Strategic Weight of Consumers and Product Differentiation

(Preliminary and Incomplete Version)

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Abstract

This paper aims to study how product market competition works on internal and external organizational governance (delegation decision, customer orientation). More specifically, we exhibit the strategic value of the strategic weight given to consumers in the customer-oriented firm’s objective function. We show that (i) in a Customer-oriented duopoly, the consumer’s optimal weight determined by both firms which practice the same delegation strategy (either both delegate, or both not delegate) decreases when products are less differentiated; (ii) in a duopoly when a customer-oriented firm competes with a shareholder-oriented firm, it puts more weight on consumer surplus in its objective function when goods are less differentiated, since consumer’s weight has a positive effect on its output. Journal of Economic Literature Classification Numbers: C72, L13, L20.

Key words: Organizational governance, Customer’s weight, Incentive scheme, Duopoly

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Introduction

In this paper, we consider a Cournot duopoly with differentiated products and treat managerial incentive and a strategic weight of consumers as specific issues of internal and external governance. We focus on a hybrid organization which takes its profit plus a share of consumer surplus into account (see also Goering, 2007, 2008a,b; Lambertini and Tampieri, 2010; Lien, 2002) and define such organization as customer-oriented. We study the interaction between product market competition and organizational governance when a customer-oriented firm competes with a shareholder-oriented (profit-maximizing) firm and when two customer-oriented firms compete.

It is argued that hybrid organizations might compensate their managers in a different way than their profit-maximizing rivals (e.g. Cai et al., 2011; Frye et al., 2006; Jegers, 2009; Mahoney and Thorne, 2005, 2006; Berrone and Gomez-Mejia, 2009; Deckop et al., 2006). For instance, the incentive contract (see Georgan, 2007; Kopel and Brand, 2012) of a customer-oriented firm is based on a weighted combination of firm’s profit and a weight given to the consumer surplus. As a result, both the profit-maximizing firm and the customer-oriented firm might prefer to hire a manager.

Moreover, in a famous work of managerial incentive, Fershtman and Judd (1987) suggest that if one firm’s manager is told to maximize sales revenue instead of profit, he will become a very aggressive seller and prove that a profit-and-sales based incentive contract induces firms to produce larger output, lower prices, and a more efficient allocation of production than the usual Cournot equilibria. Therefore, it is an efficient way to delegate decision to a
manager with a profit-and-sales based incentive contract. However, in a mixed duopoly, this is not always the case. Several works show that with such incentive contracts, only the profit maximizing firm might prefer to hire a manager whereas the other firm might prefer to be owner-managed (for mixed duopoly with a public firm, see Fernandez-Ruiz, 2009; White, 2001; for a profit maximizing firm competes with a Consumer cooperative, see Kopel and Marini, 2013).

On the base of the classical incentive scheme (see Fershtman and Judd, 1987) and the incentive contract of a customer-oriented firm (see Georing, 2007; Kopel and Brand, 2012), we propose a modified incentive scheme for a customer-oriented firm and offer two important contributions over the existing approaches to learning about managerial incentive and customer-oriented firms. First, our approach considers products to be homogenous and consumer’s weight is not optimally determined, e.g., Georing (2007), Kopel and Brand (2012). Here in this paper, we study the relationship between product differentiation and the strategic weight of consumers.

Second, our approach focus more on the delegation choice, e.g., Kopel and Brand (2012), Kopel and Marini (2013). While in this paper, the weight given to consumers in a customer-oriented firm’s objective function is treated strategically in terms of external governance. We study the strategic value of consumer’s weight within a customer-oriented firm rather than pure delegation choice.

In the model proposed in this framework, a customer-oriented firm would delegate a manager who similarly considers the interests of consumers. Moreover, the delegated manager is required to take the same weight of consumers as the
firm does, which is in reverse for the manager to show his sincerity and loyalty. This can be explained by Confucianism, "a universal ethic in which the rules and imperatives of behavior hold for all individuals" (Peter F. Drucker, Forbes, 1981). According to Peter Ferdinand Drucker (1981), Confucianism might be the most appropriate ethical approach applicable to organizations and their activities.

Besides, since there is not enough evidence showing that representative consumers take seats on the board of a hybrid organization, it is supposed in the model that the decision about the weight of consumers is made by shareholders. In other words, the consumer’s weight is strategically determined by shareholders whose basic fundamental interest is profit maximizing. We show in this paper that the consumer’s optimal weight determined by both customer-oriented firms which practice the same delegation strategy (either both delegate, or both not delegate) decreases when products are less differentiated. When one customer-oriented firm chooses to delegate, the other customer-oriented firm does not delegate, the consumer’s optimal weight that a delegate customer-oriented firm considers decreases when products are less differentiated. While for the other customer-oriented firm which does not to delegate, the relationship between product differentiation and the strategic weight of consumers is not monotonic. Yet when a customer-oriented firm which chooses to delegate competes with a shareholder-oriented firm, it would consider more weight of consumers when goods are less differentiated, regardless of the delegation situation of its rival. Moreover, when a customer-oriented firm competes with a shareholder-oriented firm, a growth of consumer’s weight which has a negative effect on good’s price and a positive effect on consumer surplus has a positive effect on the output of the customer-oriented firm but
a negative effect on that of the shareholder-oriented firm.

The rest of the paper is organized as follows. Section 1 introduces the basic model. The relationship between the optimal weight of consumers and product differentiation is discussed in Section 2. The strategic value of consumer’s weight is analyzed in Section 3. Section 4 concludes. Appendix A contains proofs of lemmas presented in the text.

1 Model

In a Cournot duopoly game, each firm $i, j$ ($i \neq j$), produces differentiated goods. The inverse demand function is given by

\[ p_i = \alpha - x_i - \gamma x_j \]  

(1)

where $0 < \gamma < 1$, which represents the degree of product differentiation (e.g., Singh and Vives, 1984). When $\gamma$ increases, goods become less differentiated.

The utility function of consumers $U (x_i, x_j) = \alpha (x_i + x_j) - \frac{1}{2} \left( x_i^2 + 2\gamma x_i x_j + x_j^2 \right)$, thereby, the consumer surplus

\[ CS = \frac{1}{2} x_i^2 + \gamma x_i x_j + \frac{1}{2} x_j^2 \]  

(2)

Moreover, it is supposed that there are no fixed costs and both firms have identical constant marginal costs $c$, with $0 < c < \alpha$.

1.1 Firm Types

We consider two types of firms by the following definition
Definition (i) One firm is called "shareholder-oriented" (i.e., S), if it considers the only interest of shareholders; (ii) one firm is called "customer-oriented" (i.e., C), if it considers not only the interests of shareholders but also that of consumers.

Clearly, the objective of a S firm is profit-maximizing, i.e.,

$$\max \pi^i = (p_i - c) x_i$$ \hspace{1cm} (3)

As for a C firm, its objective function is to maximize the sum of profit and a certain share of consumer surplus (e.g., Goering, 2007, 2008a,b; Lambertini and Tampieri, 2010; Kopel and Brand, 2012), i.e.,

$$\max V^i = \pi^i + \theta CS$$ \hspace{1cm} (4)

where the parameter $\theta \in (0, 1)$ represents the weight that a C firm $i$ puts on consumers.

1.2 Incentive Contracts

The S firm can delegate the production decision to a manager and sign an incentive contract with him. In line with the strategic incentives literature, we assume that the compensation contract of a S firm’s manager is based on a weighted average of profits $\pi_i$ and sales revenue $R_i = p_i x_i$ (see Fershtman and Judd, 1987; Kopel and Loffler, 2008; Sklivas, 1987; Kopel and Brand, 2012), i.e.,

$$\max f^i = (1 - \delta_i) \pi^i + \delta_i R_i$$

$$= (p_i - c (1 - \delta_i)) x_i$$ \hspace{1cm} (6)
where the parameter $\delta_i \in (0, 1)$ represents the incentive level of a S firm $i$. We can see that the incentive scheme works as an discount effect on the marginal cost.

The C firm can also delegate the production decision to a manager and sign a compensation contract with him. On the base of the strategic incentives literature (see Fershtman and Judd, 1987; Kopel and Loffler, 2008; Sklivas, 1987; Kopel and Brand, 2012), we assume that the compensation contract of a C firm’s manager is based on a weighted average of the firm’s objective and his manager’s objective. Under the impact of Confucian ethics, manager’s objective is supposed to be the sum of sales revenue and a certain weight of consumer surplus, i.e., $R_i + \theta CS$. To show sincerity and layerty, the manager takes the same weight of consumer surplus as the C firm does. The compensation contract of a C firm’s manager corresponds with the following objective function, i.e.,

$$\max \Omega^i = (1 - \delta_i) V^i + \delta_i (R_i + \theta CS)$$

$$= (p_i - c (1 - \delta_i)) x_i + \theta CS$$

where the incentive level $\delta_i \in (0, 1)$ of a C firm $i$ also works as an discount effect on the marginal cost.

1.3 Timing

At the very first stage, the weight ($\theta$) of consumer surplus is strategically decided by shareholders of a C firm. By the second stage, decision makers within each firm decide whether or not to sign an incentive contract with a delegated manager. For the one who chooses to delegate, the output is decided by a
manager with an incentive scheme; for the one who chooses not to delegate, the output is decided by the firm. In the third stage, firms engage in Cournot competition.

2 Product differentiation and consumer’s weight

In this section, we study the relationship between the product differentiation and consumer’s weight. We’ll show in the sub sections that the output of each firm after delegation decision will be in terms of \( \theta \) and \( \gamma \), thus the expression of each firm’s profit will also in terms of \( \theta \) and \( \gamma \). For instance, the profit of a C firm \( j \), we note \( \pi^j (x_i (\theta, \gamma), x_j (\theta, \gamma)) \). Since the weight of consumers is strategically determined by a C firm’s shareholders who care about profit. Let us define the first order derivative of \( \pi^j (x_i (\theta, \gamma), x_j (\theta, \gamma)) \) by \( g_k : (0, 1) \times (0, 1) \rightarrow R \), where \( k = \{CD, CN\} \) represents a C firm’s delegation choice: to delegate or not to delegate. Thus

\[
g_k (\theta, \gamma) = \pi^j = \pi^j_{x_i} \frac{\partial x_i}{\partial \theta} + \pi^j_{x_j} \frac{\partial x_j}{\partial \theta} . \tag{9}
\]

The comparative effect \( d\theta / d\gamma \) can be obtained by totally differentiating \( g_k (\theta, \gamma) \) with respect to \( \theta \) and \( \gamma \). Note that \( g_k (\theta, \gamma) = 0 \), therefore

\[
\frac{d\theta}{d\gamma} = - \left( \frac{\partial g_k}{\partial \gamma} \right) / \left( \frac{\partial g_k}{\partial \theta} \right) . \tag{10}
\]

2.1 Customer-oriented firm Versus Shareholder-oriented firm

In this case, a customer-oriented firm competes with a shareholder-oriented firm through the quantity produced. Both firms have situations of delegation:
delegate (we note "D") or not delegate (we note "N"). Hence, there are four sub cases to discuss.

2.1.1 SDCD sub game

When a customer-oriented firm competes with a shareholder-oriented firm, regardless of the delegation situation of each firm, it is always the customer-oriented firm which considers consumer surplus in its objective function. Thus, we denote \( \theta_j \equiv \theta \), which is the unique consumer’s weight of a customer-oriented firm \( j \). Let \( i \) represent the shareholder-oriented firm. The first order conditions \( F_{x_i}^i = 0 \) and \( \Omega_{x_j}^j = 0 \) result each manager’s reaction function as follows

\[
\begin{align*}
    x_i^S &= \frac{\alpha - c}{2} + \frac{\gamma}{2} \delta_i^S - \frac{\gamma}{2} x_j^C \\
    x_j^C &= \frac{\alpha - c}{2 - \theta} + \frac{c}{2 - \theta} \delta_j^C - \frac{\gamma(1 - \theta)}{2 - \theta} x_i^S
\end{align*}
\]  

(11)

Solving equation system (11) implies the Nash equilibrium quantities which are both represented by \( \theta, \gamma, \delta_i^S \) and \( \delta_j^C \), i.e.,

\[
\begin{align*}
    x_i^S (\theta, \gamma, \delta_i^S, \delta_j^C) &= \frac{(\theta + \gamma - 2) (\alpha - c) - c (2 - \theta) \delta_i^S + c \gamma \delta_j^C}{2\theta + \gamma^2 - \theta \gamma^2 - 4} \\
    x_j^C (\theta, \gamma, \delta_i^S, \delta_j^C) &= \frac{(\gamma - \theta \gamma - 2) (\alpha - c) - 2c \delta_j^C + c \gamma \delta_i^S (1 - \theta)}{2\theta + \gamma^2 - \theta \gamma^2 - 4}
\end{align*}
\]  

(12)

(13)

The optimal incentive of a S firm \( \delta_i^S (\theta, \gamma) \) and the optimal incentive of a C firm \( \delta_j^C (\theta, \gamma) \) are solved are follows

\[
\delta_i^S (\theta, \gamma) = \frac{\gamma^2 (\theta - 1) (-2\theta - 2\gamma - \gamma^2 + \theta \gamma^2 + 4) (c - \alpha)}{c (-2\theta - 2\gamma - \gamma^2 + \theta \gamma + \theta \gamma^2 + 4) (-2\theta + 2\gamma - \gamma^2 - \theta \gamma + \theta \gamma^2 + 4)}
\]  

(14)
\[
\delta_j^C (\theta, \gamma) = \frac{\gamma \left( G + \theta^2 \gamma^3 - \theta^3 \gamma^2 + 4 \theta \gamma - 5 \theta \gamma^2 - \theta^2 \gamma - 2 \theta \gamma^3 \right) (c - \alpha)}{c (-2 \theta - 2 \gamma - \gamma^2 + \theta \gamma + \theta \gamma^2 + 4) (-2 \theta + 2 \gamma - \gamma^2 - \theta \gamma + \theta \gamma^2 + 4)}
\]

where \( G = 4 \theta - 4 \gamma - 4 \theta^2 + 3 \gamma^2 + 3 \gamma^3 + 4 \theta^2 \gamma^2 \). Then, with the two incentives, we obtain the quantities in term of \( \theta \) and \( \gamma \), i.e.,

\[
x_i^{SD} (\theta, \gamma) = \frac{(2 - \theta) (\alpha - c) (-2 \theta - 2 \gamma - \gamma^2 + \theta \gamma^2 + 4)}{(-2 \theta + 2 \gamma - \gamma^2 - \theta \gamma + \theta \gamma^2 + 4) (-2 \theta - 2 \gamma - \gamma^2 + \theta \gamma + \theta \gamma^2 + 4)}
\]

\[
x_j^{CD} (\theta, \gamma) = \frac{(\alpha - c) (-\theta^2 \gamma + 2 \theta \gamma^2 + 4 \theta \gamma - 4 \theta - 2 \gamma^2 - 8 \gamma - 4)}{(-2 \theta + 2 \gamma - \gamma^2 - \theta \gamma + \theta \gamma^2 + 4) (-2 \theta - 2 \gamma - \gamma^2 + \theta \gamma + \theta \gamma^2 + 4)}
\]

2.1.2 SNCD sub game

In this subgame, the objective function of a S firm which does not delegate applies equation (3), i.e., \( \max \pi_i \), while the one of a C firm which delegate a manager applies equation (7 or 8), i.e., \( \max \Omega_i \). With the same method as the previous one, we get the optimal incentive

\[
\delta_j^C (\theta, \gamma) = \frac{\gamma \left( 2 \theta - 2 \gamma - \theta^2 + \gamma^2 + \theta^2 \gamma^2 + \theta \gamma - 2 \theta \gamma^2 \right) (\alpha - c)}{c (4 \theta + 4 \gamma^2 - 3 \theta \gamma^2 - 8)}
\]

and the quantities

\[
x_i^{SN} (\theta, \gamma) = \frac{(2 \theta + 2 \gamma + \gamma^2 - \theta \gamma^2 - 4) (\alpha - c)}{4 \theta + 4 \gamma^2 - 3 \theta \gamma^2 - 8},
\]

\[
x_j^{CD} (\theta, \gamma) = \frac{(2 \gamma - \theta \gamma - 4) (\alpha - c)}{4 \theta + 4 \gamma^2 - 3 \theta \gamma^2 - 8}.
\]

With the above outputs of the SDCD and the SNCD sub games, it is easy to study the sign of \(-\left( \frac{\partial g_k}{\partial \gamma} \right) / \left( \frac{\partial g_k}{\partial \theta} \right)\). We get the following lemma

**Lemma 1** For a competition between a C firm and a S firm, when the C firm delegates, regardless of the delegation situation of the S firm, we have \( \frac{\partial \theta}{\partial \gamma} > 0 \),

10
for all \((\theta, \gamma) \in (0, 1) \times (0, 1)\).

**PROOF.** see appendix.

This lemma suggests that the consumer’s optimal weight \(\theta\) that a C firm considers increases with \(\gamma\), regardless of the delegation situation of its rival, a S firm. In other words, when \(\gamma\) tends from zero towards one, goods become less differentiated, then a C firm facing a S firm as his rival, would consider more weight of consumer surplus in his delegated manager’s incentive contract. The intuition behind is simple, when we are close to a homogeneous market, the Cournot competition makes firms to increase output in order to gain the market share. As for a C firm which chooses to delegate, it may rise the share of consumer surplus to increase output, regardless of the delegation situation of its rival, a S firm.

### 2.1.3 SDCN sub game

In this sub game, the objective function of a S firm’s delegated manager applies equation (5 or 6), i.e., \(Max F_i\), while the one of a C firm which does not delegate applies equation (4), i.e., \(Max V_i\). We get the optimal incentive

\[
\delta^S_i (\theta, \gamma) = \frac{\gamma^2 (1 - \theta) (2 - \theta - \gamma) (\alpha - c)}{2c (2 - \theta) (-\theta - \gamma^2 + \theta\gamma^2 + 2)},
\]

(21)

and the quantities

\[
x^SD_i (\theta, \gamma) = \frac{(2 - \theta - \gamma) (\alpha - c)}{2 (-\theta - \gamma^2 + \theta\gamma^2 + 2)},
\]

(22)

\[
x^CN_j (\theta, \gamma) = \frac{(-\theta^2 \gamma + \theta\gamma^2 + 3\theta\gamma - 2\theta - \gamma^2 - 2\gamma + 4) (\alpha - c)}{2 (2 - \theta) (-\theta - \gamma^2 + \theta\gamma^2 + 2)}.
\]

(23)
2.1.4 SNCN sub game

In this sub game, none of the firms delegate. Without incentives, the objective function of a S firm applies equation (3), i.e., $Max\pi_i$, while the one of a C firm applies equation (4), i.e., $MaxV_i$. We obtain the quantities in terms of $\theta$ and $\gamma$

\[ x_{i}^{SN}(\theta, \gamma) = \frac{(2 - \theta - \gamma)(\alpha - c)}{\theta\gamma^2 - \gamma^2 - 2\theta + 4}, \]  
(24)

\[ x_{j}^{CN}(\theta, \gamma) = \frac{(2 - \gamma + \theta\gamma)(\alpha - c)}{\theta\gamma^2 - \gamma^2 - 2\theta + 4}. \]  
(25)

Lemma 2 For a competition between a C firm and a S firm, when the C firm does not delegate, regardless of the delegation situation of the S firm, we have a non-monotonic relationship between $\theta$ and $\gamma$.

Proof. see appendix.

This lemma suggests that the consumer’s optimal weight $\theta$ that a C firm considers first decreases then increases when $\gamma$ rises, regardless the delegation situation of its rival, a S firm. In other words, when $\gamma$ tends from zero towards one, goods become less differentiated, then facing a S firm whether delegates or not, a C firm which does not delegate would first reduce then increase the strategic weight of consumers in its objective function.

2.2 Customer-oriented Duopoly

In this case, two C firms compete through the quantity produced. There are three sub cases to discuss: (1) both firms delegate (we note "CDCD"); (2) one
delegates, the other does not delegate (we note "CDCN"); (3) both firms do not delegate (we note "CNCN").

2.2.1 CDCD sub game

In this sub game, the objective functions of both C firm’s delegated manager apply equation (7 or 8), i.e., \( \max \Omega_i \). Differentiating their objective functions with respect to \( x_i \), setting both results equal to zero, and solving for \( x_i \) yields each manager’s reaction function

\[
\begin{align*}
(2 - \theta) x_i &= \alpha - c + c\delta_i - \gamma x_j (1 - \theta) \\
(2 - \theta) x_j &= \alpha - c + c\delta_j - \gamma x_i (1 - \theta)
\end{align*}
\]  

(26)

Solving the above equation system, we get the corresponding Nash equilibrium quantities

\[
\begin{align*}
x_i (\theta, \gamma, \delta_i, \delta_j) &= \frac{\alpha - c}{\gamma - \theta - \theta\gamma + 2} + \frac{c(2 - \theta) \delta_i - c\gamma (1 - \theta) \delta_j}{(\theta - \gamma + \theta\gamma - 2)(\theta + \gamma - \theta\gamma - 2)} \\
x_j (\theta, \gamma, \delta_i, \delta_j) &= \frac{\alpha - c}{\gamma - \theta - \theta\gamma + 2} + \frac{c(2 - \theta) \delta_j - c\gamma (1 - \theta) \delta_i}{(\theta - \gamma + \theta\gamma - 2)(\theta + \gamma - \theta\gamma - 2)}
\end{align*}
\]  

(27) (28)

Then the owners of both C firms that delegate will search for the optimal incentive proportions that maximize their objective functions. It is easy to solve the optimal incentives which depend upon \( \theta \) and \( \gamma \),

\[
\delta_i^C (\theta, \gamma) = \delta_j^C (\theta, \gamma) = \frac{\gamma (1 - \theta) (\theta - \gamma + \theta\gamma) (\alpha - c)}{c(4\theta - 2\gamma - \theta^2 + \gamma^2 + \theta^2\gamma^2 + 2\theta\gamma - 2\theta\gamma^2 - 4)}
\]  

(29)

After replacing the two incentive schemes in equation (27) and (28), respectively, we obtain the quantities in term of \( \theta \) and \( \gamma \), i.e.,

\[
\begin{align*}
x_i^{CD} (\theta, \gamma) &= x_j^{CD} (\theta, \gamma) = \frac{(\theta - 2) (\alpha - c)}{4\theta - 2\gamma - \theta^2 + \gamma^2 + \theta^2\gamma^2 + 2\theta\gamma - 2\theta\gamma^2 - 4}.
\end{align*}
\]  

(30)
2.2.2 CNCN sub game

In this sub game, the objective functions of both C firms which do not delegate manager apply equation (4), i.e., \( Max V_i \). Differentiating their objective functions with respect to \( x^C_i (x^C_j) \), setting both results equal to zero, and solving the equation system, we obtain the Nash equilibrium quantities in term of \( \theta \) and \( \gamma \), i.e.,

\[
x^C_{i} (\theta, \gamma) = x^C_{j} (\theta, \gamma) = \frac{\alpha - c}{\gamma - \theta - \theta \gamma + 2}.
\] (31)

With the above outputs of the CDCD and the CNCN sub games, we get the following lemma

**Lemma 3** For a competition between two C firms, when both of them choose to delegate or both not delegate, we have \( \frac{d\theta}{d\gamma} < 0 \), for all \((\theta, \gamma) \in (0, 1) \times (0, 1)\).

**PROOF.** see appendix.

This lemma suggests that the consumer’s optimal weight \( \theta \) that both C firms consider decreases with \( \gamma \). In other words, when \( \gamma \) tends from zero towards one, goods become less differentiated, then for both C firms which delegate a manager (do not delegate), would consider less weight of consumer surplus in his delegated manager’s incentive contract (resp. objective function).

2.2.3 CDCN sub game

In this sub game, the objective function of a C firm’s delegated manager applies equation (7 or 8), i.e., \( Max \Omega_i \), while a C firm which does not delegate applies equation (4), i.e., \( Max V_i \). Thus, the optimal incentive \( \delta_i \) is in term of
\( \theta \) and \( \gamma \). The quantities also in term of \( \theta \) and \( \gamma \) are

\[
x_i^{CD}(\theta, \gamma) = \frac{(-4\theta - 2\gamma + \theta^2 + 2\theta\gamma + 4)(\alpha - c)}{-12\theta + 6\theta^2 - \theta^3 - 4\gamma^2 - 6\theta^2\gamma^2 + \theta^3\gamma^2 + 9\theta\gamma^2 + 8},
\]

(32)

\[
x_j^{CN}(\theta, \gamma) = \frac{(-\theta^2\gamma^2 - \theta^2\gamma + \theta^2 + 2\theta\gamma^2 + 3\theta\gamma - 4\theta - \gamma^2 - 2\gamma + 4)(\alpha - c)}{-12\theta + 6\theta^2 - \theta^3 - 4\gamma^2 - 6\theta^2\gamma^2 + \theta^3\gamma^2 + 9\theta\gamma^2 + 8}.
\]

(33)

Thus we obtain the following lemma:

**Lemma 4** For a competition between two C firms, when one C firm chooses to delegate, the other chooses not to, we have (i) for the one which delegate, \( \frac{\partial \theta}{\partial \gamma} < 0 \), for all \((\theta, \gamma) \in (0, 1) \times (0, 1)\); (ii) for the one which does not delegate, we have a non-monotonic relationship between \( \theta \) and \( \gamma \).

**PROOF.** see appendix.

This lemma suggests that for the C firm that chooses to delegate, the consumer’s optimal weight \( \theta \) that its shareholders considers decreases with \( \gamma \); while for the C firm that chooses not to delegate, the relationship between product differentiation and the strategic weight of consumers is not monotonic. In other words, when \( \gamma \) tends from zero towards one, goods become less differentiated, then a C firm that chooses to delegate would consider less weight of consumer surplus in his delegated manager’s incentive contract, while his rival a C firm that chooses not to delegate would first reduce then rise the weight of consumer surplus in his objective function. When \( \gamma \) tends towards one, we are close to a homogeneous market, then the strategic value of consumers would diminish for the C firm that chooses to delegate while arise for the one that does not delegate. The intuition behind is simple, when we are close to a ho-
mogeneous market, the Cournot competition makes firms to increase output in order to gain the market share. As for the C firm that delegates, it may reduce the strategic value of consumers, since the incentive scheme works as a discount on cost which induces firm to produce larger output. As for the C firm that does not delegate, without a discount on cost, it may put more weight on consumers to increase output.

3 The strategic value of consumer’s weight

In this section, we focus on the study of the impact of consumer’s weight. Thus, we analyze specifically the case when a C firm competes with a S firm. First, let us consider the important conditions of each firm’s objective function. We use a subscript $x_i (x_j, \theta)$ to denote a derivative taken with respect to $x_i$ (resp. $x_j, \theta$).

1) SN case

The first order condition to maximize the objective function of a S firm $i$ which does not delegate is

$$\pi^i_{x_i} = p^i_{x_i} x_i + p_i - c = 0, \quad (34)$$

with associated second order condition

$$\pi^i_{x_i x_i} < 0, \quad (35)$$

where, in this case, $\pi^i_{x_i x_i} = p^i_{x_i x_i} x_i + 2p^i_{x_i} = -2 < 0$, hence the second order condition is satisfied. It guarantees the uniqueness of the Cournot equilibrium. In a simultaneous-move game, the first order condition makes the Cournot equilibrium a Nash equilibrium in outputs and it is clear that the resulting
implicit function is the reaction function of a firm $i$.

Another regularity condition, called the Gale-Nikaido condition is as follows

$$\pi^i_{xixj} < 0,$$  \hspace{1cm} (36)

where $\pi^i_{xixj} = p^i_{xixj}x_i + p^j_{xj} = -\gamma < 0$, hence this condition is also satisfied (recall $0 < \gamma < 1$). The satisfaction of this condition ensures that various comparative static properties of the model are "well-behaved" (see Dixit, 1986). Moreover, it is worth mentioning that $\pi^i_{xix} = 0$.

2) SD case

The first order condition to maximize the objective function of a S firm $i$ which chooses to delegate is

$$F^i_{xix} = p^i_{xix}x_i + p_i - c\left(1 - \delta^S_i\right) = 0,$$  \hspace{1cm} (37)

with associated second order condition

$$F^i_{xixix} < 0,$$  \hspace{1cm} (38)

where, in this case, $F^i_{xix} = p^i_{xix}x_i + 2p^i_{xix} = -2 < 0$, hence the second order condition is satisfied. And the Gale-Nikaido condition is

$$F^i_{xixj} < 0,$$  \hspace{1cm} (39)

where $F^i_{xixj} = p^i_{xixj}x_i + p^j_{xj} = -\gamma < 0$, hence this condition is also satisfied. Moreover, it is worth mentioning that $F^i_{xix} = 0$.

3) CN case

The first order condition to maximize the objective function of a C firm $j$
which does not delegate is

\[ V^{ij}_{xj} = \pi^{ij}_{xj} + \theta CS_{xj} = 0, \]  

(40)

with associated second order condition

\[ V^{ij}_{xj,x_j} < 0, \]  

(41)

where, in this case, \( V^{ij}_{xj,x_j} = p^{ij}_{xj,x_j} x_j + 2p^{ij}_{x_j} + \theta CS_{xj,x_j} = -2\gamma + \theta \), hence the condition is \( \theta < 2\gamma \). And the Gale-Nikaido condition is

\[ V^{ij}_{xj,x_i} < 0, \]  

(42)

where \( V^{ij}_{xj,x_i} = p^{ij}_{xj,x_i} x_j + p^{ij}_{x_i} + \theta CS_{xj,x_i} = \theta \gamma - 1 < 0 \), the condition is satisfied, since \( \theta \in (0, 1) \). Moreover, \( V^{ij}_{x_j} = CS_{x_j} = \gamma x_i + x_j > 0 \).

4) CD case

The first order condition to maximize the objective function of a C firm \( j \) which chooses to delegate is

\[ \Omega^{ij}_{xj} = F^{ij}_{xj} + \theta CS_{xj} = 0, \]  

(43)

with associated second order condition

\[ \Omega^{ij}_{xj,x_j} < 0, \]  

(44)

where, in this case, \( \Omega^{ij}_{xj,x_j} = p^{ij}_{xj,x_j} x_j + 2p^{ij}_{x_j} + \theta CS_{xj,x_j} = -2\gamma + \theta \), hence the condition is \( \theta < 2\gamma \). And the Gale-Nikaido condition is

\[ \Omega^{ij}_{xj,x_i} < 0, \]  

(45)

where \( \Omega^{ij}_{xj,x_i} = p^{ij}_{xj,x_i} x_j + 2p^{ij}_{x_i} + \theta CS_{xj,x_i} = -\gamma + \theta \gamma < 0 \), the condition is satisfied. Moreover, \( \Omega^{ij}_{x_j,\theta} = CS_{x_j} = \gamma x_i + x_j > 0 \).
3.1 General analysis

Let us denote $\Psi^i$, a general objective function of a firm $i$, with $\Psi^i = \{\pi^i, F^i, V^i, \Omega^i\}$.

By totally differentiating the first order conditions of a $i$ firm’s objective function with respect to $x_i$, $x_j$ and $\theta$, we get

$$
\begin{bmatrix}
\frac{\Psi^i_{x_i x_i}}{x_i x_i} & \frac{\Psi^i_{x_i x_j}}{x_i x_j} \\
\frac{\Psi^j_{x_j x_i}}{x_j x_i} & \frac{\Psi^j_{x_j x_j}}{x_j x_j}
\end{bmatrix}
\begin{bmatrix}
\frac{dx_i}{d\theta} \\
\frac{dx_j}{d\theta}
\end{bmatrix}
= - \begin{bmatrix}
\frac{\Psi^i_{x_i \theta}}{x_i \theta} \\
\frac{\Psi^j_{x_j \theta}}{x_j \theta}
\end{bmatrix}.
$$

(46)

Hence, the solution is

$$
\begin{bmatrix}
\frac{dx_i}{d\theta} \\
\frac{dx_j}{d\theta}
\end{bmatrix}
= \frac{1}{D}
\begin{bmatrix}
-\frac{\Psi^j_{x_i x_j}}{x_i x_j} & \frac{\Psi^i_{x_i x_j}}{x_i x_j} \\
\frac{\Psi^j_{x_j x_i}}{x_j x_i} & -\frac{\Psi^i_{x_j x_i}}{x_j x_i}
\end{bmatrix}
\begin{bmatrix}
\frac{\Psi^i_{x_i \theta}}{x_i \theta} \\
\frac{\Psi^j_{x_j \theta}}{x_j \theta}
\end{bmatrix},
$$

(47)

where $D = \Psi^i_{x_i x_i} \Psi^j_{x_j x_j} - \Psi^i_{x_i x_j} \Psi^j_{x_j x_i} > 0$. Another way to write

$$
dx_i/d\theta = \frac{-\Psi^j_{x_i x_j} \Psi^i_{x_i \theta} + \Psi^i_{x_i x_j} \Psi^j_{x_j \theta}}{D},
$$

(48)

$$
dx_j/d\theta = \frac{\Psi^j_{x_j x_i} \Psi^i_{x_i \theta} - \Psi^i_{x_j x_i} \Psi^j_{x_j \theta}}{D}.
$$

(49)

3.2 Customer-oriented firm Versus Shareholder-oriented firm

From the previous analysis, we note that for $\Psi^i = \{\pi^i, F^i\}$, $\Psi^i_{x_i \theta} = 0$, where $i$ represents the S firm. Since a S firm does not consider consumer’s weight in its objective function, its second derivation with respect to $\theta$ is zero. This is an important information, because for all the delegation situations of a C firm
\( j \) and a S firm \( i \), we always have

\[
\frac{dx_i}{d\theta} = \frac{\Psi^i_{x_{ij}} \Psi^j_{x_i\theta}}{D} < 0, \quad (50)
\]

\[
\frac{dx_j}{d\theta} = -\frac{\Psi^j_{x_{ij}} \Psi^i_{x_j\theta}}{D} > 0, \quad (51)
\]

where \( \Psi^j = \{V^j, \Omega^j\} \) represents the objective of a C firm \( j \). Note that \( \Psi^j_{x_{ij}\theta} = CS_{x_j} = \gamma x_i + x_j > 0 \), the second order conditions \( \Psi^i_{x_{ij}x_i} < 0, \Psi^j_{x_{ij}x_i} < 0 \), and the determinant \( D > 0 \), it is easy to get the above results.

This suggests that a growth of consumer’s weight has a positive effect on the output of the C firm but a negative effect on that of a S firm. In other words, a growth of consumer’s weight within a C firm induces an increase on the output of the C firm, whereas a decrease on the output of its rival, the S firm.

For the good’s price of a S firm, we have

\[
dp_i(x_i(\theta), x_j(\theta))/d\theta = p^i_{x_i} dx_i/d\theta + p^i_{x_j} dx_j/d\theta = -dx_i/d\theta - \gamma dx_j/d\theta, \quad (52)
\]

\[
dp_j(x_i(\theta), x_j(\theta))/d\theta = -\gamma dx_i/d\theta - dx_j/d\theta. \quad (53)
\]

similarly for the good’s price of a C firm, \( dp_i(x_i(\theta), x_j(\theta))/d\theta = -\gamma dx_i/d\theta - dx_j/d\theta \). We show in appendix that \( dp_i/d\theta < 0 \) and \( dp_j/d\theta < 0 \), which means a growth of consumer’s weight has a negative effect on good’s price of both C firm and S firm.

For the consumer surplus, we always have

\[
dCS(x_i(\theta), x_j(\theta))/d\theta = CS_{x_i} dx_i/d\theta + CS_{x_j} dx_j/d\theta
\]

\[
= (x_i + \gamma x_j) dx_i/d\theta + (x_j + \gamma x_i) dx_j/d\theta. \quad (54)
\]

\[
W \text{e show in appendix that } dCS(x_i(\theta), x_j(\theta))/d\theta > 0, \text{ for all sub games (i.e., SDCD, SNCD, SDCN and SNCN).}
\]
Lemma 5  For a competition between a C firm and a S firm, a growth of consumer’s weight has (i) a positive effect on the output of the C firm but a negative effect on that of a S firm; (ii) a negative effect on good’s price of both firms; (iii) a positive effect on consumer surplus.

PROOF. see appendix.

Moreover, it is clear that with $dx_i/d\theta < 0$ and $dp_i (x_i (\theta), x_j (\theta))/d\theta < 0$, consumer’s weight also has a negative effect on the profit of a S firm. However, the case is not that obvious for a C firm. For the profit of a C firm $j$, we have

$$d\pi^j (x_i (\theta), x_j (\theta))/d\theta = \pi^j_{x_i} dx_i/d\theta + \pi^j_{x_j} dx_j/d\theta,$$

(56)

which can be positive and negative. Hence, the function of profit of a C firm is not monotonic. Thereafter, for the the social welfare

$$dW (x_i (\theta), x_j (\theta))/d\theta = W_{x_i} dx_i/d\theta + W_{x_j} dx_j/d\theta$$

$$= (p_i - c) dx_i/d\theta + (p_j - c) dx_j/d\theta,$$

(57)

(58)

where the sign of $(p_i - c) dx_i/d\theta + (p_j - c) dx_j/d\theta$ can be positive and negative, depending on the price level of each product with the marginal cost. Thus, the function of social welfare is not monotonic neither.

4 CONCLUSIONS

From this paper, we can see that the incentive scheme works as a discount on cost, whether for a S firm or for a C firm. In terms of internal governance, if both C firms practice the same delegation strategy (either both delegate, or
both not delegate), the consumer’s optimal weight decreases when products are less differentiated. We proved in this paper that consumer’s weight has a positive effect on consumer surplus but a negative effect on profit. Hence, in terms of external governance, a C firm would consider less weight of consumers, when competition is getting intense. Moreover, in a Cournot competition with differentiated products, we see that when a C firm competes with a S firm, a growth of consumer’s weight induces an increase on the output of the C firm, whereas a decrease on the output of the S firm. Moreover, a growth of consumer’s weight induces a price reduction for both firm’s goods and a rise on consumer surplus.

This paper focus on the interaction between product market competition and organizational governance. However, it does not cover asymmetric information or moral hazard problems. We will leave these issues in future research.

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A Appendix

For the proof of Lemma 1 to 4, let us recall the first order derivative of a C
firm’s profit

\[ g_k(\theta, \gamma) = \pi^j + \frac{\partial x_i}{\partial \theta} + \frac{\partial x_j}{\partial \theta}, \]

where \( k = \{CD, CN\} \). This function can also be written as

\[ g_k(\theta, \gamma) = (\alpha - c) \frac{\partial x_j}{\partial \theta} - 2x_j \frac{\partial x_j}{\partial \theta} - \gamma \left( \frac{\partial x_i}{\partial \theta} x_j + x_i \frac{\partial x_j}{\partial \theta} \right). \]
PROOF. [Lemma 1] For the SDCD sub game, we have $x_i^{SD}(\theta, \gamma) = \frac{(2-\theta)(\alpha-c)}{(h_11+\theta \gamma)h_{12}}$ and $x_j^{CD}(\theta, \gamma) = \frac{(\alpha-c)h_{13}}{(h_11+\theta \gamma)h_{12}}$, where $h_{11} = -2\theta - 2\gamma - \gamma^2 + \theta \gamma^2 + 4$, $h_{12} = -2\theta + 2\gamma - \gamma^2 - \theta \gamma + \theta \gamma^2 + 4$, $h_{13} = -\theta^2 \gamma + 2\theta \gamma^2 + 4\theta \gamma - 4\theta - 2\gamma^2 - 4\gamma + 8$.

By replacing $x_i^{SD}(\theta, \gamma)$ and $x_j^{CD}(\theta, \gamma)$ in $g_{CD}(\theta, \gamma)$, we have $\frac{\partial g_{CD}(\theta, \gamma)}{\partial \gamma} > 0$ and $\frac{\partial g_{CD}(\theta, \gamma)}{\partial \theta} < 0$, for all $(\theta, \gamma) \in (0,1) \times (0,1)$, hence $\frac{\partial \theta}{\partial \gamma} = -\frac{\left(\frac{\partial g_{\theta}}{\partial \gamma}\right)}{\left(\frac{\partial g_{\theta}}{\partial \theta}\right)} > 0$.

Similarly, for the SNCD sub game, with $x_i^{SN}(\theta, \gamma) = \frac{(2\theta+2\gamma^2-\gamma^2-4)(\alpha-c)}{4\theta+4\gamma^2-3\theta \gamma^2-8}$ and $x_j^{CD}(\theta, \gamma) = \frac{(2\gamma^2-3\gamma^2-4)(\alpha-c)}{4\theta+4\gamma^2-3\theta \gamma^2-8}$, we find $\frac{\partial g_{CD}(\theta, \gamma)}{\partial \gamma} > 0$ and $\frac{\partial g_{CD}(\theta, \gamma)}{\partial \theta} < 0$, for all $(\theta, \gamma) \in (0,1) \times (0,1)$, hence $\frac{\partial \theta}{\partial \gamma} = -\frac{\left(\frac{\partial g_{\theta}}{\partial \gamma}\right)}{\left(\frac{\partial g_{\theta}}{\partial \theta}\right)} > 0$. Thus, for the SDCD sub game and the SNCD sub game, we always have $\frac{\partial \theta}{\partial \gamma} > 0$.

PROOF. [Lemma 2] For the SDCN sub game, we have $x_i^{SD}(\theta, \gamma) = \frac{h_{21}(\alpha-c)}{2h_{22}}$, where $h_{21} = 2-\theta - \gamma^2 + \theta \gamma^2 + 2$, and $x_j^{CN}(\theta, \gamma) = \frac{-\theta^2 \gamma + h_{11}+2\theta \gamma^2}{2h_{22}(2-\theta)}$.

Thus $\frac{\partial g_{CN}(\theta, \gamma)}{\partial \theta} = -\frac{f_1(\alpha-c)^2}{2(\theta-2)^2h_{21}}$, where we denote $f_1 = A_{11} + A_{12} + A_{13}$, with $A_{11} = -64\theta + 64\gamma - 32\theta^2 + 64\theta^3 - 28\theta^4 + 4\theta^5 - 176\gamma^2 - 128\gamma^3 + 216\gamma^4 + 64\gamma^5 - 116\gamma^6 + 4\gamma^7 + 13\gamma^8 + 8\theta^2 \gamma^2 + 56\theta^2 \gamma^3 - 152 \theta^3 \gamma^2 + 150 \theta^2 \gamma^4 - 140 \theta^3 \gamma^3 + 81 \theta^4 \gamma^2$, $A_{12} = -56\theta^2 \gamma^5 + 94 \theta^3 \gamma^4 + 66 \theta^4 \gamma^3 - 13 \theta^5 \gamma^2 - 152 \theta^2 \gamma^6 + 104 \theta^3 \gamma^5 - 82 \theta^4 \gamma^4 - 10 \theta^5 \gamma^3 + 32 \theta^2 \gamma^7 - 4 \theta^3 \gamma^6 + 50 \theta^4 \gamma^5 + 16 \theta^5 \gamma^4 + 26 \theta^2 \gamma^8 - 28 \theta^3 \gamma^7 + 35 \theta^4 \gamma^6 + 8 \theta^5 \gamma^5 - 2 \theta^6 \gamma^8$, $A_{13} = +12 \theta^4 \gamma^7 - 9 \theta^5 \gamma^6 - 6 \theta^4 \gamma^8 - 2 \theta^5 \gamma^7 + 2 \theta^6 \gamma^8 - 64 \theta \gamma + 240 \theta \gamma^2 - 32 \theta \gamma^2 + 144 \theta \gamma^3 + 64 \theta \gamma^4 - 384 \theta \gamma^5 - 280 \gamma \gamma^4 - 64 \theta \gamma^5 + 4 \theta \gamma^4 + 242 \theta \gamma^6 - 180 \gamma \gamma^7 - 33 \theta \gamma^8 + 64$.

We find $f_1 > 0$ for all $(\theta, \gamma) \in (0,1) \times (0,1)$. Thus, $\frac{\partial g_{CN}(\theta, \gamma)}{\partial \theta} < 0$ for all $(\theta, \gamma) \in (0,1) \times (0,1)$.

And $\frac{\partial g_{CN}(\theta, \gamma)}{\partial \gamma} = \frac{(\alpha-c)^2 f_2}{2(\theta-2)^2(\theta - \gamma^2 + \theta \gamma^2 + 2)}$, where we denote $f_2 = A_{21} + A_{22}$, with $A_{21} = -16 \theta + 16 \gamma + 24 \theta^2 - 12 \theta^3 + 2 \theta^4 + 12 \gamma^2 - 40 \gamma^3 + 16 \gamma^4 - 2 \gamma^5 + \gamma^6 - \theta^2 \gamma^2 - 4 \theta^2 \gamma^3 - 2 \theta^3 \gamma^2 - 6 \theta^2 \gamma^4 + 20 \theta^3 \gamma^3 + \theta^4 \gamma^2 - \theta^2 \gamma^5 + 14 \theta^3 \gamma^4$, $A_{22} = -4 \theta^4 \gamma^3 + 6 \theta^2 \gamma^6 - 2 \theta^3 \gamma^5 - 4 \theta^4 \gamma^4 - 4 \theta^3 \gamma^6 + \theta^4 \gamma^5 + 4 \theta^4 \gamma^6 - 24 \theta \gamma - 4 \theta \gamma^2 + 2 \theta \gamma^2 + 64 \theta \gamma^3 - 14 \theta \gamma - 20 \theta \gamma^4 + 3 \theta \gamma^4 + 4 \theta \gamma^5 - 4 \theta \gamma^6$. We find that $f_2$ can be positive and negative under different conditions. Simi-
larly, for the SNCN sub game, we have \( \frac{\partial g_{CN}(\theta, \gamma)}{\partial \theta} = \frac{2(\gamma-2)(\gamma+1)(\gamma^2-2)h_{23}}{(-2\theta-\gamma^2+\theta\gamma+4)^2} \), where 
\( h_{23} = -4\theta - 2\gamma + 4\gamma^2 - \gamma^3 - 2\theta\gamma + 2\theta\gamma^2 + \theta\gamma^3 - 4 \). We find \( h_{22} < 0 \) for all \((\theta, \gamma) \in (0, 1) \times (0, 1)\). And \( \frac{\partial g_{CN}(\theta, \gamma)}{\partial \gamma} = \frac{f_3}{(-2\theta-\gamma^2+\theta\gamma+4)^2} \), where we denote 
\( f_3 = -32\theta + 32\gamma + 16\theta^2 - 32\gamma^3 + 20\gamma^4 - 6\gamma^5 + \gamma^6 + 4\theta^2\gamma^2 - 16\theta^2\gamma^3 - 8\theta^2\gamma^4 + 2\theta^2\gamma^5 + \theta^2\gamma^6 - 16\theta\gamma + 32\theta\gamma^2 + 24\theta^2\gamma + 16\theta^3 - 12\theta\gamma^4 + 4\theta\gamma^5 - 2\theta\gamma^6 \). We find that \( f_3 \) can be positive and negative under different conditions. Thus, for the SDCN sub game and the SNCN sub game, the sign of \( \left(\frac{\partial g_{CN}(\theta, \gamma)}{\partial \theta}\right) / \left(\frac{\partial g_{CN}(\theta, \gamma)}{\partial \gamma}\right) \) is not fixed.

**PROOF. [Lemma 3]** For the CDCD sub game, we have \( x_i^{CD}(\theta, \gamma) = x_j^{CD}(\theta, \gamma) = \frac{(\theta-2)(\alpha-c)}{h_{31}} \), where \( h_{31} = 4\theta - 2\gamma - \theta^2 + \gamma^2 + \theta^2\gamma^2 + 2\theta\gamma - 2\theta\gamma^2 - 4 \). By replacing \( x_i^{CD}(\theta, \gamma) \) and \( x_j^{CD}(\theta, \gamma) \) in \( g_{CD}(\theta, \gamma) \), we have \( g_{CD}(\theta, \gamma) = -\frac{h_{32}h_{33}(c-a)^2}{h_{31}} \), where \( h_{32} = 4\theta - 2\gamma - \theta^2 + 3\gamma^2 + \theta^2\gamma^2 - 4\theta\gamma^2 - 4 \), \( h_{33} = 2\theta + 2\gamma - \theta^2 + \gamma^2 + \theta^2\gamma^2 - 2\theta\gamma^2 \). Hence we find that \( \frac{\partial g_{CD}(\theta, \gamma)}{\partial \gamma} < 0 \) and \( \frac{\partial g_{CD}(\theta, \gamma)}{\partial \theta} < 0 \), for all \((\theta, \gamma) \in (0, 1) \times (0, 1)\), thus \( \frac{d\theta}{d\gamma} = -\left(\frac{\partial g_{CD}(\theta, \gamma)}{\partial \gamma}\right) / \left(\frac{\partial g_{CD}(\theta, \gamma)}{\partial \theta}\right) < 0 \). Similarly, for the CNCN sub game, we have \( x_i^{CN}(\theta, \gamma) = x_j^{CN}(\theta, \gamma) = \frac{\alpha-c}{\gamma - \theta - \theta\gamma^2} \). By replacing \( x_i^{CN}(\theta, \gamma) \) and \( x_j^{CN}(\theta, \gamma) \) in \( g_{CN}(\theta, \gamma) \), we find that \( \frac{\partial g_{CN}(\theta, \gamma)}{\partial \gamma} = -\frac{2\gamma+\theta^2+\gamma^2+2\theta\gamma^2}{(\theta-\gamma+\theta\gamma^2)^2} < 0 \) and \( \frac{\partial g_{CN}(\theta, \gamma)}{\partial \theta} = \frac{-2(\gamma+1)^2(c-\alpha)^2}{(\theta-\gamma+\theta\gamma^2)^2} \theta + \theta^2 + \theta^2\gamma^2 + 2\theta^2\gamma^2 + 2\theta^2\gamma^2 + 2\theta^2\gamma^2 + 2\theta^2\gamma^2 < 0 \), for all \((\theta, \gamma) \in (0, 1) \times (0, 1)\), hence \( \frac{\partial \theta}{\partial \gamma} = -\left(\frac{\partial g_{CN}(\theta, \gamma)}{\partial \gamma}\right) / \left(\frac{\partial g_{CN}(\theta, \gamma)}{\partial \theta}\right) < 0 \). Thus, for the CDCD sub game and the CNCN sub game, we always have \( \frac{d\theta}{d\gamma} < 0 \).

**PROOF. [Lemma 4]** For the CDCN sub game, we have \( x_i^{CD}(\theta, \gamma) = \frac{h_{41}(\alpha-c)}{h_{42}} \), where \( h_{41} = (-4\theta - 2\gamma + \theta^2 + 2\theta \gamma + 4) \), \( h_{42} = -12\theta + 6\theta^2 - \theta^3 - 4\gamma^2 - 6\theta^2\gamma^2 + \theta^3\gamma^2 + 9\theta\gamma^2 + 8 \), and \( x_j^{CN}(\theta, \gamma) = \frac{h_{43}(\alpha-c)}{h_{44}} \), where \( h_{43} = -\theta^2\gamma^2 - \theta^2\gamma^2 + 2\theta\gamma^2 + 3\theta\gamma - 4\theta - \gamma^2 - 2\gamma + 4 \). By replacing \( x_i^{CD}(\theta, \gamma) \) and \( x_j^{CN}(\theta, \gamma) \) in \( g_{CD}(\theta, \gamma) \), we find that \( \frac{\partial g_{CD}(\theta, \gamma)}{\partial \gamma} < 0 \) and \( \frac{\partial g_{CD}(\theta, \gamma)}{\partial \theta} < 0 \), for all \((\theta, \gamma) \in (0, 1) \times (0, 1)\), thus \( \frac{d\theta}{d\gamma} = \)}
\[- \left( \frac{\partial g_{CD}(\theta, \gamma)}{\partial \gamma} \right) / \left( \frac{\partial g_{CD}(\theta, \gamma)}{\partial \theta} \right) < 0 \] for the CD firm. By replacing \( x_{i}^{CD}(\theta, \gamma) \) and \( x_{j}^{CN}(\theta, \gamma) \) in \( g_{CN}(\theta, \gamma) \), we find that \( \frac{\partial g_{CN}(\theta, \gamma)}{\partial \theta} < 0 \), for all \((\theta, \gamma) \in (0, 1) \times (0, 1)\) and \( \frac{\partial g_{CN}(\theta, \gamma)}{\partial \gamma} \) can be positive and negative under different conditions. Thus, the sign of \( \left( \frac{\partial g_{CN}(\theta, \gamma)}{\partial \gamma} \right) / \left( \frac{\partial g_{CN}(\theta, \gamma)}{\partial \theta} \right) \) is not fixed for the CN firm.

For the proof of Lemma 5, let us recall

\[
d p_i (x_i(\theta), x_j(\theta)) / d\theta = p_{x_i}^i dx_i / d\theta + p_{x_j}^j dx_j / d\theta
\]
\[
= -dx_i / d\theta - \gamma dx_j / d\theta
\]

\[
d CS(x_i(\theta), x_j(\theta)) / d\theta = CS_{x_i} dx_i / d\theta + CS_{x_j} dx_j / d\theta
\]
\[
= (x_i + \gamma x_j) dx_i / d\theta + (x_j + \gamma x_i) dx_j / d\theta.
\]

**PROOF. [Lemma 5]** (1) Let us begin with the SDCD sub game. With \( x_{i}^{SD}(\theta, \gamma) \) and \( x_{j}^{CD}(\theta, \gamma) \), we have \( dp_i / d\theta = B_1 \gamma (\alpha - c) \) \( h_{51} = -20 - 2\gamma - \gamma^2 + \theta \gamma + \theta \gamma^2 + 4 \), \( h_{52} = -20 + 2\gamma - \gamma^2 - \theta \gamma + \theta \gamma^2 + 4 \) and \( B_1 = 32\theta + 16\gamma - 8\theta^2 + 40\gamma^2 - 20\gamma^3 - 14\gamma^4 + 5\gamma^5 + 2\gamma^6 + 18\theta^2 \gamma^2 - 7\theta^2 \gamma^3 - 12\theta^2 \gamma^4 + 3\theta^2 \gamma^5 + 2\theta^2 \gamma^6 - 16\theta \gamma - 56\theta \gamma^2 + 40\theta^2 \gamma + 24\theta \gamma^3 + 28\theta \gamma^4 - 8\theta \gamma^5 - 4\theta \gamma^6 - 32 \).

We find that \( B_1 < 0 \) for all \((\theta, \gamma) \in (0, 1) \times (0, 1)\). Hence, \( dp_i / d\theta < 0 \). And we have \( dp_j / d\theta = B_2 \gamma (\alpha - c) \) \( h_{51} = (-20 - 2\gamma - \gamma^2 + \theta \gamma + \theta \gamma^2 + 4)^2 \), \( h_{52} = (-20 + 2\gamma - \gamma^2 - \theta \gamma + \theta \gamma^2 + 4)^2 \), where we denote \( B_2 = 64\theta - 16\theta^2 + 112\gamma^2 - 60\gamma^4 + 8\gamma^6 + \gamma^7 + 36\theta^2 \gamma^2 - 24\theta^2 \gamma^4 - \theta^2 \gamma^5 + 4\theta^2 \gamma^6 + \theta^2 \gamma^7 - 128\theta \gamma^2 + 76\theta \gamma^4 + 2\theta \gamma^5 - 12\theta \gamma^6 - 2\theta \gamma^7 - 64 \). We find that \( B_2 < 0 \) for all \((\theta, \gamma) \in (0, 1) \times (0, 1)\). Hence, \( dp_j / d\theta < 0 \). Similarly for consumer surplus, we have \( d CS / d\theta = (x_i + \gamma x_j) dx_i / d\theta + (x_j + \gamma x_i) dx_j / d\theta > 0 \), for all \((\theta, \gamma) \in (0, 1) \times (0, 1)\).

(2) For the SNCD sub game, with \( x_{i}^{SN}(\theta, \gamma) \) and \( x_{j}^{CD}(\theta, \gamma) \), we have \( dp_i / d\theta = \gamma (\alpha - c) \) \( h_{51} = -6\gamma^2 + \gamma^3 + 8 \), \( h_{52} = (-4\theta^2 + 3\theta \gamma^2 + 8)^2 \), \( \gamma (\alpha - c) < 0 \) and \( dp_j / d\theta = (\gamma^2 - 2) \) \( h_{51} = (-6\gamma^2 + \gamma^3 + 8)(\alpha - c) \), \( h_{52} = (-4\theta^2 + 3\theta \gamma^2 + 8)^2 \), \( \gamma (\alpha - c) < 0 \).
for all \((\theta, \gamma) \in (0,1) \times (0,1)\). Similarly for the consumer surplus, we have
\[
dCS/d\theta = (-6\gamma^2 + \gamma^3 + 8)^2 \frac{(c-\alpha)^2}{(-4\theta - 4\gamma^2 + 3\theta \gamma^2 + 8)^2},
\]
where we find that \(-4\theta - 4\gamma^2 + 3\theta \gamma^2 + 8 > 0\) for all \((\theta, \gamma) \in (0,1) \times (0,1)\).

(3) For the SDCN sub game, with \(x^{SD}_i(\theta, \gamma)\) and \(x^{CN}_j(\theta, \gamma)\), we have
\[
dp_i/d\theta = \frac{1}{2} \gamma \frac{c-\alpha}{(\theta-2)^2} < 0 \quad \text{and} \quad dp_j/d\theta = \frac{1}{2} \gamma \frac{B_3(c-\alpha)}{(-\theta - 4\gamma^2 + 3\theta \gamma^2 + 8)^2},
\]
where we denote \(B_3 = -8\theta + 4\gamma + 2\theta^2 - 12\gamma^2 - 4\gamma^3 + 5\gamma^4 - 4\theta^2 \gamma^2 - \theta^2 \gamma^3 + 2\theta^2 \gamma^4 - 4\theta \gamma + 14\theta \gamma^2 + \theta^2 \gamma + 4\theta \gamma^3 - 6\theta \gamma^4 + 8\). We find that \(B_3 > 0\) for all \((\theta, \gamma) \in (0,1) \times (0,1)\). Hence \(dp_j/d\theta < 0\). Similarly for the consumer surplus, we have
\[
dCS/d\theta = (x_i + \gamma x_j) dx_i/d\theta + (x_j + \gamma x_i) dx_j/d\theta > 0\]
for all \((\theta, \gamma) \in (0,1) \times (0,1)\).

(4) For the SNCN sub game, with \(x^{SN}_i(\theta, \gamma)\) and \(x^{CN}_j(\theta, \gamma)\), we have
\[
dp_i/d\theta = \gamma (\alpha - c) (\gamma + 1) \frac{\gamma - 2}{h_{53}}, \quad \text{where we denote} \quad h_{53} = -2\theta - \gamma^2 + \theta \gamma^2 + 4, \quad \text{and} \quad dp_j/d\theta = (\gamma - 2) (\gamma + 1) \frac{c-\alpha}{h_{53}}.
\]
Thus \(dp_i/d\theta < 0\) and \(dp_j/d\theta < 0\) for all \((\theta, \gamma) \in (0,1) \times (0,1)\). Similarly for the consumer surplus, we have
\[
dCS/d\theta = \frac{h_{54}(c-\alpha)^2}{h_{53}^2},
\]
where \(h_{54} = 4\gamma - \gamma^2 - \theta \gamma + \theta \gamma^2 - 4, \quad h_{55} = (\gamma - 2)(\gamma + 1)^2\). We find that \(h_{54}/h_{53}^2 < 0\) for all \((\theta, \gamma) \in (0,1) \times (0,1)\). Thus \(dCS/d\theta > 0\).

From the above sub games, we can see that for each case \(dp_i/d\theta < 0\), \(dp_j/d\theta < 0\) and \(dCS/d\theta > 0\).