Axiomatic Bargaining with Inequality Aversion: Norms vs. Preferences *

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Abstract

This paper argues that social norms might play a role in bargaining situations including the ultimatum game and asks two questions. First, if social norms govern behavior instead of the standard non-cooperative equilibria, do other-regarding attitudes still impart bargaining power? Second, can one still infer differences in other-regarding preferences from differences in ultimatum game offers? To capture social norms we replace the standard non-cooperative game-theoretic solution techniques with axiomatic (or cooperative) bargaining solutions, specifically the Nash and Kalai-Smorodinsky solutions. If the Nash axioms govern the social norm then the two questions above receive affirmative answers only when the disagreement point is sufficiently asymmetric. When the Kalai-Smorodinsky axioms govern the social norm then the two questions receive partial affirmative answers: increased aversion to being behind strengthens bargaining power and can be deduced from choice differences in the laboratory, but increased aversion to being ahead has an ambiguous impact and therefore cannot be inferred from laboratory behavior. We conclude that using axioms to identify social norms separate from participant preferences proves to be a promising line of research, and that they have important implications for how experimental economists interpret their results.

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1 Introduction

Two decades of economics research, most of it experimental, suggests that concern for others payoffs matters for individual decision-making. The ultimatum game has proven particularly relevant for this literature. The proposer makes a take-it-or-leave-it offer to the responder, and the proposers offer reveals something about her attitude toward having a higher pay-off than the responder while the responders decision to reject the offer reveals something about his attitude toward having a lower payoff than the proposer. To make this reasoning complete, one must place some structure on the interactions, and that is always done using ideas from non-cooperative game theory. The literature attributes all of the heterogeneity in behavior, though, to differences in preferences.

Other-regarding preferences provide only one approach to the problem, and there may be others. Cooperation games like the prisoners dilemma and coordination games have both been modeled using an idea of social norms, which are rules about how members of a group should behave. Early work on social norms in economics used theoretical approaches. For example, Kandori (1992) examines the reasons for complying with a cooperative social norm in the prisoners dilemma, and Young (1993) examines convergence to a social norm in coordination games. More recently experimental economists have begun designing experiments to study social norms. Notably, Krupka and Weber (2013) devise an incentive-compatible method for identifying social norms and show that these norms predict behavior in other games. Gächter et al. (2013) employ their norm-elicitation methodology in a gift-exchange setting. They find that social norms have predictive power, but that a preference-based model has greater parsimony and explanatory power.

This paper departs from the literature by abandoning the non-cooperative equilibrium approach to bargaining behavior in favor of one that reflects social norms. These social norms are identified by the axioms from cooperative bargaining theory, specifically the Nash (1950) bargaining solution and the Kalai and Smorodinsky (1975) bargaining solution. The paper addresses two questions. First, in the presence of bargaining norms, do other-regarding attitudes still impart bargaining power? Second, in the presence of bargaining norms,

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2 López-Pérez (2008) links the preference-based approach to a norm-based approach by assuming that agents suffer disutility when they deviate from the social norm. López-Pérez (2010) builds an axiomatic foundation for this idea, and Kessler and Leider (2012) apply the idea to contracting, with the contract identifying the social norm.
3 Shalev (2002) and Köbberling and Peters (2003) also incorporate “behavioral” elements into axiomatic bargaining theory. Both papers examine loss-averse preferences, with Shalev using the Nash approach and Kobberling and Peters using the Kalai-Smorodinsky axioms.
do differences in laboratory subject behavior still reveal aspects of their other-regarding preferences?

We begin the paper by arguing that experimental evidence from ultimatum games and dictator games is inconsistent with the joint hypothesis that agents use non-cooperative equilibrium strategies and have other-regarding preferences that are constant across the two games. This raises several possibilities for moving forward. It could be that preferences are context-dependent and differ across the two experiments, it could be that ultimatum game play is not in equilibrium, or it could be that agents do not use non-cooperative solution techniques in bargaining settings. This paper explores the last of these premises, positing that agents use social norms in bargaining rather than non-cooperative solutions.

The task then becomes one of finding a suitable set of rules for how agents should behave in a bargaining setting. We propose that these rules can be defined by axioms, and we explore two of the most prominent axiomatic (or cooperative) bargaining solutions, those of Nash (1950) and Kalai and Smorodinsky (1975). The two solutions share three axioms in common but differ in the fourth. In the Nash axioms, bargaining power is independent of alternatives that could have been but were not selected by the bargaining mechanism. In contrast, the Kalai-Smorodinsky axioms allow bargaining power to reflect the opportunities available to each participant, with greater opportunities leading to more bargaining power. This same feature of a positive response to increased opportunities appears in another social norm, that of reciprocity (see, for example, Rabin, 1993, and Dufwenberg and Kirchsteiger, 2004). For this reason the Kalai-Smorodinsky axioms might better capture the prevailing social norm than the Nash axioms, but this paper continues to consider both bargaining models.

If, as we propose, bargaining outcomes are governed by social norms and not non-cooperative game-theoretic solutions, two questions emerge. First, do other-regarding attitudes provide a source of bargaining power or are their effects crowded out by the social norm? We answer this question using the workhorse model of inequality aversion proposed by Fehr and Schmidt (1999). One might suspect that an increased aversion to having a lower payoff than ones opponent strengthens bargaining power but an increased aversion to being ahead weakens it. We find that in a problem of bargaining over a dollar the Nash social norm crowds out these effects unless the disagreement payoffs are highly asymmetric. In contrast,
the Kalai-Smorodinsky social norm does not crowd out the effects of other-regarding attitudes on bargaining power, but only a change in the attitude toward being behind has an unambiguous sign. Changes in an agents attitudes toward being ahead could either increase or decrease bargaining power.

The second question, and one with more importance for the experimental literature, is if one can still uncover laboratory subjects other-regarding attitudes from behavior in bargaining games if play is governed by social norms. After all, one of the primary reasons for proposing inequality aversion in the first place was to explain evidence in the ultimatum game. If agents are inequality averse and one proposer in an anonymous ultimatum game makes a larger offer than another proposer, can we still surmise that the first proposer is more averse to being ahead than the second one when play is governed by a social norm? The answer is affirmative for the Nash social norm if we believe that players behave as if the disagreement payoffs are highly asymmetric, but any such heterogeneity in offers would be inconsistent with the Nash social norm if the disagreement payoffs are symmetric. Instead, the Nash social norm predicts an even split regardless of the players inequality attitudes when the disagreement payoffs are similar for the two players. The Kalai-Smorodinsky norm is much more responsive to inequality attitudes, but only allows one to surmise the proposers aversion to being ahead, and not the responders aversion to being behind.

The paper proceeds as follows. Section 2 presents the argument that laboratory data reject the joint hypothesis of stable social preferences across games and non-cooperative equilibrium play in the ultimatum game. Section 3 presents the bargaining axioms. Because the axioms are presented for utility values and the ultimatum game is presented in monetary values, Section 4 uses the Fehr-Schmidt model to transform the bargaining problem into utility values so that the bargaining solutions can be applied. Section 5 presents the results, and Section 6 offers some conclusions.

2 Laboratory games and non-cooperative equilibrium

This section explains why we consider cooperative solutions instead of non-cooperative ones in analyzing laboratory games. Economists have mostly used non-cooperative equilibrium concepts to analyze subjects’ behavior in laboratory bargaining games, but we argue that such equilibrium concepts cannot be reconciled with all of the evidence that comes from the laboratory when we also assume that social preferences are stable across the ultimatum and are measured, as modeled in Thomson (1981a) and Conley et al. (1996a). It may be that in the ultimatum game the reference function identifies an asymmetric point that favors the proposer, in which case the Nash social norm no longer crowds out social preferences. For other applications of reference functions, see Irem et al. (2012), Diskin and Felsenthal (2007), Pfingsten and Wagener (2003), and Vartiainen (2007).
dictator games. In particular, the fact that we see many more offers of 50% of the surplus in the ultimatum game than in the dictator game is at odds with any dynamic non-cooperative solution concept that prescribes that the equilibrium belief is consistent with the equilibrium behavior, and that the equilibrium strategies are best responses to beliefs.

In the standard ultimatum game, two agents bargain over the division of one dollar. The proposer proposes a division, then the responder chooses to accept or reject the proposal. The proposed division is implemented only when the responder accepts the offer. A standard dictator game differs from the ultimatum game in that the responder’s power to reject the offer is removed; the proposed division made by the dictator is always implemented. According to lab evidence, the proposals of 50% of the surplus were accepted with near certainty, in many cases with probability one. If the subjects solved the game using non-cooperative equilibrium concepts, then the proposers’ beliefs must be consistent with the near-certainty of acceptance of 50% offers.

Let $x$ denote the amount of money that the proposer offers in laboratory bargaining games and $1-x$ the amount that she keeps. Let $P(x)$ denote the probability that the proposer assigns to the acceptance of an offer of $x$.

**Proposition 1** Assume that the proposer’s preferences are constant across the ultimatum game and the dictator game. If her equilibrium offer $x^*$ is accepted with probability 1 in the ultimatum game, then she also offers $x^*$ in the dictator game.

**Proof.** By the assumption that offers of $x^*$ are accepted with probability 1, we have that $P(x^*) = 1, P'(x^*) = 0$. In the ultimatum game, the proposer offers $x$ to maximize her expected utility $P(x)U(1-x,x)$. The first order condition of her maximization problem is $P(x)[U_p(1-x,x) - U_r(1-x,x)] = P'(x)U(1-x,x)$. If she offers $x^*$, this first order condition becomes $U_p(1-x^*,x^*) - U_r(1-x^*,x^*) = 0$. In the dictator game, the dictator offers $x$ to maximize $U(1-x,x)$. The first order condition is $U_p(1-x,x) - U_r(1-x,x) = 0$, which is satisfied at $x = x^*$. Therefore, the proposer who offers $x^*$ in the ultimatum game should also offer $x^*$ in the dictator game.

However, laboratory experiments showed that offers of 50% are indeed accepted almost certainly, but there are many more offers of 50% in ultimatum games than in dictator games. The evidence rejects the joint hypothesis of that agents’ preferences are stable across games and that the agents’ solve the game using non-cooperative equilibrium. Figure 1 shows the histograms of offers in the laboratory games ran by Forsythe et al. (1994). In their experiments where subjects were paid, offers of 50% of the surplus were all accepted.

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Here the notation $U_p(\cdot)$ and $U_r(\cdot)$ denote the partial derivatives of $U(\cdot)$ with respect to its first and second arguments, respectively.

Forsythe et al. (1994) is the first study where the ultimatum and dictator games were experimented and compared. In their experiments where subjects were paid, offers of 50% of the surplus were all accepted.
where the subjects were asked to divide $5, all offers of 50% ($2.50) were accepted, but the offers of 50% were about 3 times as many in the ultimatum games as in the dictator games.

![Figure 1: Histogram of Proposals, Forsythe et al. (1994)](image)

Proposition 1 implies that when we see more 50% offers in the ultimatum game than in the dictator game, it is either that the proposers who offer 50% in the ultimatum game show disequilibrium behavior or that their preferences vary across the two games. Disequilibrium behavior could arise either because subjects have out-of-equilibrium beliefs or because they use a different solution concept. This paper considers the second possibility, that the agents solve the game using cooperative solution rather than non-cooperative solution. We solve the standard ultimatum game following axiomatic bargaining solutions. We seek to answer two questions. First, when and how are agents' social preferences connected with their bargaining power? Second, can we still infer agents' social preferences from their actions in laboratory bargaining games if social norm also plays a role in the decision-making?

3 **Axiomatic bargaining solutions as social norms**

The classic two-person bargaining problem consists of a compact convex subset $S$ of the plane representing all feasible utility payoffs achievable through bargaining, and a point $d \in S$ representing the fallback payoffs to be received by the bargainers in case of a disagreement. We only consider bargaining problems in which mutual benefits are possible; i.e. problems for which there is some $s \in S$ such that $s > d$.\(^8\) Let $\Omega$ denote the class of bargaining problems with disagreement point $d$ and feasible set $S$, a bargaining problem in $\Omega$ is denoted by $\langle d, S \rangle$.

\(^8\)We use the following vector notation through out the paper: \( s > d \) represents $s_i > d_i \forall i = \{1, 2\}$. 

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A solution $f(d, S)$ is a function defined on $\Omega$ which associates with each bargaining problem a single feasible outcome in $S$ that satisfies some prespecified conditions.

We denote the Nash solution of the bargaining problem as $f^N(d, S)$ and the Kalai-Smorodinsky solution $f^{KS}(d, S)$. Both solutions satisfy four axioms and they have the following three in common.\(^9\)

**Axiom 1 INV (Scale Invariance)** $\lambda(f(d, S)) = f(\lambda(d, S))$.

**Axiom 2 SYM (Symmetry)** If $S$ is invariant under all exchanges of agents, $f_i(d, S) = f_j(d, S)$ for all $i, j$.

**Axiom 3 PAR (Pareto Efficiency)** $f(d, S) \in PO(d, S) \equiv \{s \in S | \nexists s' \in S \text{ with } s' \geq s\}$.

INV suggests that the bargaining power of the bargainers should remain unchanged when we scale the problem up or down. SYM means that identical bargainers should receive identical outcomes. PAR claims that the outcome should achieve Pareto efficiency. The Nash solution also satisfies the axiom of independence of irrelevant alternatives (IIA), which states that the unchosen possible bargaining outcomes are irrelevant, hence removing these alternatives from the bargaining set should not change the bargaining outcome.

**Axiom 4 IIA (Independence of Irrelevant Alternatives)** If $S' \subseteq S$ and $f(d, S) \in S'$, then $f(d, S') = f(d, S)$.

Nash shows that axioms 1-4 hold if and only if the bargaining solution maximizes the product of utility gains from the disagreement point:

$$f^N(d, S) = \arg \max_{(d_1, d_2) \leq (s_1, s_2) \in S} (s_1 - d_1)(s_2 - d_2). \quad (1)$$

The Kalai-Smorodinsky solution was proposed in response to the controversy over the IIA axiom. Kalai and Smorodinsky argue that the untaken alternatives represent bargaining opportunities and should matter, especially the highest utility payoff available to each bargainer. Let $b_i(S) \equiv \max \{s_i | s \in S\}$ denotes the maximal utility level attainable by agent $i$ for every utility level attainable by agent $j$ in the feasible set $S$, Kalai and Smorodinsky introduced the following replacement for the IIA axiom:

**Axiom 5 (MON) Axiom of Monotonicity** If $S' \subseteq S$ and $b_i(S') = b_i(S)$, then $f_j(d, S') \leq f_j(d, S)$.

\(^9\)The formulation of all axioms is adopted from Thomson (1994) p. 1245-1249.
MON states that, if $S$ allows agent $j$ but not agent $i$ to achieve a higher maximal utility level than $S'$ does, then the bargaining solution should award more to agent $j$ from $S$ than it does from $S'$. A sensible bargaining outcome should reflect the improvement of opportunities for agent $j$.

Define the ideal point of $S$ as $b(S) = (b_1(S), b_2(S))$. The Kalai-Smorodinsky solution, $f^{KS}(d, S)$, is a unique solution that satisfies axiom 1-3 and 5. It is the maximal point of $S$ on the line $L(d, b(S))$ connecting $d$ to $b(S)$. Figure 2 shows the difference between the Nash solution and the Kalai-Smorodinsky solution when the disagreement point $d$ is normalized to locate at the origin.

Figure 2: Nash solution and Kalai-Smorodinsky solution

We argued in Section 2 that agents may solve bargaining games using some other approach rather than non-cooperative equilibrium concepts. One possibility is that they abide by social norms, which are shared expectations about how bargainers should behave in a given context. The axioms governing the Nash solution and the Kalai-Smorodinsky solution give normative descriptions about what “nice” outcomes should look like. They can be interpreted as identifying different social norms.

The two social norms, Nash and Kalai-Smorodinsky, differ only in their treatment of bargaining opportunities. When applying the Nash solution, the social norm is identified by IIA, which rules out the impact of unchosen allocations on the bargaining outcome. When applying the Kalai-Smorodinsky solution, the social norm is identified by MON, which claims

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10 papers about social norms?
that the bargaining outcome should reflect changes (deterioration or improvement) in the bargaining opportunities of the bargainers. Interestingly, opportunities are also considered in most theories of reciprocity. In most theories of reciprocity, agents reward their opponents for expanding their opportunities, and punish their opponents for restricting them.\footnote{Rabin (1993) measures the kindness of player \(i\)'s action toward player \(j\) according to how close \(i\)'s action allows \(j\) to come to his highest attainable payoff. Player \(i\)'s action is kinder if it allows \(j\) to obtain a higher maximal payoff. Dufwenberg and Kirchsteiger (2004) use this same notion when extending Rabin’s analysis to sequential games.} This suggests that the Kalai-Smorodinsky solution might work better in capturing the social norm in bargaining situations, but we still include the Nash solution in our analysis.

4 The Model

To model bargaining games as axiomatic bargaining problems, we realize that laboratory bargaining games are phrased in monetary payoffs while axiomatic bargaining problems are defined in utility payoffs. We get around this problem by starting with a bargaining problem defined in monetary payoffs, which we refer to as a \(\text{monetary bargaining problem}\), and transforming it to a \(\text{bargaining problem}\) defined in utility payoffs - one as defined in Nash (1950) and Kalai and Smorodinsky (1975).

We are interested in a two-person bargaining situation where two agents, 1 and 2, bargain over the division of some amount of money normalized to 1. An agreement divides the unit into the allocation \(x = (x_1, x_2) \geq 0\) with \(x_1 + x_2 \leq 1\). If the two agents fail to reach an agreement, they get their disagreement payoffs \(a = (a_1, a_2), a \in A\), where \(A\) is the set of possible allocations defined in monetary payoffs. Without losing generality we restrict our attention to disagreement monetary payoffs of the form \((a_1, 0)\) with \(0 \leq a_1 < 1\). We call the pair \(\langle a, A \rangle\) a \(\text{monetary bargaining problem}\), a term we use to distinguish the problem from the bargaining problem defined in utility payoffs. To incorporate social preferences, we transform the monetary bargaining problem into a bargaining problem defined in utility payoffs using the inequality averse utility function in Fehr and Schmidt (1999).\footnote{Like the bargaining solutions discussed below, the preferences also have an axiomatic foundation. Neilson (2006) axiomatizes a nonlinear generalization of the Fehr-Schmidt preferences in (1), and Rohde (2010) provides axioms foundation for the exact linear utility function form.}

\[ U_i(x_i, x_j) = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\}, \; i \neq j, \]  

where \(\alpha_i \geq \beta_i \geq 0\) and \(\beta_i < 1\). Agent \(i\) receives utility from her own monetary payoff \(x_i\) and disutility if her payoff differs from that of agent \(j\): her utility is reduced by \(\alpha_i(x_j - x_i)\) if her
payoff is below \( j \)'s and by \( \beta_i(x_i - x_j) \) if her payoff is above \( j \)'s.\(^{13}\) Assuming \( \alpha_i \geq \beta_i \) captures the idea that a player suffers more from disadvantageous inequality than from advantageous inequality, and \( \beta_i < 1 \) rules out agents who are ready to give up one dollar to reduce the inequity of payoffs when their payoffs are more than that of their opponents.

Let the disagreement point in utility space be \( d = (U_1(a), U_2(a)) \) and the bargaining set \( S = \{(U_1(x), U_2(x)) | x \in A \} \). Then \( a \in A \) implies that \( d \in S \), hence \( \langle d, S \rangle \) is a bargaining problem defined in utility payoffs. The class of all possible bargaining problems, \( \Omega \), is obtained by transforming all possible monetary bargaining problems by inequality averse utility functions. Therefore, all variations in \( \Omega \) arise from changing either the disagreement monetary payoffs or the parameters of the Fehr-Schmidt utility function.\(^{14}\) The bargaining solution is a mapping \( f : \Omega \rightarrow \mathbb{R}^2 \) such that \( f(d, S) \in S \).

Using the Fehr-Schmidt utility function with \( \alpha_i > 0 \) means that there are monetary allocations in \( A \) that player \( i \) likes less than the disagreement point. It would be individually irrational to choose these alternatives. The set \( \bar{S} \) differs from \( S \) by excluding elements that one or both of the players would reject. The solutions of the bargaining problems after the transformation naturally satisfy the following property.

**Axiom 6 Independence of Non-Individually Rational Alternative**

\[ f(d, S) = f(d, \bar{S}) \]

where \( \bar{S} \equiv \{s \in S | s \geq d \} \).

Figure 3 illustrates the transformation of the bargaining set from the payoff space (the left panel) to the utility space (the right panel). The areas enclosed by the dark lines represent the set of payoffs from all feasible allocations of the unit between the two players. The monetary payoffs are equitable at \((0, 0)\) and \((1/2, 1/2)\), thus the utility values of these

\(^{13}\)While the behavioral interpretation of preferences seems natural here, it is important to note that in a bargaining setting inequality aversion may arise from other sources. Malhotra (2013) relates the story of negotiations between a start-up and a venture capitalist. The start-up haggled for such a large share of the ownership that although the venture capitalist found it worthwhile to accept the deal, it no longer found it worthwhile to contribute time and effort toward the profitability of the start-up. A start-up that wishes to avoid this outcome would act as if it was averse to a deal that is too advantageous. Similarly, when two firms negotiate a joint venture, an asymmetric outcome might implicitly designate one firm as the “leader” and the other as the “follower.” For political reasons reflecting how these designations impact future behavior, both firms might wish to avoid large asymmetries, and this would lead them to act as if they are inequality averse.

\(^{14}\)We consider two effects of the parameter changes: one on the shape of the feasible set and the other on the disagreement point. Similar studies include Kalai and Smorodinsky (1975) and Kalai (1977), where they show how bargaining solutions respond to changes in the feasible set. Thomson (1987) discusses how bargaining solutions respond when disagreement point changes. Anbarci and Sun (2011) incorporate the notion of distributive justice in axiomatic bargaining solutions, though fairness consideration is captured by new axioms rather than by preferences.
allocations are also (0, 0) and (1/2, 1/2). Comparing the monetary allocation (1, 0) to (0, 0), player 1’s monetary payoff increases by 1 but her utility only increases by 1 − β₁ because she suffers from advantageous inequality, player 2’s monetary payoff remains unchanged but his utility decreases by −α₂ because he suffers from disadvantageous inequality. Therefore, monetary allocation (1, 0) corresponds to (1 − β₁, −α₂) in the utility space. Similarly, monetary allocation (0, 1) corresponds to (−α₁, 1 − β₂) in the utility space.

Without loss of generality, we normalize the bargaining problem to one with disagreement monetary payoffs a = (a₁, 0). The shaded bargaining set $\bar{A}$ includes all monetary allocations that are preferred to the disagreement monetary payoffs (a₁, 0) by at least one player when the players are inequality averse. The point $d = (a₁(1 − β₁), −a₁α₂)$ is the utility value of a, and the shaded bargaining set $\bar{S}$ is the counterpart of $\bar{A}$ in the utility space.¹⁵ Allocations in both $\bar{A}$ and $\bar{S}$ are individually rational for both players, while the allocations in white regions make at least one player worse off than the disagreement point. Lemma 1 states that the set $\bar{S}$ is a valid bargaining set.

**Lemma 1** The set $\bar{S}$ is convex, compact, and has an element $s \in \bar{S}$ for which $s > d$.

Proof in Appendix.

¹⁵We provide more details about the transformation in the Appendix.
5 Results

This section presents our main results and discusses how they help to answer the two questions we ask. We explore when and how agents’ social preferences are connected to their bargaining power, and what we can say about their social preferences based on observations from laboratory ultimatum games. In all cases, \( f^N(d, \bar{S}) \) and \( f^{KS}(d, \bar{S}) \) represent the Nash solution and the Kalai-Smorodinsky solution to bargaining problems defined in utility payoffs, while \( f^N(a, \bar{A}) \) and \( f^{KS}(a, \bar{A}) \) represent those to monetary bargaining problems.

When \( \beta_i > 1/2 \) for \( i = 1, 2 \), we say that both agents are very averse to advantageous inequality. In this case the bargaining outcome is not affected by the social norm that applies. Proposition 2 presents the result formally. When \( \beta_i < 1/2 \) for \( i = 1, 2 \), we say that both agents are moderately averse to advantageous inequality. In this case the Nash and the Kalai-Smorodinsky social norms suggest largely different results. Proposition 3 summarizes the results for the Nash solution and Proposition 4 summarizes those for the Kalai-Smorodinsky solution.

Proposition 2

\[
\beta_i > 1/2 \quad \text{for} \quad i = 1, 2 \Rightarrow f^N(d, \bar{S}) = f^N(a, \bar{A}) = f^{KS}(d, \bar{S}) = f^{KS}(a, \bar{A}) = (1/2, 1/2).
\]

Proof in Appendix.

Proposition 2 states that, if both agents are very averse to advantageous inequality, both the Nash solution and the Kalai-Smorodinsky solution to the bargaining problem would be \((1/2, 1/2)\), corresponding to an even split of the surplus in bargaining games. The result also holds when the disagreement point is asymmetric. When agent \( i \) is very averse to being ahead, she is always willing to yield some amount of money to her opponent when she has more than half of the surplus. Her utility is maximized only at \((1/2, 1/2)\) because it is strictly decreasing in her own monetary payoff.

The proposition draws a stark conclusion. If the agents are ready to sacrifice their monetary payoffs for equity when they have more than their opponent, neither their inequality attitudes nor the asymmetry of the bargaining problem can move the outcome away from the even division. Figure 4 shows that the slopes of the two segments of the bargaining frontier

\(^{16}\text{We only report our results from comparative statics. The mathematical representations of bargaining solutions, in utility and in material payoffs, are given in the Appendix.}\)
Figure 4: The preference parameters determine the slopes of the bargaining frontiers

are functions of the preference parameters.\(^\text{17}\) Figure 4 (b) shows that both segments of the bargaining frontier are upward sloping when \(\beta_i > 1/2\) for \(i = 1, 2\), making point \(\left(\frac{1}{2}, \frac{1}{2}\right)\) the only Pareto efficient allocation in the feasible set. As a result, both the Nash and the Kalai-Smorodinsky outcomes are purely driven by the axiom of Pareto efficiency. Inequality attitudes do not provide bargaining power because the bargaining opportunities do not matter other than the only Pareto efficient outcome.

The more interesting case is when the agents are moderately averse to advantageous inequality. As shown in Figure 4 (a), both segments of the bargaining frontier are downward sloping when \(\beta_i > 1/2\) for \(i = 1, 2\). All points on the bargaining frontier are Pareto efficient, hence bargaining opportunities may play a role. Our conclusions in this case depend on the social norm that applies. Proposition 3 summaries the results from using the Nash solution.

**Proposition 3**

*Suppose \(\beta_i < 1/2\) for \(i = 1, 2\), and define \(\bar{a}_1 = (\alpha_2 + \beta_1)/(\alpha_2(1 - 2\beta_1) + (1 + 2\alpha_2)(1 - \beta_1))\). Then*

\(^\text{17}\)To see this, first note that the northwest segment corresponds to the case when \(x_2 > x_1\). The utilities of the two players are \(U_2 = x_2 - \beta_2(x_2 - x_1)\) and \(U_1 = x_1 - \alpha_1(x_2 - x_1)\). Also, \(x_1 + x_2 = 1\). Solve the three equations simultaneously and write \(U_2\) as a function of \(U_1\), we have that

\[
U_2 = \frac{1 + \alpha_1 - \beta_2}{1 + 2\alpha_1} - \frac{1 - 2\beta_2}{1 + 2\alpha_1} U_1.
\]

The slope of the northwest segment is given by \(\frac{dU_2}{dU_1} = -\frac{1 - 2\beta_2}{1 + 2\alpha_1}\). Similarly, the slope of the southeast segment is given by \(\frac{dU_2}{dU_1} = -\frac{1 + 2\alpha_2}{1 - 2\beta_1}\).
(1) if $0 \leq a_1 < \bar{a}_1$, $f^N(d, \bar{S}) = f^N(a, \bar{A}) = (1/2, 1/2)$, and
(2) if $\bar{a}_1 < a_1 < 1$, $\partial x_1^N/\partial \alpha_2 < 0$, and $\partial x_1^N/\partial \beta_1 < 0$.

Proof in Appendix.

The Nash outcome is $(1/2, 1/2)$ and remains unresponsive to changes in agents’ inequality attitudes until the disagreement payoffs become very asymmetric. Figure 5 shows how asymmetric a bargaining problem has to be for any uneven split to become a possible outcome. The figure shows, for different values of $a_1$, the combinations of $\alpha_2$ and $\beta_1$ for which the tangency occurs exactly at $(1/2, 1/2)$. The region above a curve corresponds to combinations of $\alpha_2$ and $\beta_1$ for which the Nash outcome remains at $(1/2, 1/2)$, while the region below it corresponds to those for which the bargaining outcome strictly favors agent 1. For any given $a_1$, a decrease in either $\alpha_2$ or $\beta_1$ would increase player 1’s bargaining power and yield a division that allocates more than half to player 1. When the bargaining problem is asymmetric enough, the Nash outcome does respond to changes in the players’ inequality attitudes. Specifically, player 1 gets more either when she becomes less averse to advantageous inequality or when her opponent becomes less averse to disadvantageous inequality.

Figure 5: The contours of $\bar{a}_1$

If the ultimatum game is governed by the Nash social norm, we would be able to identify the proposer’s attitude toward advantageous inequality and the responder’s attitude toward
disadvantageous inequality using appropriate observations. For example, we consider two proposers, if one offers 30% while the other offers 40% of the surplus, then the proposer who offers 40% must be more averse to advantageous inequality than the proposer who offers 30%. When we consider two responders, if they were both offered 30% of the surplus but one accepts the offer while the other rejects it, then the responder who rejects the offer must be more averse to disadvantageous inequality than the one who accepts it.

When the Kalai-Smorodinsky social norm applies, the bargaining outcome is always responsive to changes in the preference parameters. However, we would fail to identify the attitude toward disadvantageous inequality of the player who has disadvantageous disagreement payoff. Proposition 4 summarizes the results.

**Proposition 4** Suppose $\beta_i < \frac{1}{2}$ for $i = 1, 2$. Then

1. $\frac{\partial x_{KS}^1}{\partial \alpha_1} > 0$, $\frac{\partial x_{KS}^1}{\partial \beta_1} < 0$, $\frac{\partial x_{KS}^1}{\partial \beta_2} > 0$, and
2. $\frac{\partial x_{KS}^1}{\partial \alpha_2} < 0$ when $a_1 = 0$, but its sign is ambiguous when $a_1 > 0$.

Proof in Appendix.

When the Kalai-Smorodinsky social norm is applied to a bargaining problem with symmetric disagreement payoffs, the bargaining power of player 1 increases when she becomes more averse to disadvantageous inequality or less averse to advantageous inequality, or when her opponent becomes less averse to disadvantageous inequality or more averse to advantageous inequality. However, when the disagreement payoffs are in favor of player 1, the bargaining power of player 1 could either increase or decrease when player 2 becomes more averse to disadvantageous inequality.

When the disagreement point is to player 2’s disadvantageous, he could become more or less agreeable when he becomes more averse to disadvantageous inequality. Figure 6 shows what happens. The left panel shows the Kalai-Smorodinsky outcome before $\alpha_2$ changes. The right panel shows what happens when player 2 becomes more averse to disadvantageous inequality ($\alpha_2$ increases). On one hand, player 2 dislikes the fallback payoff more. This moves the disagreement point $d$ downward to $d'$. As a result, the new feasible set includes more allocations that are to player 1’s advantage. This change makes player 2 more agreeable, hence increases the bargaining power of player 1. On the other hand, player 2 also dislikes more all allocations that assign more than half to player 1. This rotates clockwise the segment of the bargaining frontier below the 45-degree line. As a result, the new feasible set excludes many allocations that are in favor of player 1, making player 2 less agreeable and decreasing the bargaining power of player 1. The change of the bargaining outcome depends on which one of the two forces dominates the other.
If the ultimatum game is governed by the Kalai-Smorodinsky social norm, we would still be able to identify the proposer’s attitude toward advantageous inequality. A proposer who offers more than the others is more averse to advantageous inequality than the others. However, when we consider two responders, one accepts an offer of 30% while the other rejects it, we cannot determine which responder is more averse to disadvantageous inequality. The responder’s aversion to disadvantageous inequality is no longer identifiable when the disagreement point is to his disadvantage.

It is worth mentioning that an ultimatum bargaining problem can be asymmetric in nature even when two players both get nothing when they disagree. The way in which the surplus is earned, entitlement, or framing can introduce asymmetry to a seemingly symmetric ultimatum game. Experiments have shown that the proposers on average offer much smaller share of the surplus to the responders when they earned the surplus or the right to be proposers, or when they are “sellers” instead of “proposers” in ultimatum games.\(^{18}\) The asymmetry can be captured by using asymmetric reference functions as in Thomson (1981b) and Conley et al. (1996b).

\(^{18}\text{More details can be found in Hoffman et al. (1994) and Hoffman and Spitzer (1985).}\)
6 Conclusion

This paper separates social norms from other-regarding preferences in bargaining situations by using axiomatic, rather than non-cooperative, solutions. The axioms of the bargaining solution, specifically those of the Nash and Kalai-Smorodinsky solutions, identify the social norms, while the other-regarding preferences are governed by the Fehr-Schmidt model of inequality aversion. The purpose of the paper is to determine how the inequality attitudes and social norms interact.

The Nash axioms allow the bargaining outcome to depend on the disagreement point but no other unchosen elements of the bargaining set. Because inequality attitudes only come into play when allocations are asymmetric, if the disagreement point is symmetric the Nash bargaining solution leads to an even split regardless of any differences in the agents’ inequality attitudes. While it is not surprising that inequality aversion pushes outcomes toward an even split, it is perhaps more surprising that these attitudes push the outcome all the way to an even split for large portions of the parameter space when the disagreement payoffs are unequal. In contrast, without inequality aversion the Nash outcome would be asymmetric whenever the disagreement point is.

The Kalai-Smorodinsky solution allows the bargaining norm to react to changes in agents’ best opportunities, and these opportunities are, by their very nature, asymmetric. As a consequence inequality attitudes do impact the Kalai-Smorodinsky outcome, with bargaining power arising from higher aversion to disadvantageous inequality, at least when the agent in question has a higher disagreement payoff than his rival, and from lower aversion to advantageous inequality. However, the impact of attitudes toward disadvantageous inequality is ambiguous for the player who is disadvantaged by the disagreement payoff. This occurs because increased inequality aversion makes him less happy with outcomes that favor his advantaged rival, but it also makes him less happy with the disagreement allocation, and either of these effects could dominate.

The unambiguous effects of inequality attitudes on bargaining power are consistent with intuition, but two lessons can still be drawn from this paper. One is that for these intuitive results to hold the social bargaining norm must take into account some asymmetric opportunities. The other is that counterintuitive effects can also hold, namely that a disadvantaged bargainer becoming more averse to disadvantageous inequality may actually reduce his bargaining power. Because of this, one cannot directly infer inequality attitudes from bargaining outcomes, but must instead infer them from other allocation tasks.

This paper raises the possibility of a new research approach to social norms. In much of the prior literature social norms have arisen out of equilibria of the underlying repeated noncooperative game, or they have been incorporated directly into preferences so that agents
experience disutility when they deviate from the norm. For example, Kandori (1992) uses an infinitely-repeated game to model social norms, and in that model defections by one agent trigger sanctions by others in the same community. Akerlof and Kranton (2000) take the alternative approach of building social norms into the utility function through the construct of identity. In their model, agents are prescribed with an identity and deviations from what the identity consider as appropriate behavior generates disutility for the individual, making norms individually self-enforcing. This paper, in contrast, argues for an axiomatic approach to social norms, identifying properties of a “nice” outcome from a social point of view and then finding that outcome. Other-regarding preferences can still exist, but separately from the social norm. This makes exploration of the interaction between the two worthwhile.

References


Appendix

1. Transformation of the bargaining set:

Figure 7: The Transformation of the Bargaining Set from $A$ to $S$

At disagreement point $(a_1, 0)$, $x_1 > x_2$ so the utility functions of the two players are given by:

$$U_1(x_1, x_2; \beta_1) = x_1 - \beta_1(x_1 - x_2)$$
$$U_2(x_1, x_2; \alpha_2) = x_2 - \alpha_2(x_1 - x_2)$$

Solving

$$x_1 - \beta_1(x_1 - x_2) = a_1(1 - \beta_1) = x_1 - \alpha_1(x_2 - x_1)$$

We have that the indifference curve of agent 1 is

$$x_2 = \left\{ \begin{array}{ll}
-x_1 \frac{1-\beta_1}{\beta_1} + a_1 \frac{1-\beta_1}{\beta_1} & \text{if } x_1 > x_2; \\
x_1 & \text{if } x_1 = x_2; \\
x_1 \frac{1+\alpha_1}{\alpha_1} - a_1 \frac{1-\beta_1}{\alpha_1} & \text{if } x_1 < x_2.
\end{array} \right.$$ 

Solving

$$x_2 - \beta_2(x_2 - x_1) = -a_1 \alpha_2 = x_2 - \alpha_2(x_1 - x_2)$$

We have that the indifference curve of agent 2 is

$$x_2 = \left\{ \begin{array}{ll}
-x_1 \frac{\beta_2}{1-\beta_2} - a_1 \alpha_2 & \text{if } x_2 > x_1; \\
x_1 & \text{if } x_2 = x_1; \\
x_1 \frac{\alpha_2}{1+\alpha_2} - a_1 \frac{\alpha_2}{1+\alpha_2} & \text{if } x_2 < x_1.
\end{array} \right.$$
The boundary of set ¯A consists of a segment of player 1’s indifference curve (ap2p3), a segment of player 2’s indifference curve (ap3), and a segment of the bargaining frontier (p1p3). The individually rational bargaining set defined in material payoffs, ¯A, with disagreement point a, is the convex hull of points a, p1, p2, p3, and the point (1/2, 1/2), where

\[
a = (a_1, 0)\\p_1 = \left(\frac{a_1}{1+2a_1}, a_1 \frac{1-\beta_1}{1+2a_1} - a_1 \frac{1-\beta_1}{1+2a_1}\right)\\p_2 = (a_1 (1-\beta_1), a_1 (1-\beta_1))\\p_3 = \left(\frac{1+\alpha_2}{1+2a_2} + \frac{a_2}{1+2a_2} - a_1 \frac{\alpha_2}{1+2a_2}\right)
\]

The transformed bargaining set, i.e. the bargaining set defined in utility payoffs, S is the convex hull of points d, U(p1), U(p2), U(p3), and the point (1/2, 1/2), where

\[
d = (a_1 (1-\beta_1), -a_1 \alpha_2)\\U(p_1) = \left(\frac{1+\alpha_2-\beta_1-a_1 \alpha_2 (1-2\beta_1)}{1+2a_2}, -a_1 \alpha_2\right)\\U(p_2) = (a_1 (1-\beta_1), a_1 (1-\beta_1))\\U(p_3) = \left(a_1 (1-\beta_1), \frac{1+\alpha_2-\beta_1-a_1 (1-\beta_1)(1-2\beta_2)}{1+2a_2}\right)
\]

2. Proof of Lemma 1

- (1) Given the monetary disagreement point (a1, a2), consider the allocation \(z = \left(\frac{1}{2}(1 + a_1 - a_2), \frac{1}{2}(1 - a_1 + a_2)\right)\). Note that \(z_1 + z_2 = 1\) and \(z_1, z_2 > 0\) because \(a_i - a_j > -1\). Therefore \(z \in A\). It remains to show that \(U_i(z) > U_i(a)\) for \(i = 1, 2\). Assume, without loss of generality, that \(a_1 \leq a_2\). Then \(z_1 < z_2\) by construction. We have

\[
U_1(z) = z_1 - a_1 (z_2 - z_1) = \frac{1}{2} (1 + a_1 - a_2) - a_1 (a_2 - a_1)
\]

and

\[
U_1(a) = a_1 - a_1 (a_2 - a_1).
\]

Therefore,

\[
U_1(z) - U_1(a) = \frac{1}{2} (1 + a_1 - a_2) - a_1 - \frac{1}{2} (1 - a_1 - a_2) > 0.
\]

Similarly,

\[
U_2(z) - U_2(a) = \frac{1}{2} (1 - a_1 - a_2) > 0.
\]

Therefore \((U_1(z), U_2(z)) \in S\).
(2) The set $\bar{S}$ is a quadrilateral (refer to Figure 7). Two of the sides are vertical and horizontal segments extending from the disagreement point $d$. If $\beta_1 \neq \frac{1}{2}$, then $\bar{S}$ can be described by the equation:

$$
0 \leq s_2 \leq \begin{cases} 
\frac{1+\alpha_1-\beta_2}{1+2\alpha_1}s_1 & \text{if } 0 \leq s_1 \leq \frac{1}{2} \\
\frac{1+\alpha_2-\beta_1}{1-2\beta_1}s_1 & \frac{1}{2} < s_1 \leq 1
\end{cases}
$$

The set $\bar{S}$ fails to be convex if both boundary segments are downward sloping and the segment corresponding to $s_1 < \frac{1}{2}$ is steeper than the one corresponding to $s_1 > \frac{1}{2}$. They are both downward sloping if $\beta_1, \beta_2 < \frac{1}{2}$. Writing out the condition on the slopes, one sees that

$$
\frac{1-2\beta_2}{1+2\alpha_1} \leq 1 \leq \frac{1+2\alpha_2}{1-2\beta_1}
$$

and so $\bar{S}$ must be convex.

3. The solutions

$$
f^N(\bar{S}, d)\\
f^{KS}(\bar{S}, d)
$$

4. Proof of Proposition 2

5. Proof of Proposition 3

We need to prove that if $\beta_i < 1/2$ ($i \in \{1, 2\}$) and $0 < a_1 < (\alpha_2 + \beta_1)/(\alpha_2(1 - 2\beta_1) + (1 + 2\alpha_2)(1 - \beta_1))$ then $f^N(d, \bar{S}) = f^N(a, A) = (1/2, 1/2)$. First note that the slope of the isoquant, $(s_1 - a_1(1 - \beta_1))(s_2 + a_1\alpha_2)$ evaluated at the kink $(1/2, 1/2)$ is given by

$$
\left. \frac{ds_2}{ds_1} \right|_{(1/2,1/2)} = -\frac{1 + 2a_1\alpha_2}{1 - 2a_1(1 - \beta_1)}. 
$$

For the solution to stay at the kink we need that the slope of the isoquant to be steeper than the segment of the bargaining frontier above the 45-degree line and flatter than that below the 45-degree line. i.e.

$$
(1 - 2\beta_2)/(1 + 2\alpha_1) < (1 + 2a_1\alpha_2)/(1 - 2a_1(1 - \beta_1)) < (1 + 2\alpha_2)/(1 - 2\beta_1)
$$

It is easy to see that the first inequality holds as long as $a_1 > 0$ when $\beta_i < 1/2$ ($i \in \{1, 2\}$). Specifically, $0 < a_1 < 1/(2 - 2\beta_1)$ yields that $0 < 2a_1(1 - \beta_1) < 1$, so
0 < 1 − 2\alpha_1(1 − \beta_1) < 1 < 1 + 2\alpha_1. In addition, 0 < 1 − 2\beta_2 < 1 + 2\alpha_1\alpha_2, hence the first part of the inequality holds. The second inequality specify the upper bound of \( a_1, \bar{a}_1 \).

\[
(1 + 2\alpha_1\alpha_2)/(1 − 2\alpha_1(1 − \beta_1)) < (1 + 2\alpha_2)/(1 − 2\beta_1)
\]

\[
\Rightarrow \quad a_1 < \frac{\alpha_2 + \beta_1}{\alpha_2(1 − 2\beta_1) + (1 + 2\alpha_2)(1 − \beta_1)}
\]

So

\[
\bar{a}_1 = \frac{\alpha_2 + \beta_1}{\alpha_2(1 − 2\beta_1) + (1 + 2\alpha_2)(1 − \beta_1)}
\]

For given preference parameters the Nash solution stays at the kink as long as the asymmetry is not sufficient in yielding \( a_1 > \bar{a}_1 \).

6. Proof of Proposition 4

First we prove that

\[
\frac{\partial x^K_1}{\partial \alpha_1} > 0, \quad \frac{\partial x^K_1}{\partial \beta_1} < 0, \quad \frac{\partial x^K_1}{\partial \beta_2} > 0.
\]

Then we show that \( \frac{\partial x^K_1}{\partial \alpha_2} < 0 \) when \( a_1 = 0 \), and that its sign is ambiguous when \( a_1 > 0 \).

The disagreement point and the bliss point are given by

\[
d = (a_1(1 − \beta_1), −a_1\alpha_2) = (d_1, d_2)
\]

and

\[
b(\bar{S}) = \left( \frac{(1 + \alpha_2 − \beta_1) − a_1\alpha_2(1 − 2\beta_1)}{1 + 2\alpha_2}, \frac{(1 + \alpha_1 − \beta_2) − a_1(1 − \beta_1)(1 − 2\beta_2)}{1 + 2\alpha_1} \right) = (b_1, b_2).
\]

When \( a_1 = 0 \), they become

\[
d \bigg|_{a_1=0} = (0, 0)
\]

and

\[
b(\bar{S}) \bigg|_{a_1=0} = \left( \frac{(1 + \alpha_2 − \beta_1)}{1 + 2\alpha_2}, \frac{(1 + \alpha_1 − \beta_2)}{1 + 2\alpha_1} \right).
\]

\[
\frac{\partial d_1}{\partial \alpha_2} = 0 \quad \text{and} \quad \frac{\partial d_2}{\partial \alpha_2} = −a_1 < 0.
\]

This means that the disagreement point moves downward when \( \alpha_2 \) increases. However,

\[
\frac{\partial b_1}{\partial \alpha_2} = −(1 + a_1)(1 − 2\beta_1)(1 + 2\alpha_2)^{-2} < 0 \quad \text{and} \quad \frac{\partial b_2}{\partial \alpha_2} = 0,
\]

which means that the bliss point moves left-ward when \( \alpha_2 \) increases. As a result, the new line \( L(d, b(\bar{S})) \) becomes steeper but assumes a lower position comparing to the one before the increase of \( \alpha_2 \). The change in \( s_1 \), therefore the change in \( x_1 \), is ambiguous.