Time Inconsistency and Naivety of Consumer within Hotelling Competition

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Abstract

In this paper, we look at firms competing à la Hotelling with time inconsistent consumers, both sophisticated and naives. The previous literature claims that a monopoly offering a two-part tariff contract to such consumers do not exploit the time inconsistency bias but the naivety bias. Moreover, with perfect competition, firms do not succeed anymore to exploit the naivety bias even if an allocation inefficiency stays. We adapt DellaVigna and Malmendier (2004) framework to study the impact of an imperfect competition between firms offering a two-part tariff contract and the introduction of transport costs on the exploitation by the firms of this two bias. These costs can be introduced at the contract period or at the consumption period. At the contract period, we observe that the competition even imperfect erases the exploitation of the naivety bias but the introduction of the transport costs conducts to a distortion between consistent and time inconsistent consumers. Indeed, this last consumer gives more weight to the transport costs which leads to a higher market power for firms. At the consumption period, transport costs generate an heterogeneity between consumers, firms are no longer able to provide a perfect commitment to sophisticated consumers. Consequently, allocation with both sophisticated and naive are not efficient.

Keywords: Two-part tariff contract; Hotelling competition; time inconsistency; quasi-hyperbolic discounting; naivety.

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**Introduction**

An important and growing empirical and experimental literature highlights several biases of rationality in consumer decision. These results have an important impact on the interaction between firms and those consumers. Especially, it is interesting to wonder if firms are exploiting this bias or if these bias have an impact on the social welfare; if the market still allows efficient allocations.

In this article, we focus on consumers with time inconsistency problem and naivety bias. Various consumption goods involve costs and benefits delayed in time. It is the case with leisure goods (tobacco, alcohol, gambling . . .) or investment goods (gym, cultural goods, saving . . .) and facing the opportunity to consume these goods, consumer can have self-control problems due to a present bias (ie a very strong preference for the present). That behavior can lead consumer to over consume leisure goods or under consume investment bias. Indeed, there is a strong temptation to postpone or even never realize a activity with immediate cost if one has a strong preference for the present and the benefits are future. However, later, one can regret that choice because it is not optimal for her long term utility.

On the contrary, for the same reason it is easy to be tempted by an activity or a good with an immediate benefits even if one knows that it will definitely cost her the next day.

Although consumer can have a naivety problem ie thinks she will resist to the temptation of consumption when she will face the choice to consume (eg saying ”I am going to a chocolate salon but I will not eat one”) or on the contrary make some resolutions about her consumption and never respect them (eg saying ”This year, I will go to the gym club three times a week”).

A way to take into account these biases is to adapt the utility function. There are two main methods to treat time inconsistency in the literature. The first one is the multi selves model (Strotz 1956, Peleg and Yaari 1973). The time inconsistent agent changes her utility between today and tomorrow. Today she has the utility \( u \) whereas tomorrow she will have the utility \( v \).

The second way to include time inconsistency bias is with quasi-hyperbolic preferences (Phelps and Pollak 1968, Laibson 1997, Gul and Pesendorfer 2001). From that \((\beta, \delta)\) model, the agent has two discount rates, the traditional long-term one, \( \delta \) and the short-term one, \( \beta \), which represents the present bias. The utility in a period 0 is,

\[
U_0 = u_0 + \sum_{t=1}^{n} \beta^t \delta^t u_t
\]

A time inconsistent consumer can be distinguished between a sophisticated one and a naive one (O’Donoghue and Rabin 2001). The sophisticated is perfectly aware of her time consistent problem whereas the naive thinks wrongly she is time consistent. In the two previous models, the naive believes her short term discount rate \( \beta \) is equal to 1 (like the consistent) or she can be partially naive and having some belief on her short term discount rate \( \hat{\beta} \) with \( \beta < \hat{\beta} < 1 \). In the first model, the naive believes his utility will stay \( u \) instead of \( v \) in the next period or if she is only partially naive, believes her utility may be \( u \) with a

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1The consistent agent does not change and keeps the utility \( u \) in the future.
probability $\theta$ or $v$ with a probability $(1 - \theta)$.

Several empirical or experimental studies have proved the existence of time inconsistent and naivety bias in different industries.

Many experimental studies show that the annual discount rate decreases with time with monetary rewards and explain it thanks to quasi-hyperbolic preferences (Thaler 1981, Benzión et al. 1989, Chapman 1996, Andersen et al. 2008). Other lab or field experiments find evidence of present bias (Augenblick et al. 2013) and naivety (Acland and Levy 2013, Ariely and Wertenbroch 2002) thanks to real effort costs.

Empirical studies based on several industries tend to point out that consumers have time inconsistency and naivety problem: phone market (Miravete 2003, Grubb 2009), gym attendance (DellaVigna and Malmendier 2006), credit market (Heidhues and Köszegi 2010).

Thus, it is interesting to study the interaction between these consumers and firms. DellaVigna and Malmendier (2004) use the Phelps and Pollaks design i.e. a $(\beta, \delta)$ model to study the interaction between a monopoly and such consumers with a two-part tariff contract. Their main results are monopoly exploits the naivety bias of consumer but not the time inconsistency problem. Indeed, she offers at sophisticated a perfect commitment device and thanks to this, she is able to consume as a consistent one. On the contrary, the contract offered to naive consumer leads to a negative surplus for consumer and a lower welfare due to inefficient allocations. These results are robust with perfect competition.

Eliaz and Spiegler generalize this analysis with Strotz framework, unspecified non-linear contract and heterogeneity on the naivety degree (Eliaz and Spiegler 2006, Eliaz and Spiegler 2008).

In this paper, we will focus on time inconsistent consumers (both sophisticated and naive) who consume an investment good. Furthermore, there are two firms which offer a two-part tariff contract and compete à la Hotelling. Indeed, with a monopoly, there is no exploitation of the time inconsistency problem if the consumer is sophisticated only of the naivety bias and with perfect competition, the naivety is not exploited anymore but the inefficiency problem stays. Thus, we want to study the impact of the introduction of transport cost first at the contracting period and then at the consumption period on the firm profit. We wonder if the time inconsistency bias still has not impact on the firm profit and if an imperfect competition erases the exploitation of naivety problem.

We use a model very similar to DellaVigna and Malmendier one. There is three periods. In the first period, firms propose a two-part tariff contract and consumers accept or not. At the second period, agents choose whether to consume or not. If they do, they support the costs of the consumption and have the benefits at the last period.

Adding some transport costs introduces an heterogeneity in taste of consumers. We can assume that these costs will be supported when consumers accept the contract at the first period or support it if they consume at the second period.

Within this framework, we find that the introduction of the transport cost at the contract allows firms to exploit the time inconsistency problem. Indeed, transport costs represent

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2Grubb (2009) found almost the same results with overconfidence of consumers.
a market power for the firm and since we assume firms rational, there is a difference in discounting these transport costs between consumers and firms. Time inconsistent consumers overweight these costs and give to firms a higher market power. Also, the introduction of competition, even imperfect, prevents firms to exploit the naivety bias. However, the allocation inefficiency problem stays.

The introduction of transport costs at the consumption period has more impact. It induces an heterogeneity in the consumption probability. The immediate result which follows is the incapacity of the firms to provide a perfect commitment for sophisticated consumers. Then, it implies a non optimal welfare with sophisticated compared to consistent ones. Moreover, as the transport cost play a role in the consumption probability, the profit made facing a sophisticated and a naive will not be the same.

In section 1, we present the main results from DM with a monopoly and these results extended to perfect competition. In section 2, we introduce a competition la Hotelling with transport costs at the contract period. Then, we look the impact of the introduction of transport costs at the consumption period (section 3) and finally we conclude.

1 Framework with Monopoly and Perfect Competition

This section stands to present the principal results for a monopoly offering a two-part tariff contract at time-inconsistent consumers. This model is a simplified version of DellaVigna and Malmendier’s one (DM therefore) (DellaVigna and Malmendier 2006).

Results can be easily extrapolated to perfect competition (DellaVigna and Malmendier 2006, Heidhues and Köszegi 2010, Huck and Zhou 2011).

1.1 The setting

Timing and payoffs The monopoly offers a two-part tariff contract \((L,p)\) in period 0 \((P_0)\) at a representative consumer. This agent can accept or rejects the contract. If she accepts, she will pay the lump sum \(L\) in period 1 \((P_1)\). In \(P_1\), consumer faces her consumption cost \(c\) and decide or not to consume given \(p\) determined by the contract she accepted in \(P_0\). If she consumes, she supports \(c\) and \(p\) and will have the consumption benefit \(b\) at period 2 \((P_2)\). If she does not consume, she has no more payoff. Figure 1 summarizes decisions and payoffs.
Period 0 | Period 1 | Period 2
---|---|---
Monopoly offers \((L,p)\) contract | Consumer accepts | Payoffs: \(-c-p\) | Agent consumes | b
Consumer rejects | Payoffs: \(-L\) | Payoffs: 0 | Agent does not consume | 0

\[ \text{Figure 1: Timing of the model} \]

The consumer supports the cost \(c\) when she consumes. This cost is unknown to both firm and consumer when contracting at \(P_0\), only the distribution of \(c\) is common knowledge. We denote by \(F\) its cumulative distribution function and by \(f\) its probability density function\(^3\).

**Consumer** There is a unique representative consumer. This consumer can experiment both time inconsistency and naivety. We model, following DM, this self-control problem by using quasi-hyperbolic discounting preferences. With time inconsistency problem, her utility in \(P_1\) is

\[ U_1 = u_1 + \sum_{t=2}^{n} \beta \delta^t u_t. \]

We add a short term discount factor \(\beta\) between the present and the period right after "today" to the traditional long term discount factor \(\delta\). A time inconsistent person can be sophisticated (fully aware about her time inconsistency problem) or (partially) naive. If she is naive, she has a wrong belief about her short term discount rate. She thinks she will be more patient that she will actually be. In \(P_0\), she thinks her utility in \(P_1\) will be

\[ U_1 = u_1 + \sum_{t=2}^{n} \hat{\beta} \delta^t u_t \]

with \(\hat{\beta} \in [\beta, 1]\). When she is sophisticated, she has the right belief about her impatience (\(\hat{\beta} = \beta\)).

We study the case with a naive consumer but it is easy to replace \(\hat{\beta}\) by \(\beta\) to have the behavior for the sophisticated consumer and both \(\hat{\beta}\) and \(\beta\) by one when the consumer is consistent.

In this framework, the consumer in \(P_0\) accepts the contract if her expected utility is superior to her reserved utility \(\pi\). This expected utility depends on the lump-sum \(L\) but also on the utility she expects to have with the consumption.

She expects to consume in \(P_1\) for all \(c < \hat{\beta} \delta b - p\), ie with a probability \(F(\hat{\beta} \delta - p)\). We can already note that her expected probability is superior to her real probability to consume. Indeed, she will actually consume if \(c < \beta \delta b - p\). The difference \(F(\hat{\beta} \delta - p) - F(\beta \delta - p)\) is a

\[ \text{To guarantee the existence of the contracts, DM used "Assumption ABP: There exists a pair \((M,z) \in R^2\) such as } f(y'') \leq M f(y') \text{ for all } y' \text{ and } y'', \text{ with } z < |y'| < |y''| \text{ and } y', y'' > 0\text{."} \]

\[ \text{When consumer is consistent, then } \beta = 1. \]
measurement of the naivety degree. The expected utility of the consumption depends also on \( c \) and it is equal to \( \delta b - p - c \) for all \( c \). At the end, the agent expected utility in \( P_0 \) is:

\[
\beta \delta [-L + \int_{-\infty}^{\hat{\delta}b-p} (\delta b - p - c) dF(c)]
\] (1)

Here, this expected utility is discounted with the real short-term discount rate \( \beta \) and not her belief on it since it is the utility she is facing and not her anticipated one. Only her consumption probability is anticipated and depends on her belief on the short-term discount rate.

**Monopoly** The monopoly is considered being time consistent. She does not add the short turn discount rate by discounting between today and tomorrow. She tries to maximize her profit under the participation constraint of the consumer. We consider she is facing one representative consumer type.

Her profit includes the lump-sum if the consumer accepts the contract and the consumption price if she consumes with the (real) probability \( F(\beta \delta b - p) \). Without lost of generality, we consider the monopoly have no cost. Thus, the monopoly program can be written as,

\[
\max_{L,p} \delta [L + pF(\beta \delta b - p)]
\]

s.t.

\[
\beta \delta [-L + \int_{-\infty}^{\hat{\delta}b-p} (\delta b - p - c) dF(c)] \geq \beta \delta \bar{u}
\]

### 1.2 Optimal pricing

It is easy to see that the participation constraint is binding since otherwise the monopoly can arise his profit by increasing the lump-sum. Then we obtain the consumption price thanks to the first order condition of monopoly program facing naive consumer. These conditions give us the following optimal prices,

\[
\left\{
\begin{aligned}
L^* &= \int_{-\infty}^{\hat{\delta}b-p} (\delta b - p - c) dF(c) - \bar{u} \\
p^* &= -(1 - \hat{\beta})\delta b \frac{f(\hat{\delta}b - p^*)}{f(\beta \delta b - p^*)} - \frac{F(\hat{\delta}b - p^*) - F(\beta \delta b - p^*)}{f(\beta \delta b - p^*)} \\
\end{aligned}
\right.
\] (2)

DM proves the existence of such contract thanks to the ABP assumption.

From these previous prices, we can deduce properties on prices for each type of consumer.
**Proposition 0a (DM prices properties)**

1. For consistent consumer, the two-part tariff is set traditionally. The consumption price equals to marginal cost (here 0) and the lump-sum allows to catch all the consumer surplus.

2. For the sophisticated consumer, the consumption price is below the marginal cost and the lump-sum is higher than for the consistent to compensate. It implies a perfect commitment for the sophisticated; the consumption probability is the same than the consistent.

3. For the naive consumer, the consumption price is also set below the marginal price but the lump-sum is even higher since she overestimates her consumption.

**Proof.** See DM (2006)

For a consistent consumer, since \((1 - \hat{\beta}) = 0\) and \(F(\hat{\beta}\delta b - p^*) - F(\beta\delta b - p^*) = 0\), the monopoly sets the marginal price at the marginal cost (here 0) and the lump-sum is equal to \(L^* = \int_{-\infty}^{\delta b} (\delta b - c) dF(c) - \bar{u}\).

Facing the sophisticated consumer, monopoly offers \(p^* = -(1 - \hat{\beta})\delta b\) since \(F(\hat{\beta}\delta b - p^*) - F(\beta\delta b - p^*) = 0\) and \(\frac{f(\hat{\beta}\delta b - p^*)}{f(\beta\delta b - p^*)} = 1\). Thus, the consumption price allows a perfect commitment to the sophisticated since \(F(\beta\delta b - p^*) = F(\delta b)\) ie the same probability to consume than for the consistent. Moreover, \(L^*\) can be rewritten as \(L^* = \int_{-\infty}^{\delta b} (\delta b - c) dF(c) - p^* F(\delta b) - \bar{u}\) so the monopoly is recovering the loss induced by a below marginal cost price in the lump-sum.

For a naive consumer, the monopoly has two reasons to offer a low consumption price. The first one is the same that for the sophisticated. The second one is explicated by consumer overestimating her consumption probability. Since, she believes she will consume more often than she would actually do, the monopoly can arise the lump-sum by recovering the price weighted by the belief on the consumption probability (which is higher than the real one) and decrease the marginal price to attract naive people. The naive consumer looses because of lower benefits she will actually do with her (lower) real consumption probability. The return of investment represented by the consumption is not enough to compensate her higher lump-sum.

### 1.3 Profit and welfare

**Profit** The monopoly makes profit thanks to the optimal lump-sum which binds the participation constraint and thanks to the optimal consumption price weighed by the real consumption probability. \(\pi^* = \delta[L^* + p^* F(\beta\delta b - p^*)]\). Substituting \(L\) by \(L^*\),

\[
\pi^* = \delta \left[ \int_{-\infty}^{\beta\delta b - p^*} (\delta b - c) dF(c) - \bar{u} + \int_{\beta\delta b - p^*}^{\hat{\beta}\delta b - p^*} (\delta b - p^* - c) F(c) \right] \tag{3}
\]

The first part of equation (3) corresponds to the social surplus generated by the interaction between the monopoly and the consumer. This part represents the expected utility of the consumption without the consumption price. Indeed, this price is payed by the consumer but received by the monopoly so at the end, he has no consequence on the social surplus.

The second part of (3) corresponds to the additional fictive surplus the naive consumer
beliefs she will make with her consumption and the monopoly manages to capture. Since, this consumption does not happen and so the interaction, the consumption price is taken into account in this surplus. This fictive surplus does not exist when the monopoly faces consistent or sophisticated consumer.

**Consumer surplus and social welfare** As we saw with the expression (1), the utility of the consumer in \( P_0 \) facing the optimal contract proposed by the monopoly is,

\[
U_0 = \beta \delta [-L^* + \int_{-\infty}^{\beta \delta b - p^*} (\delta b - p^* - c) dF(c)]
\]

Since \( L^* = \int_{-\infty}^{\beta \delta b - p^*} (\delta b - p^* - c) dF(c) - \bar{u} \), for all the consumer \( U_0 = 0 \). With this result, the social welfare \( W \) arises with the profit. In that case, one conclusion would be that the social welfare arises with the naivety degree.

However, this measure does not consider the fact that the consumer has wrong belief on her future short term discount rate. Also, we can wonder if there is a loss of surplus between the sophisticated consumer who uses a short-term discount rate and a consistent one.

One manner to deal with these problem is to consider for all consumers their long term utility (O’Donoghue and Rabin 2001). In this case, the short-term discount rate is not taken into account anymore since we suppose that in the long term utility function, there is no more present bias so \( \beta = 1 \) as the belief on the short-term discount rate. We are only interested in the real consumer surplus and not her anticipated one.

With this assumption, the long-term consumer surplus is,

\[
CS_0 = \delta [-L^* + \int_{-\infty}^{\beta \delta b - p^*} (\delta b - p^* - c) dF(c)]
\]

Substituting \( L \) and rewritten, equation (7) gives us,

\[
CS_i = \delta \left[ -\int_{\beta \delta b - p^*}^{\beta \delta b - p^*} (\delta b - c - p^*) dF(c) + \bar{u} \right]
\]

Or,

\[
CS_i = \delta \left[ \int_{-\infty}^{\beta \delta b - p^*} (\delta b - c) dF(c) \right] - \pi_i
\]

With \( i \), the consumer type and \( \pi_i \), the monopoly profit facing the type \( i \). The social surplus is the profit added to the consumer surplus,

\[
W_i = CS_i + \pi_i = \delta \left[ \int_{-\infty}^{\beta \delta b - p^*} (\delta b - c) dF(c) \right]
\]
Proposition 0b (DM profit and welfare properties)

1. For the consistent and the sophisticated consumer, profit, consumer surplus and welfare are the same because of the perfect commitment. Time inconsistency does not change the welfare and its sharing.

2. For naïve consumer, the welfare is lower because of the non-optimality of \( p \) and its sharing between the monopoly and the consumer is different. Consumer surplus is negative due to an overestimation of the consumption. This fictive surplus leads to additional profit for the monopoly.


1.4 Perfect competition

Thanks to the results with the monopoly cases and the reservation utility, we can extrapolate to perfect competition case.

Proposition 0c (Perfect competition)

1. The consumption price is the same as monopoly case for all the consumer types. Perfect competition decreases the lump-sum to set the profit equals to 0. The lump-sum is equal to 0 for consistent consumer and superior to 0 for the inconsistent one.

2. Perfect competition does not change the social welfare but its sharing.

3. For naïve consumer, there remains an inefficiency effect of the naivety but its impact is lower. The loss in consumer surplus due to monopoly power becomes larger as naivety increases (for fixed \( \beta \)).

The competition has an impact through the reservation utility \( \bar{u} \). In the expression (2) of the optimal prices, in one hand the marginal price does not depend on \( \bar{u} \), it results that the consumption price is the same in both perfect competition case and monopoly one. In the other hand, \( \bar{u} \) decreases the lump-sum \( L \). \( \bar{u} \) is determined as to equate profits to 0 in the case of perfect competition.

The social welfare, in expression (5) does not depend on \( \bar{u} \) ie it is not affected by the monopoly power. However, its sharing will change since both profit and consumer surplus depends on \( \bar{u} \). Moreover, for naïve consumer, the degree of \( \bar{u} \) does not affect the efficiency problem since the consumption probability stays the same. The welfare is still affected by this problem. Moreover, the naivety degree arises the monopoly power on the consumer surplus. Her losses arises with the naivety degree.

We studied the case with the monopoly case and the perfect competition case. However, it is interesting to study the imperfect competition case, especially Hotelling case. Indeed, the transport cost, whether supported at the moment consumers choose their contract or at the moment they have to choose to consume, can affect the welfare and its sharing differently. We study this impact in the next two sections.
2 Hotelling With Transport cost at the Contract Period

In this section, we study the Hotelling competition case with transport costs at the contract period. Firms may offer products or services with different characteristics or qualities. The transport cost represents the difference between the consumer most preferable contract and the contracts offered by firms.

2.1 Framework

In order to study the impact of the transport costs on pricing and welfare, we keep a framework close to the monopoly case of DM. The timing of choices and payoffs stay the same, the contract is still a two-part tariff and the cost and benefits of the consumption have the same properties. However, now, the consumer has to support a cost $t$ at the moment she chooses her contract.

Moreover, we have a representative segment of demand $[0,1]$ in which consumers are uniformly distributed. We have two symmetric firms (A, B) located on each part of the segment: A in 0 and B in 1. The consumer supports a transport cost depending on her situation on the segment demand i.e. her distance to the firms. We suppose this cost is linear, $t(x) = tx$ with $t$ the marginal cost and $x$ the distance between the consumer and the firm.

**Consumers and demand** Consumer chooses to consume at firm 1 or firm 2 and her utility depends on the contract offered by firm $i$. The utility of the consumer located in $x$ and who has chosen the firm A is

$$U_A - tx = \beta \delta [-L_A + \int_{-\infty}^{\beta b - p_A} (\delta b - p_A - c) dF(c)] - tx$$

And her utility if she chooses the firm B is

$$U_B - t(1-x) = \beta \delta [-L_B + \int_{-\infty}^{\beta b - p_B} (\delta b - p_B - c) dF(c)] - t(1-x)$$

These utilities are the same as DM case minus the cost of localization for the consumer.

The demand to the firm $i$ is determined by the marginal consumer $\bar{x}$ who is indifferent between consuming at the firm A and at the firm B i.e $U_A - t\bar{x} = U_B - t(1 - \bar{x})$. It follows

$$\begin{align*}
D_A = \bar{x} &= \frac{U_A - U_B}{2t} + \frac{1}{2} \\
D_B = 1 - \bar{x} &= \frac{U_B - U_A}{2t} + \frac{1}{2}
\end{align*}$$
Firms Firms are in strategic interaction and maximize their profit regarding the consumer utility but also the other firm behavior through the demand function. The firm $i$ program is

$$\max_{L_i,p_i} \pi_i = \delta [L_i + p_i F(\beta \delta b - p_i)] D_i$$ (6)

With $D_i = \frac{U_i - U_{i-1}}{2t} + \frac{1}{2}$ or,

$$D_i = \frac{\beta \delta [L_{i-1} - L_i + \int_{-\infty}^{p_i} (\delta b - p_i - c) dF(c) - \int_{-\infty}^{p_{i-1}} (\delta b - p_{i-1} - c) dF(c)]}{2t} + \frac{1}{2}$$ (7)

### 2.2 Optimal contracts

The first order condition to the program (6) considering the demand (7) gives the optimal contract offered by $i$

$$\left\{ \begin{array}{l}
\hat{p}_i^* = -(1 - \hat{\beta}) \delta b \frac{f(\hat{\beta} \delta b - \hat{p}_i^*)}{f(\hat{\beta} \delta b - \hat{p}_i^*)} - \frac{F(\hat{\beta} \delta b - \hat{p}_i^*) - F(\beta \delta b - \hat{p}_i^*)}{f(\beta \delta b - \hat{p}_i^*)} \\
\hat{L}_i^* = \frac{t}{\beta \delta} + F(\delta \beta b - \hat{p}_i^*) \left[ (1 - \hat{\beta}) \delta b \frac{f(\hat{\beta} \delta b - \hat{p}_i^*)}{f(\beta \delta b - \hat{p}_i^*)} - \frac{F(\hat{\beta} \delta b - \hat{p}_i^*) - F(\beta \delta b - \hat{p}_i^*)}{f(\beta \delta b - \hat{p}_i^*)} \right]
\end{array} \right.$$ (8)

Or,

$$\left\{ \begin{array}{l}
\hat{p}_i^* = -(1 - \hat{\beta}) \delta b \frac{f(\hat{\beta} \delta b - \hat{p}_i^*)}{f(\beta \delta b - \hat{p}_i^*)} - \frac{F(\hat{\beta} \delta b - \hat{p}_i^*) - F(\beta \delta b - \hat{p}_i^*)}{f(\beta \delta b - \hat{p}_i^*)} \\
\hat{L}_i^* = \frac{t}{\beta \delta} - F(\delta \beta b - \hat{p}_i^*) p^*
\end{array} \right.$$ (8)

As firms are symmetric, the same contract is offered by both firm A and B. Moreover, the marginal consumer is located in the middle of the segment demand, $\bar{x} = 1 - \bar{x} = D_A = D_B = \frac{1}{2}$. By symmetry, we study only the firm $i$.

**Proposition 1 (Pricing with transport cost at contract period)**

1. The consumption price is the same than in DM case with the same properties. This price is equal to zero with consistent consumer and negative (below marginal cost) for sophisticated and naive one.

2. The lump-sum is set such as it allows to recover the loss due to a negative marginal price and firms can make some profit thanks to her market power through the transport costs.

3. The market power of the firm arises with the degree of time inconsistency of the consumer.

The marginal price does not depend on $t$ and has the same properties than in DM case. In particular, the proof of the existence of the contract is the same than in DM case. Although this price still allows perfect commitment to sophisticated consumer, ie the consumption probability is the same for sophisticated consumer than for consistent one.

However, transport costs have an impact on the lump-sum. The lump-sum is composed of
discounted transport costs and the marginal price weighted by the consumption probability. The transport costs reflect the firms' power. This result is very close to Armstrong and Vickers' one studying two-part tariffs in duopoly competition\(^5\). However, since the market power is captured at the consumption period and the transport costs supported at the contract period (a period before), these costs are evaluated by consumers with time-inconsistency, i.e., divided by \(\delta\) and \(\beta\). It follows that the market power is more important for the firms when the degree of time inconsistency arises (when \(\beta\) decreases). Nevertheless, since these costs are already paid at the consumption period, the market power does not depend on the naivety. Furthermore, firms recover only the consumption price with the real consumption price. In contrast to DM model, they are no longer able to capture the consumer fictive surplus.

### 2.3 Profit and welfare

**Profit** The firm \(i\) makes profit thanks to the lump-sum and the marginal price weighed by the real consumption probability for the served portion of the segment demand. This profit at equilibrium is,

\[
\pi_i^* = \delta [L_i^* + p_i^* F(\beta \delta b - p_i^*)] D_i
\]

Firms are symmetric, it follows that demand is equal to \(\frac{1}{2}\) for each firm. Moreover, we see with expression (8) that the lump-sum is a linear function of the marginal price.

\[
\pi_i^* = \frac{t}{2\beta}
\]

Again, we find a result very close to Armstrong and Vickers (2010). They claimed the sum of all the firms' profits in the industry is equal to \(t\). The difference here is the \(t\) weighted by the short-run discount rate. Indeed, the firms and an inconsistent consumer do not evaluate the transport cost with the same way. Firms use only \(\delta\) whereas consumers use \(\beta \delta\). Consumer put more weight on this transport cost than firms and in this way give more market power to the firms.

As we claim in proposition 1 (4), firms can no longer exploit the consumer naivety with her contract.

**Consumer surplus and social welfare** Since the utility of the consumer depends on her transport cost, each consumer has a different utility. Thus we calculate the total surplus\(^6\),

\[
CS = \int_0^{\tilde{x}} (sc_1 - tx)dx + \int_{\tilde{x}}^1 (sc_2 - t(1 - x))dx \tag{9}
\]

With \(\tilde{x} = 1/2\), the marginal consumer and \(sc_i = \delta [-L_i^* + \int_{-\infty}^{\beta \delta b - p_i^*} (\delta b - p_i^* - c)dF(c)]\) the surplus without transport costs of all the consumer who turn to the firm 1. Since firms are


\(^6\) Here, since the consumers are uniformly distributed on the demand segment \([0,1]\), the total surplus is equal to the average surplus.
symmetric, we have \( s_1 = s_2 \). We remind that we take into account the long-term surplus of the consumer, ie with the real probability of consumption and discounted only with the long-term discount rate.

By replacing \( L^* \) and rewrite (9),

\[
CS = \delta \left[ \int_{-\infty}^{\beta b - p_i^*} (\delta b - c) dF(c) \right] - \frac{t(4 + \beta)}{4\beta}
\]

Or,

\[
CS = \delta \left[ \int_{-\infty}^{\beta b - p_i^*} (\delta b - c) dF(c) \right] - \frac{t}{4} - 2\pi_i
\]

It follows the total welfare,

\[
W = CS_i + 2\pi_i = \delta \left[ \int_{-\infty}^{\beta b - p_i^*} (\delta b - c) dF(c) \right] - \frac{t}{4}
\]

This total welfare is the same as DM framework minus the impact of the transport costs. Since, these costs do not depend on the consumer time inconsistency or naivety and marginal prices are the same, we have the same results on the welfare than for DM. The interesting thing is the sharing of this welfare.

**Proposition 2** (Profit and welfare with transport cost at contract period)

1. Time inconsistency increases the profit and decreases the consumer surplus but not change the total welfare.
2. Naivety has no impact on the profit but since the marginal price is not optimal the degree of naivety decreases the consumer surplus and so the social welfare.

In this section, we saw that the introduction of transport costs supported by consumers at the contract period leads to a distortion in the firm profit between the consistent and the sophisticated consumer. Indeed, the time inconsistency and the difference between the evaluation of transport cost between firms and consumers lead to a greater power market for the firm.

The introduction of competition even imperfect removes the exploitation of naivety by the firm but not the inefficiency problem since the consumption price stays non optimal.

### 3 Hotelling With Transport cost at the Consumption Period

#### 3.1 Framework

In this section, we study the case in which the transport costs is supported by consumers when they consume. In this framework, we assume firms even competing, may exploit the naivety bias since the transport cost will impact the probability consumption.
Indeed, consumers have to choose one of the firms contract in $P_0$ and she anticipates she consumes only if $\delta \hat{\beta}b > c + p_i + tx$ ie $\delta \hat{\beta}b - p_i - tx > c$, it follows the believed consumption probability $F(\delta \hat{\beta}b - p_i - tx)$. The real one is $F(\delta \hat{\beta}b - p_i - tx)$. Thus each consumer has a different consumption probability.

In this section, we make more restrictive assumptions on the effort cost density function.

**Assumption** $f(c)$ is positive, decreasing in $c$ and determined on $[0, +\infty]$.

A way to interpret this assumption is saying that each consumer has a different taste ie cost for the investment good consumption captured by the transport cost but some hazards can affect her cost when they consume.\(^7\)

The utility of the consumer located in $x$ is

$$U_x = \delta \beta[-L_i + \int_{-\infty}^{\delta \hat{\beta}b - p_i - tx} (\delta b - p_i - tx - c) dF(c)]$$

The transport cost play a role both in the consumption probability and in the evaluation of the consumption benefits. The marginal consumer $\tilde{x}$ who determine the demand to the firm is characterized by $U_i = U_{-i}$ ie

$$\delta \beta[-L_i + \int_{-\infty}^{\delta \hat{\beta}b - p_i - t\tilde{x}} (\delta b - p_i - t\tilde{x} - c) dF(c)] = \delta \beta[-L_{-i} + \int_{-\infty}^{\delta \hat{\beta}b - p_{-i} - t(1-\tilde{x})} (\delta b - p_{-i} - t(1 - \tilde{x}) - c) dF(c)]$$

Or,

$$L_i - L_{-i} = \int_{-\infty}^{\delta \hat{\beta}b - p_i - t\tilde{x}} (\delta b - p_i - t\tilde{x} - c) dF(c) - \int_{-\infty}^{\delta \hat{\beta}b - p_{-i} - t(1-\tilde{x})} (\delta b - p_{-i} - t(1 - \tilde{x}) - c) dF(c) \quad (10)$$

We can not determine $\tilde{x}$ but we will use the expression (10) to determine the optimal contract and we know that $\tilde{x} = \frac{1}{2}$ since firms are symmetric.

Facing a consumer $x$, the firm $i$ realize the profit $\pi_i = \delta[L_i + p_i F(\delta \beta b - p_i - tx)]$. However, since the consumption probability is different for each consumer due to the different localization of consumer, the total profit is

$$\pi_i = \delta[L_i + \int_0^{P_i} (p_i F(\delta \beta b - p_i - tx)) dx] D_i \quad (11)$$

With $D_i = \tilde{x}$

\(^7\)We can take the example of sport. Suppose a consumer want to make some exercises, he can choose between firm A which is a rugby club and B which is an indoor soccer club. She supports a "transport" cost to consume rugby or soccer depending on her taste. This cost is common knowledge and we suppose that taste do not vary with time and is different for each consumer. However, facing the decision whether to consume, she can face some extra unusual costs: if she is sick, if it is raining, if she has an important work to do for the next day . . .
To simplify future equations, we denote:
\[ X_i = \delta \hat{\beta} b - p_i - tx \]
\[ \hat{X}_i = \delta \hat{\beta} b - p_i - t\hat{x} \]

Thus, the firm \( i \) program is

\[ \max_{L_i, p_i} \pi_i = \delta [L_i + p_i \int_0^{\hat{x}} (F(X_i)) dx] \hat{x} \]

With \( \hat{x} \) such as,

\[ L_i - L_{-i} = \delta b (1 - \hat{\beta}) (F(\hat{X}_i) - F(\hat{X}_{-\hat{i}})) + \int_{\hat{X}_{-\hat{i}}}^{\hat{X}_i} F(c) dc \]

### 3.2 Optimal contracts

Using the partial derivatives of the demand with respect to the two prices and the symmetry assumption, we obtain the partial derivatives of the profit with respect to the prices:

\[
\frac{\partial \Pi_i}{\partial L_i} = \delta \left[ \frac{\partial \hat{x}}{\partial L_i} [L_i + p_i \int_0^{\hat{x}} F(X_i) dx] + \hat{x} [1 + p_i \frac{\partial \hat{x}}{\partial L_i} F(X_i)] \right]
\]

\[
\frac{\partial \Pi_i}{\partial p_i} = \frac{\delta}{2} (F(\delta \beta b - p_i) - F(X_i)) \left[ -\frac{p_i}{t} - \delta b (1 - \hat{\beta}) \frac{f(\hat{X}_i)}{F(\delta \beta b - p_i) - F(X_i)} + \int_0^{\hat{x}} F(X_i) dx - F(\hat{X}_i) \right]
\]

The first order condition gives us the optimal prices

\[
\begin{cases}
  p_i^* = t \left[ -\delta b (1 - \hat{\beta}) \frac{f(\hat{X}_i)}{F(\delta \beta b - p_i) - F(X_i)} + \int_0^{\hat{x}} F(X_i) dx - F(\hat{X}_i) \right]
  \\
  L_i^* = -p_i \left( \int_0^{\hat{x}} F(X_i) dx + \frac{1}{2} F(X_i) \right) + t \left[ \delta b (1 - \hat{\beta}) f(\hat{X}_i) + F(\hat{X}_i) \right]
\end{cases}
\]

**PROPOSITION 3 (Existence)**

There exist a contract \((L^*, p^*)\) which is a profit maximizing contract.

**Proof.** The second order conditions are very difficult to verify. To ensure that this contract is a maximum, we can instead confine \( p^* \) in a defined interval and show that if \( p \) is inferior than this interval, the profit is increasing in \( p \) and if it is superior, the profit is decreasing in \( p \). Thus by continuity of the profit function, there exists a price \( p^* \) which represents a local maximum between the boundary of the interval. This interval \([M, \bar{M}]\) is such that,

\[
M = -2M \frac{1 + \delta b (1 - \hat{\beta}) f(0)}{t f(0)}
\]

\[
\bar{M} = 1
\]

**Assumption** There exist \( M \) such that for all \( y' < y \), \( f(y') < M f(y) \) with \( M > 1 \).

\[\text{see calculation in annex A1 A2 A3}\]

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The calculation of this interval is in annex. ■

In this framework, since the consumption probability is not the same for all the consumer served by the firm, it is difficult for the firm to exert her market power. Also, the marginal price takes into account that difference in the consumption probability. It induces that the marginal price is different than in DM framework and it is not evident that we will find the same properties.

To better understand the different effects of time inconsistency and naivety bias, we propose to study separately the different contracts proposed for each type of consumer: the consistent one, the sophisticated one and the naive one.

**Consistent consumer** Facing consistent consumers, firms set the contract,

\[
\begin{aligned}
p_c^* &= t \left[ \int_0^\bar{x} F(\delta b - p_c^* - tx)dx - F(\delta b - p_c^* - t/2) \right] \\
L_c^* &= -p_c^* \left( \int_0^\bar{x} F(\delta b - p_c^* - tx)dx + \frac{1}{2} F(\delta b - p_c^* - t/2) \right) + t \left[ F(\delta b - p_c^* - t/2) \right] 
\end{aligned}
\] (12)

With this expression, it is difficult to determine the sign of the marginal price and to interpret it. However, we can focus on the lump-sum. Its composition is similar than previous cases. Indeed, the first part of this lump-sum stands to compensate the marginal price and the second part ensures that firms exert their market power.

However, the probability of consumption is not the same for each consumer. The closest one does not have any transport cost so he has the highest consumption probability. On the other side, the marginal consumer supports the highest transport cost so he has the lower consumption probability. With the expression (12), we see that the market power of firms depending on transport cost is weighed by the marginal consumption probability ie firms have only a market power for the lowest consumption probability.

Nonetheless, in the first part of the lump-sum expression, the marginal price is compensated thanks to a probability included between the marginal one and the closest one.

**Proof.** \( \int_0^\bar{x} F(\delta b - p_c^* - tx)dx > \frac{F(\delta b - p_c^* - t/2)}{2} \) so,

\[
\int_0^\bar{x} F(\delta b - p_c^* - tx)dx + \frac{1}{2} F(\delta b - p_c^* - t/2) > F(\delta b - p_c^* - t/2)
\]

And,

\[
\int_0^\bar{x} F(\delta b - p_c^* - tx)dx < \frac{F(\delta b - p_c^*)}{2} \) so,

\[
\int_0^\bar{x} F(\delta b - p_c^* - tx)dx + \frac{1}{2} F(\delta b - p_c^* - t/2) < F(\delta b - p_c^*)
\]

■
**Sophisticated consumer**  
Facing sophisticated consumers, firms set the contract,

\[
\begin{align*}
    p^*_s &= t \left[ -\delta b (1 - \beta) \frac{f(X_i)}{F(\delta \beta b - p^*_s) - F(X_i)} + \int_0^{\bar{x}} F(X_i) dx - F(X_i) \right] \\
    L^*_s &= -p^*_s \left( \int_0^{\bar{x}} F(X_i) dx + \frac{1}{2} F(X_i) \right) + t \left[ \delta b (1 - \beta) f(X_i) + F(X_i) \right]
\end{align*}
\]

For this consumer, we can reduce the existence interval and can discuss according to the new boundary of this interval the sign of the marginal price. We keep the lower boundary \( M \) but we can reduce the higher boundary \( \bar{M} \). We find that,

\[ p^*_s < 1 - \frac{2 \delta b (1 - \beta)}{t} \]

Thanks to this boundary, we can claim that the optimal marginal price is negative if this boundary is negative. One condition is,

\[ \beta < \frac{\delta b - \frac{1}{2}}{\delta b} \]

This condition can also be written as

\[ \frac{t}{2} < \delta b (1 - \beta) \]

Even if we can not find conditions where the profit is positive, we see that with a sufficiently high time inconsistency degree or sufficiently low transport costs, the marginal price is sure to be negative.

Intuitively, this result is consistent with the previous results. Indeed, the time inconsistency degree tends to decrease the marginal price to offer at the consumer a commitment whereas the transport costs offer to firms a market power.

The analysis of the lump sum is really closed to the consistent one. The marginal price is weighed by a probability between the closest consumer one and the marginal consumer one. The market power is exerted thanks to transport cost weighed by the marginal consumer probability but in this case also with \( \delta b (1 - \beta) f(X_i) \) which represents the effect of the time inconsistency degree on the market power of the firm \(^{10}\).

\(^{9}\)see calculation in annex A5

\(^{10}\)It is not obvious that time-inconsistency will arise the market power since it has an impact on the consumption probability but also on the price which has an impact on the probability.
Proposition 4

With Hotelling competition, firms facing sophisticated consumer offer a two-part tariff contract with the following properties

1. The marginal price \( p_s^* \) decreases with \( \beta \) ie when the time inconsistency degree increases\(^{11}\).

2. Perfect commitment for sophisticated thanks to the marginal price is not allowed anymore.

Proof. To prove (1), we used the partial derivative of profit with respect to the marginal price.

\[
\frac{\partial \Pi_i}{\partial p_i} = \frac{\delta}{2} \left( F(\delta \beta b - p_i) - F(X_i) \right) \left[ - \frac{p_i}{t} - \delta b (1 - \beta) \frac{f(X_i)}{F(\delta \beta b - p_i) - F(X_i)} + \int_0^x \frac{F(X_i)dx}{F(\delta \beta b - p_i) - F(X_i)} \right]
\]

If we consider the profit as a function of \( \beta \) and \( p(\beta) \) and we know that first order conditions are verified then

\[
\frac{\partial}{\partial \beta} \left( \frac{\partial \Pi_i}{\partial p_i}(\beta, p_i(\beta)) \right) + \frac{\partial p_i(\beta)}{\partial \beta} \cdot \frac{\partial}{\partial p_i} \left( \frac{\partial \Pi_i}{\partial p_i}(\beta, p_i(\beta)) \right) = 0
\]

Now, we know that the second order conditions are respected ie \( \frac{\partial^2 \Pi_i}{\partial \beta \partial p_i} < 0 \) in \( p_s^* \). Then the sign of \( \frac{\partial^2 \Pi_i}{\partial \beta \partial p_i} \) gives us the sign of \( \frac{\partial p_i}{\partial \beta} \) ie the variation of the marginal price with respect to \( \beta \). We have,

\[
\frac{\partial}{\partial \beta} \frac{\partial \Pi_i}{\partial p_i} = \frac{\delta}{2} \left[ - \frac{p_i}{t} \left( \delta b (f(\delta \beta b - p_i) - f(X_i)) \right) - (\delta b)^2 (1 - \beta) f'(X_i) + \frac{\delta b}{t} (F(\delta \beta b - p_i) - F(X_i)) \right]
\]

The third term is positive because \( F \) is increasing. The second term is positive too because \( f' < 0 \). For the last term, we have \( f(\delta \beta b - p_i) - f(X_i) < 0 \) because \( f \) is decreasing so the sign of this term depends on the sign of the marginal price. If the price is positive, then it is positive. If the marginal price is negative, we show in annex A5 that if we assume \( f \) is linear, we find that this last term is also positive. We can conclude that \( \frac{\partial p_i}{\partial \beta} > 0 \).

This results is again very closed to previous ones. The intuition is the time inconsistency reduces the consumption probability so firms have incentives to reduces the marginal prices to maintain the consumption probability. Especially with a two-part tariffs where she can compensate the marginal price in the lump-sum.

Perfect commitment happens if the consumption probability of the sophisticated consumer is the same than consumption probability of the consistent one ie if \( F(\delta \beta b - p_s^* - tx) = F(\delta b - p_s^* - tx) \) or, given the monotony of \( F \), if \( \delta \beta b - p_s^* = \delta b - p_c^* \). Thus, the condition for the perfect commitment is,

\[
\int_0^x f(X_i)dx = f(X_i)
\]

This condition is too particular to be validated in the general case. We can conclude there is no perfect commitment.

\(^{11}\)At least when \( p_s^* \) is positive or negative and \( f(c) \) linear
The intuition behind this comes from the fact that each consumer has a different consumption probability and the firm can not offer a perfect commitment to each consumer.

**Naive consumer** Facing naive consumers, firms set the contract,

\[
\begin{align*}
  p_i &= t \left[ -\delta b(1 - \hat{\beta}) \frac{f(\hat{X}_i)}{F(\delta \beta b - p_i) - F(X_i)} + \int_0^{\hat{x}} F(X_i)dx - F(\hat{X}_i) \right] \\
  L_i &= -p_i \left( \int_0^{\hat{x}} F(X_i)dx + \frac{1}{2} F(X_i) \right) + t \left[ \delta b(1 - \hat{\beta}) f(\hat{X}_i) + F(\hat{X}_i) \right]
\end{align*}
\]

As with consistent and sophisticated consumer, it is hard to claim the sign of the marginal price.

However, we observe that if the lump-sum is composed by the same two parts. The second part is different than for the consistent and the sophisticated. indeed, the transport cost is now weighed by the belief on the consumption probability and not the real one. On the contrary of the previous case with transport costs at the contract period, the naivety is now impacting the market power of the firms.

By studying the variation of the marginal price regarding the time inconsistency degree and naivety degree, we can highlight some results.

**Proposition 5**

*With Hotelling competition, firms facing naive consumers offer a two-part tariff contract with the following properties*

1. The marginal price \( p^*_s \) increases with \( \hat{\beta} \) (with constant \( \beta \)) ie when the naivety degree increases.
2. Time inconsistency has an ambiguous impact on the marginal price. If the optimal price is positive, then the time inconsistency degree decreases the marginal price however if this price is negative, the time inconsistency degree increases this price.

**Proof.** To prove this proposition, we use the same technique as previous ie to prove (1), we use,

\[
\frac{\partial}{\partial \hat{\beta}} \frac{\partial \Pi}{\partial p_i} = \frac{\delta}{2} [ -\delta b f'(\hat{X}_i) ]
\]

This equation is the same sign as the partial derivative of the marginal price with respect to the belief on the short term discount rate \( \hat{\beta} \) (with a constant \( \beta \)). Since, \( f \) is decreasing, it is easy to saw that the expression (44) is positive and conclude that the naivety degree arises the marginal price.

To prove (2), we have,

\[
\frac{\partial}{\partial \beta} \frac{\partial \Pi}{\partial p_i} = \frac{\delta}{2} [ -\delta b \left[ (f(\delta \beta b - p_i)) - f(X_i) \right] + \frac{(F(\delta \beta b - p_i) - F(X_i))}{t} ]
\]

The second term is positive but the first term depends on the sign of \( p^*_n \). If the marginal price is positive, the first term is positive and so the equation. It follows that the time inconsistency degree decreases the marginal price. However, if \( p^*_n \) is negative, we show in
the annex A5 that this expression can not be positive, it follows that in that case the time inconsistency degree decreases the marginal price.

### 3.3 Profit and welfare

**Profit** At the equilibrium, by replacing the optimal lump-sum in the expression (11) of the profits, we obtain,

\[ \pi_i = \delta \left[ \frac{1}{2} F(X_i) + t \beta_1 (1 - \hat{\beta}) f(\hat{X}_i) + F(\hat{X}_i) \right] \]

Or by replacing the optimal marginal price,

\[ \pi_i = \delta t \left[ \beta_1 (1 - \hat{\beta}) f(\hat{X}_i) + F(\hat{X}_i) \right] \]

To improve the understanding of the profit function and the impact of the localization, we can compare the profit firms realize with a consumer located in \( x \) to the profit with \( \tilde{x} \).

With the marginal consumer, the profit is

\[ \pi_{\tilde{x}} = \delta \left[ L_i + p_i (\beta_1 - p_i - t\tilde{x}) \right] \]

ie,

\[ \pi_{\tilde{x}} = \delta \left[ \frac{1}{2} F(X_{\tilde{x}}) - \int_0^{\tilde{x}} (F(X_i))dx \right] + t \beta_1 (1 - \hat{\beta}) f(\hat{X}_i) + F(\hat{X}_i) \]

And with any consumer \( x \), \( \pi_x = \delta \left[ L_i + p_i (\beta_1 - p_i - tx) \right] \)

ie,

\[ \pi_x = \delta \left[ \frac{1}{2} F(X_i) - \int_0^{\tilde{x}} (F(X_i))dx \right] + \int_{X_i}^{\tilde{x}} p_i dF(c) + t \beta_1 (1 - \hat{\beta}) f(\hat{X}_i) + F(\hat{X}_i) \]

(13)

In both expressions, through the first term, the marginal price has a negative effect on the profit if it is positive and a positive one if it is negative since \( \frac{F(X_{\tilde{x}})}{2} - \int_0^{\tilde{x}} (F(X_i))dx < 0 \).

The negative effect of this price (if it is positive) decreasing the consumption probability \((-p_i \int_0^{\tilde{x}} (F(X_i))dx < 0\)) is higher than the profit made thanks to this price with the marginal consumer \((p_i \frac{F(X_i)}{2})\). However, we see thanks to expression (13) that since a consumer \( x \) has a higher consumption probability to consume than a marginal one, firm receive \( p \) with a highest probability so this price is a positive source of profit if it is positive.

Generally, the marginal price has 3 different impacts on the profit. First, when the marginal price increases, the consumption probability decreases but it increase the profit made when there is consumption and finally it can decrease the demand at the firm giving the strategic interaction between firms.

**Consumer surplus** As we did for the firm profit, it is more interesting to study the surplus of any consumer located in \( x \) and the surplus of the marginal consumer.
The marginal consumer surplus is,
\[
SC_{\tilde{x}} = \delta \left[ \int_{-\infty}^{X_i} (\delta b - \frac{t}{2} - c) dF(c) + p_i^* \int_{0}^{\tilde{x}} F(X_i) dx - \frac{p_i^*}{2} F(X_i) \right] - t(\delta b(1 - \hat{\beta}) f(\hat{X}_i) + F(\hat{X}_i))
\]

And the surplus of one consumer \(x\) is,
\[
SC_x = \delta \left[ \int_{-\infty}^{X_i} (\delta b - tx - c) dF(c) + p_i^* \int_{0}^{\tilde{x}} F(X_i) dx - \frac{p_i^*}{2} F(X_i) - \int_{X_i}^{X_i} p_i^* dF(c) \right] - \frac{t}{2}(\delta b(1 - \hat{\beta}) f(\hat{X}_i) + F(\hat{X}_i))
\]

Or,
\[
SC_x = \delta \int_{-\infty}^{X_i} (\delta b - tx - c) dF(c) - \pi_x
\]

It follows the social welfare for one consumer \(x\),
\[
W_x = SC_x + \pi_x = \int_{-\infty}^{\delta b - p_i^* - tx} (\delta b - tx - c) dF(c)
\]

Thus the consumer surplus is the total surplus generated by the interaction between this consumer and one firm minus the firm profit.

The social welfare depends on the consumption probability trough the marginal price and since the price are different for each consumer type and does not allow the perfect commitment, the profit, the consumer surplus and so the social welfare of these three types are different.

**Proposition 6**
The introduction of Hotelling competition and transport cost at the period of consumption generates,

1. Because of the heterogeneity in the consumption probabilities, a perfect commitment is impossible for the sophisticated. That implies different profit, consumer surplus and social welfare between a consistent and a sophisticated consumer. We can conjecture that the consumption probability is lower with the sophisticated and so the social welfare.
2. Because of wrong beliefs on consumption probability, profit, consumer surplus and social welfare are different than for sophisticated. Again, we can conjecture that the social welfare is not optimal since the price and so the consumption probability are not optimal for the naive consumer.
4 Conclusion

We introduce imperfect competition and transport costs in DM model. These costs allows us to introduce an heterogeneity on the taste of consumers. It can be introduced at the contract period ie consumers support it one time by choosing between the two firms the best contract for them or at the consumption period ie consumers support it only if they consume.

When transport costs are introduced at the contract period, they do not impact the consumption. The marginal price proposed to each type of consumers is the same than DM framework. The lump-sum stands to recover loses due to negative marginal price in case of time inconsistent consumer and to exert a market power due to transport costs. Thereby, firms are still able to offer to sophisticated consumers a perfect commitment, their consumption probability stays the same than consistent consumers one. However, time inconsistent overweigh the transport costs with their short term discount run which gives more power market to firms. In that sense, because of the difference in evaluation of these costs between the firm and these consumers, firms can exploit the time inconsistency bias. Thus, firm profit increases (and consumer surplus decreases) when the degree of time inconsistency increases.

Facing naive consumers, we find that the introduction of even imperfect competition is enough to prevent the exploitation of the naivety bias by firms. However as perfect competition, the problem of allocation inefficiency is not solve which implies that naivety bias reduces the social welfare, even if it has no impact on the profit.

When transport costs are introduced at the consumption period, they impact the consumption probability. The consumer heterogeneity stands both in the difference between consumption probabilities and in the difference between consumption utility. This mainly implies that firms are no longer able to provide a perfect commitment to sophisticated consumers. Thereby, we can conjecture the social welfare is lower with sophisticated consumers than consistent ones.

Moreover, within this framework, we observe that the time inconsistency degree decreases the marginal price whereas the naivety degree increases it. The intuition is that firms have incentives to decrease the price to attract time inconsistent consumers while the naivety tends to give them more market power at the consumption period.

In conclusion, we can claim that within this framework both time inconsistency bias and naivety one can have impact on consumer surplus and social welfare. This result reminds the significance of the time inconsistency bias. Indeed, sophisticated people are often considered as rational since they try to commit themselves in order to act like a consistent. Furthermore, the literature we studied in the first section point out that time inconsistency, with a two-part tariff contract, has no impact on consumer surplus and social welfare. Thereby, one can conclude that considering this bias is not important in front of the naivety one.
References


Annex

A1-Marginal consumer

Characteristics of the marginal consumer

The consumer \( \bar{x} \) is indifferent between the firm \( i \) and the firm \(-i\) if \( U_i^\bar{x} = U_{-i}^\bar{x} \). Or if:

\[
\delta \beta \left[ -L_i + \int_{-\infty}^{\hat{X}_i} (\delta b - p_i - t \bar{x} - c) dF(c) \right] = \delta \beta \left[ -L_{-i} + \int_{-\infty}^{\hat{X}_{-i}} (\delta b - p_{-i} - t(1 - \bar{x}) - c) dF(c) \right]
\]

We denote \( \hat{X}_i = \delta \hat{b} - p_i - t \bar{x} \) et \( \hat{X}_{-i} = \delta \hat{b} - p_{-i} - t(1 - \bar{x}) \).

So,

\[
L_i - L_{-i} = \int_{-\infty}^{\hat{X}_i} (\delta b - p_i - t \bar{x} - c) dF(c) - \int_{-\infty}^{\hat{X}_{-i}} (\delta b - p_{-i} - t + t \bar{x} - c) dF(c)
\]

These integrals can be rewritten as,

\[
\int_{-\infty}^{\hat{X}_i} (\delta b - p_i - t \bar{x} - c) dF(c) = \left[ (\delta b - p_i - t \bar{x} - c) F(c) \right]_{-\infty}^{\hat{X}_i} + \int_{-\infty}^{\hat{X}_i} F(c) dc
\]

\[
= \delta b (1 - \hat{\beta}) F(\hat{X}_i) + \int_{-\infty}^{\hat{X}_i} F(c) dc
\]

By symmetry,

\[
\int_{-\infty}^{\hat{X}_{-i}} (\delta b - p_{-i} - t + t \bar{x} - c) dF(c) = \delta b (1 - \hat{\beta}) F(\hat{X}_{-i}) + \int_{-\infty}^{\hat{X}_{-i}} F(c) dc
\]

Thus, the marginal consumer is such that,

\[
L_i - L_{-i} = \delta b (1 - \hat{\beta}) (F(\hat{X}_i) - F(\hat{X}_{-i})) + \int_{\hat{X}_{-i}}^{\hat{X}_i} F(c) dc
\]  (14)

We can not formally explicit the expression of this consumer however we can use the expression (14) to find the demand variation with respect to the prices \( p_i \) and \( L_i \) thanks to derivatives.

Since, firms are symmetric and we assume there are localized in each part of the demand segment, the marginal consumer is situated in the middle of this segment, ie \( \bar{x} = \frac{1}{2} \).

Derivatives of the marginal consumer with respect to the prices

To calculate the optimal contracts, it would be necessary to know the derivative of the demand (ie the marginal consumer) with respect to the prices. (We only study the case of the firm \( i \), results for the other firm are simply found by symmetry).

Derivative of the demand function with respect to the lump-sum
\[
\frac{\partial}{\partial L_i} (L_i - L_{-i}) = \frac{\partial}{\partial L_i} (\delta b(1 - \hat{\beta})(F(\hat{X}_i) - F(\hat{X}_{-i}))) + \frac{\partial}{\partial L_i} \int_{\hat{X}_{-i}}^{\hat{X}_i} F(c) dc
\]

\[
1 = \delta (1 - \hat{\beta}) \left[ \frac{\partial \hat{X}_i}{\partial L_i} f(\hat{X}_i) - \frac{\partial \hat{X}_{-i}}{\partial L_i} f(\hat{X}_{-i}) \right] + \frac{\partial}{\partial L_i} \int_{\hat{X}_{-i}}^{\hat{X}_i} F(c) dc
\]  

(15)

We have \(\frac{\partial \hat{X}_i}{\partial L_i} = -t \frac{\partial \tilde{x}}{\partial L_i}\) et \(\frac{\partial \hat{X}_{-i}}{\partial L_i} = t \frac{\partial \tilde{x}}{\partial L_i}\)  
And,

\[
\frac{\partial}{\partial L_i} \int_{\hat{X}_{-i}}^{\hat{X}_i} F(c) dc = \frac{\partial}{\partial L_i} \left[ P(c) \right]_{\hat{X}_{-i}}^{\hat{X}_i} \\
= \frac{\partial}{\partial L_i} [P(\hat{X}_i) - P(\hat{X}_{-i})] \\
= \frac{\partial \hat{X}_i}{\partial L_i} F(\hat{X}_i) - \frac{\partial \hat{X}_{-i}}{\partial L_i} F(\hat{X}_{-i}) \\
= -t \frac{\partial \tilde{x}}{\partial L_i} (F(\hat{X}_i) + F(\hat{X}_{-i}))
\]

With \(P(c)\) : primitive of \(F(c)\).
Thus, the expression (15) can be rewritten,

\[
1 = \delta b(1 - \hat{\beta}) \left[ -t \frac{\partial \tilde{x}}{\partial L_i} (f(\hat{X}_i) + f(\hat{X}_{-i})) \right] - t \frac{\partial \tilde{x}}{\partial L_i} (F(\hat{X}_i) + F(\hat{X}_{-i}))
\]

\[
1 = -t \frac{\partial \tilde{x}}{\partial L_i} \left[ \delta (1 - \hat{\beta}) (f(\hat{X}_i) + f(\hat{X}_{-i})) + F(\hat{X}_i) + F(\hat{X}_{-i}) \right]
\]

So,

\[
\frac{\partial \tilde{x}}{\partial L_i} = -\frac{1}{t \left[ \delta b(1 - \hat{\beta}) (f(\hat{X}_i) + f(\hat{X}_{-i})) + F(\hat{X}_i) + F(\hat{X}_{-i}) \right]}
\]

Or, since firms are symmetric,

\[
\frac{\partial \tilde{x}}{\partial L_i} = -\frac{1}{2t \left[ \delta b(1 - \hat{\beta}) f(\hat{X}_i) + F(\hat{X}_i) \right]}
\]  

(16)

Derivative of the demand function with respect to the consumption price

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\[ \frac{\partial}{\partial p_i}(L_i - L_{-i}) = \frac{\partial}{\partial p_i}(\delta b(1 - \hat{\beta})(F(\hat{X}_i - F(\hat{X}_{-i}))) + \int_{\hat{X}_{-i}}^{X_i} F(c)dc) \]

\[ 0 = \delta(1 - \hat{\beta})[\frac{\partial \hat{X}_i}{\partial p_i}f(\hat{X}_i) - \frac{\partial \hat{X}_{-i}}{\partial p_i}f(\hat{X}_{-i})] + \frac{\partial}{\partial p_i} \int_{\hat{X}_{-i}}^{X_i} F(c)dc \]  

(17)

We have \( \frac{\partial \hat{X}_i}{\partial p_i} = -1 - t \frac{\partial \hat{x}}{\partial p_i} \) et \( \frac{\partial \hat{X}_{-i}}{\partial p_i} = t \frac{\partial \hat{x}}{\partial p_i} \)

And

\[ \frac{\partial}{\partial p_i} \int_{\hat{X}_{-i}}^{X_i} F(c)dc = \frac{\partial}{\partial p_i} [F(c)]_{\hat{X} \rightarrow \hat{X}_{-i}} \]

\[ = \frac{\partial}{\partial p_i} [P(\hat{X}_i) - P(\hat{X}_{-i})] \]

\[ = \frac{\partial \hat{X}_i}{\partial p_i}F(\hat{X}_i) - \frac{\partial \hat{X}_{-i}}{\partial p_i}F(\hat{X}_{-i}) \]

\[ = (1 - t \frac{\partial \hat{x}}{\partial p_i})(F(\hat{X}_i) + F(\hat{X}_{-i})) - F(\hat{X}_i) \]

Thus, the expression (17) can be written as,

\[ 0 = \delta b(1 - \hat{\beta})[-t \frac{\partial \hat{x}}{\partial p_i}(f(\hat{X}_i) + f(\hat{X}_{-i})) - f(\hat{X}_i)] - t \frac{\partial \hat{x}}{\partial p_i}(F(\hat{X}_i) + F(\hat{X}_{-i}) - F(\hat{X}_i)) \]

\[-\delta b(1 - \hat{\beta})f(\hat{X}_i) - F(\hat{X}_i)) = t \frac{\partial \hat{x}}{\partial p_i} \delta b(1 - \hat{\beta})(f(\hat{X}_i) + f(\hat{X}_{-i}))) + F(\hat{X}_i) + F(\hat{X}_{-i})) \]

So,

\[ \frac{\partial \hat{x}}{\partial p_i} = \frac{-\delta b(1 - \hat{\beta})f(\hat{X}_i) - F(\hat{X}_i))}{t[\delta b(1 - \hat{\beta})(f(\hat{X}_i) + f(\hat{X}_{-i}))) + F(\hat{X}_i) + F(\hat{X}_{-i})]} \]

Or, since firms are symmetric,

\[ \frac{\partial \hat{x}}{\partial p_i} = -\frac{1}{2t} \]

Finally, we can notice that \( \frac{\partial \hat{x}}{\partial p_i} = \delta b(1 - \hat{\beta})f(\hat{X}_i) - F(\hat{X}_i), \frac{\partial \hat{x}}{\partial p_i}. \)

Or, \( \frac{\partial \hat{x}}{\partial L_i} = \frac{1}{\delta b(1 - \hat{\beta})f(\hat{X}_i) - F(\hat{X}_i)} \frac{\partial \hat{x}}{\partial p_i}. \)
A2-Calculation of the lump-sum

To calculate the equilibrium prices, we use the profit,

$$\Pi_i = \delta [L_i + p_i \int_0^{\tilde{x}} F(X_i)dx]$$

Avec $X_i = \delta \beta b - p_i - tx$.

The derivative of the profit with respect to the lump-sum is,

$$\frac{\partial \Pi_i}{\partial L_i} = \delta \left[ \frac{\partial \tilde{x}}{\partial L_i} [L_i + p_i \int_0^{\tilde{x}} F(X_i)dx] + \tilde{x} [1 + p_i \frac{\partial}{\partial L_i} \int_0^{\tilde{x}} F(X_i)dx]\right]$$  \hspace{1cm} (18)

As a reminder, $\frac{\partial \hat{X}_i}{\partial L_i} = -t \frac{\partial \tilde{x}}{\partial L_i}$ et $\frac{\partial \hat{X}_i}{\partial \tilde{x}} = t \frac{\partial \tilde{x}}{\partial L_i}$

And,

$$\frac{\partial }{\partial L_i} \int_0^{\tilde{x}} F(X_i)dx = \frac{\partial }{\partial L_i} \left[ \frac{P(X_i)}{-t} \right]_0^{\tilde{x}}$$

$$= \frac{\partial }{\partial L_i} \left[ -\frac{1}{t} (P(X_i) - P(\delta \beta b - p_i)) \right]$$

$$= -\frac{1}{t} \frac{\partial X_i}{\partial L_i} F(X_i)$$

$$= \frac{\partial \tilde{x}}{\partial L_i} F(X_i)$$

By replacing in (18), we have,

$$\frac{\partial \Pi_i}{\partial L_i} = \delta \left[ \frac{\partial \tilde{x}}{\partial L_i} [L_i + p_i \int_0^{\tilde{x}} F(X_i)dx] + \tilde{x} [1 + p_i \frac{\partial}{\partial L_i} \int_0^{\tilde{x}} F(X_i)dx]\right]$$

The first order conditions give us,

$$\frac{\partial \Pi_i}{\partial L_i} = 0$$

$$\delta \left[ \frac{\partial \tilde{x}}{\partial L_i} [L_i + p_i \int_0^{\tilde{x}} F(X_i)dx] + \tilde{x} [1 + p_i \frac{\partial}{\partial L_i} \int_0^{\tilde{x}} F(X_i)dx]\right] = 0$$

Or,

$$L_i = -p_i \left( \int_0^{\tilde{x}} F(X_i)dx + \frac{F(X_i)}{2} \right) + t \left[ \delta (1 - \hat{\beta}) f(\hat{X}_i) + F(\hat{X}_i) \right]$$
A3-Calculation of the marginal price

The derivative of the profit with respect to the marginal is,

$$\frac{\partial \Pi}{\partial p_i} = \delta \left[ \frac{\partial \bar{x}}{\partial p_i} [L_i + p_i \int_0^{\bar{x}} F(X_i) dx] + \bar{x} \left[ \int_0^{\bar{x}} F(X_i) dx + p_i \left( \frac{\partial}{\partial p_i} \int_0^{\bar{x}} F(X_i) dx \right) \right] \right]$$

(19)

As a reminder, we have $\frac{\partial \bar{x}}{\partial p_i} = -1 - t \frac{\partial \bar{x}}{\partial p_i}$ et $\frac{\partial \bar{x}}{\partial \beta b} = t \frac{\partial \bar{x}}{\partial p_i}$

And,

$$\frac{\partial}{\partial p_i} \int_0^{\bar{x}} F(X_i) dx = \frac{\partial}{\partial p_i} \left[ - \frac{1}{t} \right] \left[ P(X_i) - P(\delta \beta b - p_i) \right]$$

$$\frac{\partial}{\partial p_i} \left[ - \frac{1}{t} \right] \left[ X_i F(\delta \beta b) + F(\delta \beta b - p_i) \right]$$

$$= \frac{F(\delta \beta b - p_i)}{t} + \frac{\partial \bar{x}}{\partial L_i} F(X_i)$$

Replacing in (19),

$$\frac{\partial \Pi}{\partial p_i} = \delta \left[ \frac{\partial \bar{x}}{\partial p_i} [L_i + p_i \int_0^{\bar{x}} F(X_i) dx] + \bar{x} \left[ \int_0^{\bar{x}} F(X_i) dx + p_i \left( \frac{F(X_i) - F(\delta \beta b - p_i)}{t} + \frac{\partial \bar{x}}{\partial p_i} F(X_i) \right) \right] \right]$$

(20)

If we replace $L_i$ in the expression (20), we have,

$$\frac{\partial \Pi}{\partial p_i} = \delta \left[ \frac{\partial \bar{x}}{\partial p_i} \left[ p_i \left( \int_0^{\bar{x}} F(X_i) dx + \frac{F(X_i)}{2} \right) + \bar{x} \left( \frac{\delta (1 - \beta) f(\bar{X}_i) + F(\bar{X}_i)}{t} \right) + \frac{\partial \bar{x}}{\partial p_i} F(X_i) \right] \right]$$

$$+ \bar{x} \left[ \int_0^{\bar{x}} F(X_i) dx + p_i \left( \frac{F(X_i) - F(\delta \beta b - p_i)}{t} + \frac{\partial \bar{x}}{\partial p_i} F(X_i) \right) \right]$$

$$= \delta \left[ p_i \left( - \frac{\partial \bar{x}}{\partial p_i} \frac{F(X_i)}{2} + \bar{x} \left( \frac{F(X_i) - F(\delta \beta b - p_i)}{t} + \frac{\partial \bar{x}}{\partial p_i} F(X_i) \right) \right) \right]$$

$$+ \frac{\partial \bar{x}}{\partial p_i} t \left[ \delta (1 - \beta) f(\bar{X}_i) + F(\bar{X}_i) \right] + \bar{x} \left[ \int_0^{\bar{x}} F(X_i) dx \right]$$

$$= \delta \left[ p_i \left( \frac{F(X_i)}{4t} + \frac{F(X_i) - F(\delta \beta b - p_i)}{2t} - \frac{F(X_i)}{4t} \right) \right]$$

$$- \delta b (1 - \beta) f(\bar{X}_i) + F(\bar{X}_i) + \int_0^{\bar{x}} F(X_i) dx$$

Thus,

$$\frac{\partial \Pi}{\partial p_i} = \delta \left( \frac{F(\delta \beta b - p_i) - F(X_i)}{2} \right) \left[ - \frac{p_i}{t} - \delta b (1 - \beta) \frac{f(\bar{X}_i)}{F(\delta \beta b - p_i) - F(X_i)} + \int_0^{\bar{x}} F(X_i) dx - F(\bar{X}_i) \right]$$

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The first order conditions give us,

\[
\frac{\partial \Pi_i}{\partial p_i} = 0
\]

\[
0 = \frac{\delta}{2}(F(\delta \beta b - p_i) - F(X_i)) \left[ -\frac{p_i}{t} - \delta b(1 - \hat{\beta}) \frac{f(\hat{X}_i)}{F(\delta \beta b - p_i) - F(X_i)} + \int_0^{\hat{x}} F(X_i)dx - F(\hat{X}_i) \right]
\]

It implies,

\[
p_i = t \left[ -\delta b(1 - \hat{\beta}) \frac{f(\hat{X}_i)}{F(\delta \beta b - p_i) - F(X_i)} + \int_0^{\hat{x}} F(X_i)dx - F(\hat{X}_i) \right]
\]
A4-Existence of the contract

To prove the existence of this contract, we use assumptions on the density probability of the consumption costs. We assume, \( f \) is positive, decreasing and defined on \([0, +\infty[\). This function has a maximum in \( c = 0 \). Also, we assume that, 

*If \( x < y \) then it exists \( M \) such that \( f(x) < Mf(y) \) with \( M > 1 \).*

As a reminder, the derivate of the profit with respect to the marginal price is,

\[
\frac{\partial \Pi_i}{\partial p_i} = \frac{\delta}{2} (F(\delta \beta b - p_i) - F(X_i)) \left[ -\frac{p_i}{t} - \delta b(1 - \hat{\beta}) \frac{f(\hat{X}_i)}{F(\delta \beta b - p_i) - F(X_i)} + \int_0^{\hat{X}_i} F(x)dx - F(\hat{X}_i) \right]
\]

Thus, if

\[
p_i > t \left[ -\delta b(1 - \hat{\beta}) \frac{f(\hat{X}_i)}{F(\delta \beta b - p_i) - F(X_i)} + \int_0^{\hat{X}_i} F(x)dx - F(\hat{X}_i) \right]
\] (21)

Then \( \frac{\partial \Pi_i}{\partial p_i} < 0 \) implying the profit is decreasing with respect to the marginal price so the firm may decrease her marginal price to arise her profit.

Moreover if,

\[
p_i < t \left[ -\delta b(1 - \hat{\beta}) \frac{f(\hat{X}_i)}{F(\delta \beta b - p_i) - F(X_i)} + \int_0^{\hat{X}_i} F(x)dx - F(\hat{X}_i) \right]
\] (22)

then \( \frac{\partial \Pi_i}{\partial p_i} > 0 \), implying the profit is increasing with respect to the marginal price so the firm may increase her marginal price to arise her profit.

However to verify the existence of this contract and that the optimal marginal price is a maximum, it is necessary that we find a real boundaries in which the optimal marginal price will be included. Thereby, we have to find an inferior finite boundary \( \underline{M} \) and a superior finite boundary \( \overline{M} \) to the right part of inequality (21) and (22). The price \( p_i \) which maximize the profit will be included in the interval \([\underline{M}, \overline{M}]\) by continuity of the function. Indeed if the profit is increasing (respectively decreasing) with respect to the marginal price when this price is higher (respectively lower) than \( \overline{M} \) (resp. \( \underline{M} \)) then the firm has incentives to decrease (resp. increase) her marginal price to arise her profit. Thereby, there exists equilibrium price included in this interval by continuity. We can construct these boundaries.

**Construction of the superior finite boundary \( \overline{M} \)**

Find a superior finite boundary tho the right part of inequalities (21) and (22) is equivalent to find a superior finite boundary to,

\[
\int_0^{\hat{X}_i} F(x)dx - F(\hat{X}_i)
\]

\[
\frac{F(\delta \beta b - p_i) - F(X_i)}{F(\delta \beta b - p_i) - F(X_i)}
\]

Indeed, we know that \( F(c) \) is a positive increasing function, that \( f(c) \) is a positive function and \( \hat{\beta} \leq 1 \). It follows that the first term of the right part of the inequality is negative.
We denote \( A = F(\delta \beta b - p_i) - F(X_i) \).

We have,

\[
\int_0^{\hat{x}} F(X_i)dx = \frac{P(\delta \beta b - p_i) - P(X_i)}{t}
\]

With \( P \) the primitive of \( F \)

Since \( F(c) \) is strictly increasing and positive then,

\[
P(\delta \beta b - p_i) - P(X_i) < (\delta \beta b - p_i - (\delta \beta b - p_i - \frac{t}{2}))F(\delta \beta b - p_i)
\]

\[
< \frac{t}{2} F(\delta \beta b - p_i)
\]

It follows,

\[
\int_0^{\hat{x}} F(X_i)dx < \frac{F(\delta \beta b - p_i)}{2} < F(\delta \beta b - p_i)
\]

So, we have,

\[
\frac{\int_0^{\hat{x}} F(X_i)dx - F(\hat{X}_i)}{A} < \frac{F(\delta \beta b - p_i) - F(\hat{X}_i)}{A}
\]

As \( A = F(\delta \beta b - p_i) - F(X_i) \) and \( F(X_i) < F(\hat{X}_i) \) then \( F(\delta \beta b - p_i) - F(\hat{X}_i) < A \).

It follows,

\[
\frac{\int_0^{\hat{x}} F(X_i)dx - F(\hat{X}_i)}{A} < 1
\]

Thereby, \( \bar{M} = 1 \)

**Construction of the inferior boundary \( \bar{M} \)**

Find a inferior finite boundary tho the right part of inequalities (21) and (22) is equivalent to find a inferior finite boundary to,

\[
-\delta b(1 - \hat{\beta}) \frac{f(\hat{X}_i)}{A} - \frac{F(\hat{X}_i)}{A}
\]

Since \( F(c) \) is a positive function \( \frac{\int_0^{\hat{x}} F(X_i)dx}{A} \) is positive.

First, we have,

\[
A = F(\delta \beta b - p_i) - F(X_i) > \frac{t}{2} F(\delta \beta b - p_i)
\]

So,

\[
\frac{1}{A} < \frac{2}{tf(\delta \beta b - p_i)}
\]
Or
\[ \frac{1}{A} < \frac{2M}{tf(0)} \]

For the second term of (23), it is easy to see that \(-F(\hat{X}_i) > -1\). It follows for this second term,
\[ -\frac{F(\hat{X}_i)}{A} > -\frac{2M}{tf(0)} \quad (24) \]

For the first term, we have,
\[ \delta b(1 - \hat{\beta}) \frac{f(\hat{X}_i)}{A} < \delta b(1 - \hat{\beta}) \frac{2Mf(\hat{X}_i)}{tf(0)} < \frac{2M\delta b(1 - \hat{\beta})}{t} \]

It follows,
\[ -\delta b(1 - \hat{\beta}) \frac{f(\hat{X}_i)}{A} > -\frac{2M\delta b(1 - \hat{\beta})}{t} \quad (25) \]

Thereby, according to (24) and (25),
\[ M = -2M \frac{1 + \delta b(1 - \hat{\beta})f(0)}{tf(0)} \]
A5-Prices studies according to the consumer type

Sophisticated consumer

Superior finite boundary $M_s$

We have shown that $\frac{\int_0^\infty F(x)dx - F(\hat{X}_i)}{A} < 1$. It results,

$$\frac{\int_0^\infty F(x)dx - F(\hat{X}_i)}{A} - \delta b(1 - \beta) \frac{f(X_i)}{A} < 1 - \delta b(1 - \beta) \frac{f(X_i)}{A}$$

Since $A < \frac{1}{2} f(X_i)$ then $\delta b(1 - \beta) \frac{f(X_i)}{A} > \frac{2\delta (1 - \beta)}{t}$ so,

$$1 - \delta b(1 - \beta) \frac{f(X_i)}{A} < 1 - \frac{2\delta (1 - \beta)}{t}$$

It follows that $M_s$ such that $p^*_s < M_s$ is equal to,

$$M_s = 1 - \frac{2\delta (1 - \beta)}{t}$$

Variation of the consumption price with respect to time inconsistency degree

We have,

$$\frac{\partial}{\partial \beta} \frac{\partial \Pi_i}{\partial p_i} = \frac{\delta}{2} \text{d} \left[ - \frac{p_i}{t} \left( f(\delta \beta b - p_i) - f(X_i) \right) \right] - \delta b(1 - \beta) f'(X_i) + \int_0^\infty f(X_i)dx Big$$

and we have shown that $\frac{\partial p_i}{\partial \beta}$ has the same sign that $\frac{\partial \Pi_i}{\partial p_i}$.

It is easy to see that $f(c)$ decreasing and a positive consumption price implies $\frac{\partial p_i}{\partial \beta} > 0$. However, if this price is negative, it is more difficult to determine the sign of this expression. To go beyond this difficulty, we can assume take the case where the function $f(c)$ is linear and therefore $f'(c) = S$ with $S$ a negative real number.

Thanks to this hypothesis, we can show that $\frac{\partial \Pi_i}{\partial p_i}$ is also positive for a negative price.

$$\int_0^\infty f(X_i)dx = \frac{F(X_0) - F(X_i)}{t} > \frac{1}{2} f(X_0) = \frac{1}{2} (f(0) + (\delta \beta b - p_i)S)$$

With $X_0 = \delta \beta b - p_i$

So,

$$\int_0^\infty f(X_i)dx - \delta b(1 - \beta) f'(X_i) > \frac{f(0)}{2} + \frac{\delta \beta b}{2} - \frac{p_i}{2} S - \delta b(1 - \beta) S$$

$$> \frac{f(0)}{2} - S(\delta b(1 - \frac{3}{2} \beta) + \frac{p_i}{2})$$

Thereby,

$$\frac{f(X_0) - f(X_i)}{t} = \frac{1}{2} S$$
Thus, according to (26) and (27),
\[
\int_0^\tilde{x} f(X_i)dx - \delta b(1-\beta)f'(X_i) - p_i \frac{f(X_0) - f(X_i)}{t} > \frac{f(0)}{2} - S(\delta b(1 - \frac{3}{2} \beta) + \frac{p_i}{2}) - \frac{p_i}{2} S \\
> \frac{f(0)}{2} - S(\delta b(1 - \frac{3}{2} \beta) + p_i)
\]

It results that \( \int_0^\tilde{x} f(X_i)dx - \delta b(1-\beta)f'(X_i) \) will be positive if \( \delta b(1 - \frac{3}{2} \beta) + p_i > 0 \). As,
\[
p_i > M = -2M \frac{1+\delta b(1-\beta)f(0)}{tf(0)}
\]

Then,
\[
\delta b(1 - \frac{3}{2} \beta) + p_i > \delta b(1 - \frac{3}{2} \beta) - 2M \frac{1+\delta b(1-\beta)f(0)}{tf(0)} \\
> \delta b(1 - \frac{2M}{t} - \beta(\frac{3}{2} - \frac{2M}{t})) - \frac{2M}{tf(0)}
\]

Therefore \( \delta b(1 - \frac{3}{2} \beta) + p_i \) will be positive if \( \delta b(1 - \frac{2M}{t} - \beta(\frac{3}{2} - \frac{2M}{t})) - \frac{2M}{tf(0)} \) is positive ie if,
\[
1 - \frac{2M}{t} - \beta(\frac{3}{2} - \frac{2M}{t}) > \frac{2M}{\delta btf(0)} \\
\beta(\frac{3t - 4M}{2t}) < \frac{tf(0) - 2M(f(0) + 1)}{\delta bf(0)}
\]

If we assume reasonably that \( t < \frac{4}{3} M \) then,
\[
\beta > \frac{2(tf(0)\delta b - 2M(f(0)\delta b + 1))}{f(0)(3t + 4M)\delta b}
\]

Yet we have seen that the price is negative if \( \beta \) is lower then \( \frac{\delta b - \frac{3}{2}}{\delta b} \). Consequently, it is
necessary that,

\[ \frac{\delta b - \frac{t}{2}}{\delta b} > \frac{2(tf(0)\delta b - 2M(0))(f(0)\delta b + 1))}{f(0)(3t + 4M)\delta b} \]

\[ -\frac{t}{\delta b} > \frac{2tf(0)\delta b - 4M(f(0)\delta b + 1) - \delta bf(0)(3t + 4M)}{f(0)(3t + 4M)\delta b} \]

\[ \frac{t}{2} < -\frac{\delta bf(0)[2t - 4M - 3t - 4M] - 4M}{f(0)(3t + 4M)} \]

\[ t(\frac{1}{2} - \frac{\delta b}{3t + 4M}) < \frac{4M(2\delta bf(0) + 1)}{f(0)(3t + 4M)} \]

\[ t(\frac{f(0)(3t + 4M)}{2} - \delta bf(0)) < 4M(2\delta bf(0) + 1) \]

\[ f(0)\frac{3}{2}t^2 + t(f(0)(2M - \delta b)) - 4M(2\delta bf(0) + 1) < 0 \]

This last condition is verified is \( t \) is between the two polynomial roots. This roots are,

\[ \begin{align*}
  t_1 &= -\frac{\sqrt{4f(0)^2M^2 + 4\delta bf(0)^2 + 24f(0)M + (\delta b)2f(0)^2 + 2f(0)M - \delta bf(0)}}{3f(0)} \\
  t_2 &= \frac{\sqrt{4f(0)^2M^2 + 4\delta bf(0)^2 + 24f(0)M + (\delta b)2f(0)^2 - 2f(0)M + \delta bf(0)}}{3f(0)}
\end{align*} \]

Since we have as condition for a negative prive \( \frac{t}{2} < \delta b(1 - \beta) \) with \( t > 0 \), we can reasonably think that the condition is verified. It results that we have \( \frac{\partial}{\partial \beta} \frac{\partial \Pi}{\partial p_i} \) positive as \( \frac{\partial p_i}{\partial \beta} \).

An other simplier proof

If \( f(c) \) linear then,

\[ \frac{F(X_0) - F(X_i)}{t} = \frac{1}{2}(\frac{f(X_i) + f(X_0)}{2}) \]

Since,

\[ \frac{f(X_i)}{2} = \frac{\delta \beta b - p_i - \frac{t}{2}}{2} S \]

And,

\[ \frac{f(X_0)}{2} = \frac{\delta \beta b - p_i}{2} S \]

Thereby,

\[ \frac{F(X_0) - F(X_i)}{t} = (\frac{\delta \beta b - p_i}{2} - \frac{t}{8})S \]

Moreover,

\[ -\frac{p_i}{t} (f(X_0) - f(X_i)) = -\frac{p_i}{2} S \]
Il follows that,

\[-\frac{p_i}{t} (f(X_0) - f(X_i)) - \delta b(1 - \beta)S + \frac{F(X_0) - F(X_i)}{t} = S(-\delta b(1 + \frac{2\beta}{2}) - \frac{t}{8} - p_i)\]

This is positive for \((-\delta b(1 - \frac{3\beta}{2}) - \frac{t}{8} - p_i) < 0\) thus for all \(p_i > -\delta b(1 - \frac{3\beta}{2}) - \frac{t}{8}\). Since this last term is positive if \(\frac{t}{2} > -4\delta b(1 - \frac{3}{2})\) and we assume that \(t > 0\), this condition will always be true for \(\beta < \frac{2}{3}\). As we saw that the price is negative for a low \(\beta\), we can generalize saying that for a linear \(f\), \(\frac{\partial p_i}{\partial \beta}\) will be positive even for negative price.

**Naive consumer**

**Variation of the consumption price with respect to time inconsistency degree**

We have,

\[
\frac{\partial}{\partial \beta} \frac{\partial \Pi}{\partial p_i} = \frac{\delta}{2} \delta b \left[ -\frac{p_i}{t} (f(\delta \beta b - p_i) - f(X_i)) + \int_{0}^{\hat{i}} f(X_i) dx \right]
\]

We already have,

\[-\frac{p_i}{t} (f(\delta \beta b - p_i) - f(X_i)) + \int_{0}^{\hat{i}} f(X_i) dx = P(\frac{\delta \beta b}{2} - \frac{t}{8} - p_i)\]

Thereby, \(\frac{\partial p_i}{\partial \beta}\) would be positive if \(\frac{\delta \beta b}{2} - \frac{t}{8} - p_i < 0\) thus if \(p_i > \frac{\delta \beta b}{2} - \frac{t}{8}\) and we consider that \(\frac{\delta \beta b}{2} - \frac{t}{8}\) can not be negative as \(\delta \beta b - \frac{t}{2}\) can not be negative, however, the agent has no incentive to consume.