis. Finally (Bruneau, Delatte, & Fouquau, 2012) suggest that the probability of default is a nonlinear function of fundamentals and driven by self-fulfilling speculation⁶. They highlight the market perception of risks influenced notably by the sovereign CDS market. The perception of risks would then be a key point for explaining sovereign spreads though in practice government defaults are extremely rare events for advanced economies (Buiter & Rahbari, 2013).

Yet, this literature has been mainly empirical and needs solid theoretical foundations. The aim of this paper is to fill in this gap. Amid the different features highlighted by the empirical literature, we consider that a model describing the determinants of sovereign debt spread should encompass the following characteristics: the role of macroeconomic fundamentals (not only fiscal position but also business cycle), risk pricing (related to risk aversion of market) and self-fulfilling prophecies. Some tentative have already been proposed (Bruneau et al., 2012) or (Argyrou & Tsoukalas, 2011), inspired by exchange rate crises literature. The second generation currency crises models describe games where there are interactions between expectations of market participants and decisions made by the central bank regarding the peg (Obstfled, 1996) or (Sachs, Tornell, & Velasco, 1996). Central banks face a trade-off between unemployment and devaluation. The incentive to exit the peg increases with the unemployment rate. Speculators are aware of this trade-off and may ask for higher interest rates to offset the risk of devaluation which raises the unemployment rate and triggers the collapse of the exchange rate regime.

In the current context, Eurozone countries face the same kind of trade-off. Governments may either commit and implement restrictive fiscal policies or default on debt. The cost of the commitment strategy increases when interest rates increase or when the fiscal multipliers are high. This leaves the opportunity for speculators to drive the economy towards a bad equilibrium in which the sovereign debt risk premium increases forcing the government to renege its commitment. Compared to Sachs, Tornell, & Velasco, (1996), we introduce information asymmetries in the model. The cost of default is not known to financial markets. By this way, we may introduce the possibility that governments do not default although risk premiums on bond yield is high⁷.

⁶ See also (De Grauwe & Ji, 2013).
⁷ Which is not the case in a one-period model with rational expectations. Once the random shock has occurred, government and speculators may compute the cost of the commitment strategy and the cost of default and choose accordingly the optimal strategy.
The rest of the paper is organized as follows. The portfolio choice of investors is shortly described in the first section. The model without information asymmetries is described in the second section. The introduction of information asymmetries is made in the third section. The fourth section draw some economic policy conclusions based on the main results of the model. The fifth section concludes.

A model of portfolio choice with default

In this section, we develop a model to describe the main components of the risk premium. This is a simple model of portfolio choices where investors allocate their wealth between a risk-free asset and a risky asset which is a domestic asset issued by the government. It takes into account risk aversion and a probability that the government default on public debt. Such kind of models can be based on term structure models of interest rates to account for long maturity issuances. A general model of term structure is developed following (Shiller, 1979), and completed with a portfolio choice model.

The expected one-period holding yield $H_t^n$ of a risky asset, a bond, maturing in $n$ periods, given by equation (1.1) is equal to expected price of the bond at time $t + 1$, which amounts to $P_{t+1}^n$ with a probability $1 - Prob_d$ that the government do not default on debt, and amounts to the expected value of the bond if the government defaults at time $t + 1$, approximated by $(1 - d^e)P_t^n$ plus a coupon payment $C$ at the end of the period, minus the price of the bond at time $t$, divided by the $P_t^n$. We assume that $d^e$ is the expected capital loss – or discount – on the bond price and $Prob_d$ is with the probability of default. Both terms are supposed to be exogenous for individual investors. It is clear that the capital loss is not known a priori. In the case of sovereign default, once it is announced by the government, an open discussion is generally opened between the borrower and its creditor to fix the amount of the discounted payment. The exogeneity of the probability of default may be justified either by the atomistic weight of an individual investor or by the fact that this probability simply reflects the historical probability of default, which is very low for advanced economies.

$$H_t^n = \frac{C + (1 - Prob_d)(E_tP_{t+1}^{n-1} + Prob_d(1 - d^e)P_t^n - P_t^n)}{P_t^n}$$

The yield to maturity $R_t^n$ on an $n$-period bond is determined by the requirement that the price $P_t^n$ of the bond is the present value of coupons and principal discounted by $R_t^n$:

$$P_t^n = \frac{C}{R_t^n} + \frac{R_t^n - C}{R_t^n (1 + R_t^n)^n}$$

Substituting (1.2) in (1.1), linearizing around $C = R_t^n = R_{t+1}^{n-1} = \bar{R}$ and $Prob_d = \bar{Prob}_d$, simplifying and rearranging gives:

$$H_t^n = \frac{R_t^n - \gamma_n E_t R_t^n}{1 - \gamma_n} = Prob_d. d_{t+1}^e$$

8 Such an approach is close to (Bernoth et al., 2006).
9 It may also be noticed that sovereign default are not ruled out by a supranational authority or court having the ability to resolve failing government and even less to proceed to liquidation. Then, the amount of capital losses may always be disputed.
With \( \gamma_n = \gamma \left[ \frac{1 - \text{Prob}_d}{\gamma (1 - \text{Prob}_d + R)} \right]^{\gamma n - 1} \) and \( \gamma = \frac{1}{1 + \bar{R}} \).

\( E_t R_{t+1}^n \) is the expected yield to maturity at time \( t + 1 \), with variance \( \sigma^2_{R^n} \). The investor divides his investment fund into fractions \( \omega_1 \) which is allocated to the purchase of long-term risky bonds and \( \omega_2 \) allocated to the purchase of risk-free short-term bonds that pay \( r_t \). The expected return of its portfolio has mean:

\[ (1.4) \quad \rho_t = \omega_1 (1 + H_t) + \omega_2 (1 + \tau_t) \]

And variance:

\[ (1.5) \quad S_t^2 = \omega_1 \left[ \frac{\gamma_n^2 \sigma_R^2}{(1 - \gamma_n)^2} + (\text{Prob}_d)^2 . \sigma_{d^e}^2 \right] \]

The risk of the portfolio notably depends only positively on the probability of default and on \( \sigma^2_{d^e} \), that can be viewed as an indicator of the uncertainty around the amount of capital loss in case of default. For sake of simplicity we assume that \( R_{t+1}^n \) and \( d_{t+1}^e \) are not correlated but this hypothesis shall be discussed: in case of default, the price of the bond would fall around \((1 - d^e) P^n_t\) and the yield to maturity would go up. In the worst case, \( (1.5) \) would then underestimate the variance of the expected return of the portfolio.

Assume the existence of a utility function \( U \) that orders the risk-averse investor’s preferences according to the values of the couple \((\rho_t, S_t^2)\). The optimal choice for \( \omega \) is obtained by maximising \( U(\rho_t, S_t^2) \) under the constraint \( \omega_1 + \omega_2 = 1 \). First order conditions give:

\[ (1.6) \quad \frac{\partial U}{\partial \rho} (1 + \tau_t) = \frac{\partial U}{\partial \rho} (1 + H_t) + 2 \frac{\partial U}{\partial S^2} \omega_1 . \left[ \frac{\gamma_n^2 \sigma_R^2}{(1 - \gamma_n)^2} + (\text{Prob}_d)^2 . \sigma_{d^e}^2 \right] \]

and

\[ (1.7) \quad \omega_1 + \omega_2 = 1 \]

The optimal share of long-term risky bonds is then:

\[ (1.8) \quad \omega_1 = \left[ \frac{v(1 - \gamma_n)^2}{\gamma_n^2 \sigma_R^2 + (1 - \gamma_n)^2 (\text{Prob}_d)^2 . \sigma_{d^e}^2} \right] [H_t - \tau_t] \]

with \( v = -\frac{1}{2} \frac{\partial U / \partial \rho}{\partial U / \partial S^2} \).

For \( \frac{\partial U}{\partial S^2} < 0 \), the investor is risk-averse as utility decreases when the risk of the portfolio increases. The share of the risky asset increases with the return of the risky asset and decreases, for risk-averse investor, with the risk of the risky asset, the probability of default and the variance of the capital loss. The yield to maturity on the risky bond is then given by:

\[ (1.9) \quad R_t = (1 - \gamma_n) [H_t + \phi_t] + \gamma_n . R_{t+1} \]

with \( \phi_t = \text{Prob}_d . d_{t+1}^e + \frac{1}{v} \left[ \frac{\gamma_n^2 \sigma_R^2}{(1 - \gamma_n)^2} + (\text{Prob}_d)^2 . \sigma_{d^e}^2 \right] \omega_1 \)
Let’s consider now that, at the market equilibrium for the bond asset, demand by investors is equal to supply of bond. For sake of simplicity, we may suppose the behaviour of a representative investor whose wealth-to-GDP is $w$. The demand for the government bond – the risky asset here – is simply: $\omega_1 w$. The total debt-to-GDP ratio is debt is given by $b$, so that the risk premium is finally given by:

$$\phi_t = \text{Prob}_d. d_t^{e} + \frac{1}{v} \left[ \frac{\gamma_n^2 \sigma_R^2}{(1-\gamma_n)^2} + (\text{Prob}_d^2) \sigma_d^2 \right]^\frac{b}{w}$$

Here it only adds the fact that risk premium also depend on the debt-to-GDP ratio.

Iterating (1.9) through the future, it gives:

$$R_t = \frac{1-\gamma}{1-\gamma_n} \sum_{j=0}^{n-1} \gamma^j E_t [r_{t+j} + \phi_{t+j}]$$

The long-term risky bond yield to maturity is equal to a weighted sum of expected future short-term interest rates and a risk premium that depends on the degree of risk aversion, the volatility on the risky bond market, a probability that the government will default in the future and the expected capital loss in case of default. If investors are risk neutral, then the premium only depends on the default and the expected discount. If expected premium and expected short term rate are constant, then equation (1.10) simply becomes $R_t = r_t + \phi_t$.

**A model of default on public debt with self-fulfilling features**

We now develop a simple macroeconomic model, encompassing the effect of fiscal policy on economic activity and public debt dynamics. The model also takes into account the interest rates (including a risk premium) effects on debt and economic dynamics. Besides, the policy-maker, here the government, has the choice to default partly on public debt.

Consider an economy in which the output gap $og$ positively depends on the fiscal impulse $FI$, interest rate and a random demand shock. The impact of fiscal policy depends on the size of the fiscal multiplier identified by the parameter $k$. The interest rate $r + \phi$ – including the risk premium – has a negative on output gap.

$$og_t = k.FI_t - \alpha(r_t + \phi_t) + \mu_t$$

Debt dynamics depends on past debt including debt burden, cyclical component of taxes and public expenditures $\varphi$. $og_t$, the fiscal impulse and the value of repudiated debt $d_t. b_{t-1}$ in case of default:

$$b_t = (1 + r_t + \phi_t) b_{t-1} - \varphi \cdot og_t + FI_t - d_t. b_{t-1}$$

We assume $0 \leq d_t \leq 1$ since other case would have no sense in this type of model.
Based on the previous portfolio choice model, the risk premium is given by equation a positive function of default probability, expected discount and risk aversion:

\[
\phi_t = \text{Prob}_d, d_t^e + \tilde{\phi}_t
\]

where \(\tilde{\phi}_t = \frac{1}{\nu} \left[ \gamma_n^2 \sigma^2 + (\text{Prob}_d)^2 \cdot \sigma^2 \right] \omega_1 + (\text{Prob}_d)^2 \cdot \sigma^2 \cdot \omega_1 \) increases with risk aversion and uncertainty.

The authorities’ objective is to minimize:

\[
\frac{1}{2} \left( og_t^2 + \lambda (b_t - \bar{b})^2 + \delta d_t^2 \right)
\]

Objective function (2.4) indicates that the policymaker dislikes a wide open output gap (since it implies high inflation or high unemployment), as well as a debt to far from a target \(\bar{b}\) (think of the 60% target for EMU member countries) and repudiation of a part of the public debt. She sets \(FI_t\) and \(d_t\) to minimize (2.4) subject to equations (2.1), (2.2) and (2.3) and the market’s expectation of the extent of default on public debt.

Under discretion, the policymaker sets \(FI_t\) and \(d_t\) optimally, taking \(d_t\) as given. The solution to this problem is

\[
og_t = -\frac{\lambda x (1-\varphi_k)}{k} B_t, (b_t - \bar{b}) = zB_t \text{ and } d_t = \frac{\lambda x b_t - \mu}{\delta} B_t
\]

where \(Z = \frac{\delta k^2}{\delta k^2 + \lambda k^2 b_t^2 + \lambda \delta (1-\varphi_k)^2} \).

and \(B_t = (b_{t-1} - \bar{b}) + (r_t + \text{Prob}_d, d_t^e + \tilde{\phi}_t) (b_{t-1} + \frac{\alpha}{k}) - \frac{\mu_t}{k} \).

Using (2.1) the fiscal impulse is then \(FI_t = -\frac{\lambda x (1-\varphi_k)}{k^2} B_t + \frac{\alpha (r_t + \text{Prob}_d, d_t^e + \tilde{\phi}_t) - \mu_t}{k} \).

The policymaker sets the fiscal impulse to minimize its loss function: a high level of debt, a high debt burden (enlarged by high risk premium) and positive shocks on the output gap imply negative fiscal impulses. Conversely, a negative shock on output gap implies a positive fiscal impulse to stabilize it.

Using (2.5), the loss for the policymaker is

\[
L^{d} = \frac{1}{2} \lambda, z, B_t^2
\]

where the subscript \(d\) stands for “default”. If in addition, we impose the perfect foresight condition that \(d_t^e = d_t\) then we obtain from (2.5) that

\[
d_t^e = \frac{\lambda x b_t - \mu}{\delta - \lambda x b_t, \text{Prob}_d, d_t + \tilde{\phi}_t, \frac{\alpha}{k}} \left( b_{t-1} - \bar{b} \right) + \left( r_t + \tilde{\phi}_t \right) \left( b_{t-1} + \frac{\alpha}{k} \right) - \frac{\mu}{k} \)

Now suppose that the policymaker is able to commit and is supposed not to default. Then, \(d_t = 0\) and we have:
\[(b_t - \bar{b}) = \tilde{z}.B_t \text{ and } \alpha g_t = \frac{-\lambda \tilde{z}(1 - \varphi k)}{k}B_t\]

where \(\tilde{z} = \frac{k^2}{k^2 + \lambda(1 - \varphi k)^2}\)

and the corresponding loss is

\[(2.9) \quad L^e = \frac{1}{2} \lambda . \tilde{z}.B_t^2\]

The government always have an incentive to default, as \(\tilde{z} > z\) since \(\lambda . k^2 . b_{t-1}^2 > 0\). Next, if we assume that the government faces an additional fixed private cost \(c > 0\) when it defaults on its debt.

If expectations of the discount in case of default is given by \(d^e\), the government finds it optimal to default if and only if \(L^d + c < L^e\). Using (2.6) and (2.9) we get

\[(2.10) \quad (b_{t-1} - \bar{b}) + (r_t + Prob_d\cdot d^e + \tilde{\phi}_t)(b_{t-1} + \frac{\alpha}{k}) - \frac{\mu_t}{k} > K\]

where \(K = [2c]^{1/2}[\lambda(\tilde{z} - z)]^{-1/2}\). A default will then occur in equilibrium if:

- inherited debt is sufficiently high relative to the debt target,
- debt burden is sufficiently high, meaning notably that a restrictive monetary policy, by increasing the risk-free interest rate, increases the risk of default,
- a sufficiently high negative shock on output gap occurs, especially when the fiscal multiplier is low,
- risk aversion is sufficiently high, which would be translated in higher risk premium
- expectations of default and are sufficiently high.

Financial markets understand the temptation summarized by equation (2.10) and then rationally determine expectations of default.

If agents expect \(d^e = 0\), it is a rational equilibrium only if

\[(2.11) \quad (b_{t-1} - \bar{b}) + (r_t + \tilde{\phi}_t)(b_{t-1} + \frac{\alpha}{k}) - \frac{\mu_t}{k} \leq K\]

If agents expect a default, the rational equilibrium condition is obtained by combining equations (2.7) and (2.10). This condition holds if \((b_{t-1} - \bar{b}) + (r_t + \tilde{\phi}_t)(b_{t-1} + \frac{\alpha}{k}) - \frac{\mu_t}{k} > \frac{K}{1 + n.\text{Prob}_d(b_{t-1} + \frac{\alpha}{k})}\) where

\[n = \frac{\lambda x b_{t-1}}{\delta - \lambda x b_{t-1} \cdot \text{Prob}_d(b_{t-1} + \frac{\alpha}{k})}\]
Following Figure 1, since default occurrence depends on expectations, we can isolate three cases\(^\text{10}\). In the first one, fundamentals (inherited debt, debt burden and shock) are too low to entail a default, whatever the level of expected default. For levels of fundamentals \((b_{t-1} - \bar{b}) + (r_t + \phi_t)(b_{t-1} + \frac{\alpha}{k} - \mu_t + \frac{\alpha}{k})\), no larger than \(rac{K}{1 + n \cdot \text{Prob}_d(b_{t-1} + \frac{\alpha}{k})}\), only one equilibrium with no default is possible: attaching a positive probability to default cannot be rational, since no default will occur whatever agents expected. The government enjoys full credibility.

In the other extreme case, fundamentals are too high / bad (greater than \(K\)). Whatever the level of expected default, the government will always default on debt.

In the intermediary case, there are multiple equilibria: one in which the market expects the default and the default occurs, and one in which the market expects no default and the government does not default. For levels of fundamentals in the range \([\frac{K}{1 + n \cdot \text{Prob}_d(b_{t-1} + \frac{\alpha}{k})}, K]\), if the market expects a default of size \(\frac{\lambda \cdot b_{t-1}}{\delta - \lambda \cdot b_{t-1} \cdot \text{Prob}_d(b_{t-1} + \frac{\alpha}{k})} \cdot (b_{t-1} - \bar{b}) + (r_t + \phi_t)(b_{t-1} + \frac{\alpha}{k} - \mu_t + \frac{\alpha}{k})\), then the policymaker will validate such expectations; on the contrary, if markets expect no default, then the government does not default in fine. Credibility of the government is partial and depends on expectations.

The size of these three zones depends on the value of \(K\). It can be shown that \(K\) increases with the cost of default, the weights of public debt and of the amount of default in the loss function of the government, the impact of interest rate on the output gap. Then for higher cost of default, the area of full credibility increases and the zone where the government has no credibility is pushed away. Nonetheless, it must also be noted that the area of partial credibility widens meaning that the risk of multiple equilibria amplifies. Concerning the value of fiscal multiplier, it can be shown that \(K\) decreases with \(k\). Then, in time of crisis, recent evidence showing that fiscal multiplier are higher imply that the government may lose credibility and enter the intermediate area where expectations of defaults can force the government to default on public debt even if the fundamentals have not deteriorated. Finally, it must be stressed that once the shock is revealed here, there is no more uncertainty. Financial markets are fully aware of fundamentals and of the

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\(^{10}\) There is then a clear connection with the cases highlighted by (Sachs et al., 1996).
trade-off faced by government. Then, if they expect capital losses, \( d_t^e > 0 \), the rational solution leads to a default on public debt.

**The case for differentiated costs of default**

In the model above, we assumed that financial markets perfectly know the fixed cost of default \( c \). Consequently, the part of the risk premium \( (Prob_{d_t} d_t^e) \) associated with a potential default is either null \( (d_t^e = 0 \text{ and there is no default}) \) or positive \( (d_t^e > 0 \text{ and the government defaults}) \). Another point is that it is not clear why markets would suddenly shift from equilibrium with credibility to equilibrium without credibility, forcing the government to default when economy is in the partial credibility zone. Actually, as stressed by (Buiter & Rahbari, 2013), sovereign defaults are always a political choice. But it is a costly choice as even if no supranational authority can enforce the government to pay its debt, it must yet face a loss of credibility, a restricted ability to raise funds in the future, political costs, costs needed to recapitalize the banking sector… that may not be proportional to the extent of the default or to any other macroeconomic variable. Conversely, governments have indeed always the ability to increase taxes or cut public expenditures to honour its debt. But the needed consolidation may not be socially or politically accepted. There is then a trade-off between economic, social and political costs of consolidation and the cost of default. This trade-off is highly uncertain as neither the social or political cost of consolidation, nor the costs in terms of reputation are observable. According to us, the fact that financial markets don’t know the government’ fixed cost can explain such a changing behaviour.

Assume that markets expect a fixed cost lower than the government’s one \( \hat{c} < c \). Default conditions do not change for the government, but do change from financial markets point of view. Now financial markets expect a default if

\[
(3.1) \quad (b_{t-1} - \bar{b}) + (r_t + Prob_{d_t} d_t^e + \bar{\phi}_t) \left( b_{t-1} + \frac{a}{k} \right) - \frac{\mu_t}{k} > \bar{K}
\]

where \( \bar{K} = [2\hat{c}]^{1/2} [\lambda (\underline{z} - \bar{z})]^{-1/2} \)

Then, two cases deserve attention according to the credibility hypothesis.

The first situation corresponds to the case where the government is the area of full credibility whereas financial markets expecting a lower cost of default are in the partial credibility zone. They may then expect default if:

\[
(3.2) \quad (b_{t-1} - \bar{b}) + (r_t + \bar{\phi}_t) \left( b_{t-1} + \frac{a}{k} \right) - \frac{\mu_t}{k} > \frac{\bar{K}}{1+nProb_{d_t}(b_{t-1}+\frac{a}{k})}
\]

If condition (3.2) is met but if
\begin{equation}
(b_{t-1} - \bar{b}) + \left( r_t + \phi_t \right) (b_{t-1} + \frac{\alpha}{k}) - \frac{\mu_t}{k} \leq \frac{K}{1+n\cdot \text{Prob}_d(b_{t-1} + \frac{\alpha}{k})},
\end{equation}

then the government does not default whereas the default premium may be positive. The loss of the government is indeed lower when not defaulting than when defaulting. In such a situation, fundamentals are not sufficiently “deteriorated” to force the government to default, but as the expected cost of default is weaker for financial markets, they may then expect capital losses, increasing the risk premium.

The second situation is represented by the possibility that:

\begin{equation}
(3.3) \quad (b_{t-1} - \bar{b}) + \left( r_t + \phi_t \right) (b_{t-1} + \frac{\alpha}{k}) - \frac{\mu_t}{k} \geq \bar{R}
\end{equation}

In that case, the market expects default. But the default occurs as soon as:

\begin{equation}
(3.4) \quad (b_{t-1} - \bar{b}) + \left( r_t + \phi_t \right) (b_{t-1} + \frac{\alpha}{k}) - \frac{\mu_t}{k} > \frac{K}{1+n\cdot \text{Prob}_d(b_{t-1} + \frac{\alpha}{k})}.
\end{equation}

In that case, the government may be partially credible but forced into default as financial markets consider that the cost of default is weak. Here, the range of multiple equilibria has been reduced, meaning that, for the same value of fundamentals, the possibility that financial markets systematically expect default is increased. Then, the situation of default would occur more frequently or rapidly. Let’s suppose for example that we start from a situation of sound macroeconomic fundamentals as illustrated in the previous case. The government has no incentive to default even if, due to a weaker expected cost of default, a risk premium may appear in the financial markets. Consider then the occurrence of a strong adverse shock deteriorating the economic situation. Such a shock may push the government in the partial credibility area, so that default may become an optimal solution if it is expected by financial markets. The default scenario may become certain if at the same time, the adverse shock push fundemandals such that condition (3.3) is now met.

On the contrary, if \( \bar{R} \) sufficiently low, then \( \bar{R} < \frac{K}{1+n\cdot \text{Prob}_d(b_{t-1} + \frac{\alpha}{k})} \) and the government does not default. In that case, there is a huge gap between the expected costs of default of government and financial markets.

Finally, we may also consider a final situation where \( K < \bar{R} \). Here, the expected cost of default is higher for financial markets than for financial markets: \( \hat{c} > c \). Here, a default may occur even if it is not expected by financial markets. There is not risk premium but the value of fundamentals for which the government
considers that it is preferable to default is so low that it chooses to default. This is also true for:
\[ K < \frac{R}{1 + n \cdot \text{Prob}_{d}(b_{t-1} + \frac{\sigma^2}{2})} \]

Considering this, what is the best strategy for the government to avoid the default and to decrease the risk premium? Actually, the government should convince financial markets that the cost of default is very high. A higher value for \( \bar{R} \) will indeed reduce the probability of default occurrence.

**Conclusion**

In this paper, we have developed a simple analytical model to illustrate the interconnections between fundamentals, risk premium incorporated on the sovereign spread and the risk of default. The model is inspired by exchange rate crises models of (Obstfeld, 1996) and (Sachs et al., 1996) and adapted to a situation where government decide rationally to default or not taking into account the risk premium, the output gap and the cost of implementing restrictive fiscal policy. The situation of some European countries during the recent sovereign debt crisis is clearly well illustrated by this kind of models. They notably suggest that self-fulfilling prophecies may force the government to default. Here, it is worth reminding that default is always and finally the consequence of a political choice where the government face a trade-off between implementing more austerity to reduce public debt or defaulting. In situations where fundamentals are deteriorated, either because the debt burden has increased, the risk aversion of financial markets have increased or a negative shocks has occurred, the credibility of the government may decrease and become partial. There are then multiple equilibria and the government is forced to default of this is the situation expected by financial markets. This kind of vicious circle is clearly representative of what occurred for European countries in the recent period. Yet, default is costly. The reputation of government may seriously undermined in case of default. The access to financial markets funding may be restricted for a sustained period. Besides, the threshold over which austerity measures become socially unsustainable is clearly unknown. The ability to raise taxes or to cut spending is part of the trade-off but is uncertain and can only be expected. There is then no reason that expectation of financial investors matches with the supposed cost of default used by the government when the rational choice is made. Multiple situations may occur, enriching the conclusions of the model of (Sachs et al., 1996) as we illustrate cases where financial markets expect default and capital losses whereas no default occurs since the cost of default is high for government and the value of fundamentals is good enough so that the government does not find it optimal to default.

Clearly, considering the uncertainty of the cost of default opens new issues regarding the possibility to influence private expectations of the cost of default. There is an incentive to convince financial markets that the cost of default is high. Besides, the role of institutions clearly matters. In the European context, the sovereign spreads were certainly fuelled by the debate on a possible exit of countries from the monetary union. The adoption of the TSCG has contributed to reinforce the viability of EMU. The role of the ECB and the announcement of the OMT have probably played an even greater role to lessen the sovereign spreads. By convincing the financial markets that it would stand ready to intervene to purchase public
debt, it has clearly sent a signal that default was less likely and more costly as it would have denied the ability of government to benefit from the support of the central bank. More research are yet needed to formalize more thoroughly this issue.

References


