The banking crisis with interbank market freeze

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Abstract: This paper studies banking crises characterized by interbank market freeze, fire sale and contagion in a model with collateralized interbank loans. We examine the role of the interbank market in spreading and amplifying crises by distinguishing three sources of liquidity risks, i.e., panic-induced run, gambling behavior and foreign sovereign debt crisis. Our results underline that insufficient bank capital and/or liquidity reserves could lead to malfunctioning of the interbank market. However, implementing more restrictive regulations to reinforce banks’ resilience to shocks could hamper the role of banks as financial intermediary. Therefore, the government’s crisis management is crucial, particularly for the member-states of a monetary union, in ensuring the stability of the banking system and the well-functioning of the interbank market and it is efficient as long as the scope of bailout is credible in the sense of not compromising the soundness of its budgetary positions.

Key words: Banking crisis, interbank market, capital ratio, multiple equilibria, bank run, gambling asset, asymmetric information, and sovereign debt crisis.

JEL Classification: E43, G01, G11, G12, G18, G02, G28.

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1. Introduction

The recent global financial crisis has clearly revealed the fragility of the banking system strongly relying on the interbank market to alleviate liquidity shortfall and reduce funding liquidity risk. In particular, following the collapse of Lehman Brothers and Bear Sterns in 2008 and the advent of the Greek sovereign debt crisis in late 2009, several episodes of severe turbulences have been observed in interbank markets around the world, forcing central banks and fiscal authorities to carry out massive liquidity injections in attempting to restore the normal functioning of the banking system. In the Euro zone, national governments have the responsibility of supervising domestic banks and bailing out the latter while they are themselves subject to additional financial pressures given that they have abandoned monetary sovereignty to the European central bank.

The malfunctioning of the interbank market amplifies the banking system’s massive illiquidity resulting from the lack of synchronicity between liquidity needs and liquidity availability, characterized by interbank market freeze, fire sale, contagion and eventually insolvencies and bailouts (Tirole, 2009). Banks’ engagement in maturity transformation has been impressively increased in the pre-crisis period. This is made possible by the existence of a highly developed interbank market, through which banks having a liquidity surplus lend to those enduring a liquidity deficit. The interbank market, by allowing banks to bridge temporary liquidity mismatches, permits them to keep less liquidity reserves. Before the eruption of global financial crisis, banks have extensively expanded their balance sheets, leading to a highly leveraged and less liquid banking system whose reliance on wholesale funding sources such as interbank loans has been excessively (Adrian and Shin, 2008).

Given that the safe types of assets are usually employed in the transactions, a complete interbank market is quite competitive and efficient in bridging short-term liquidity gaps and achieving the optimal resource allocation of the banking system (Allen and Gale, 2000). However, financially fragile banks and the interbank market are extremely risk-sensitive to sudden shocks. The lending banks may rationally suspend loans to borrowing ones in expecting that the latter will fail so as to avoid the counterparty risk. An aggregate liquidity shock can immediately give rise to the
freezing of the funding market, either for the reason that lending banks suffer a liquidity shortage themselves or they have concern about the solvency of the borrower.

The recent twin banking and sovereign debt crises in euro-zone countries have aroused a broad attention among economists and policymakers (Lane, 2012; Moro, 2013). In the Euro zone, the elimination of national currencies implied an increased importance of national fiscal policies as a tool for countercyclical macroeconomic policy (Wyplosz 1997; Gali and Monacelli, 2008). In particular, since banking regulation remained a national responsibility, individual governments had to bear the risks of a banking crisis and the direct and indirect costs associated with it. The fiscal bailouts undertaken by governments with unsustainable public debt cannot stop an unfolding banking crisis and will contrarily aggravate the panic among market participants. Consequently, the normal conditions of the interbank market will not be restored unless the government has a large enough fiscal cushion that provides sufficient scope for policy manipulation during crisis times (Attinasi et al., 2010).

The main purpose of this paper is to identify the bright side and the dark side of the interbank lending market in ensuring the stability of the banking sector, to examine the capacity of a government in rescuing troubled banks, and to provide some suggestions for the future reform of the banking regulation. We are mostly interested in the functioning of the EMU interbank markets during the 2010-2012 Euro zone crises. Therefore, to the difference of most works on the interbank market that focus on the effectiveness of the monetary policy during a banking crisis, we highlight the role played by the national government and the linkage between banks’ balance sheet and the government’s budget.

We develop a theoretical framework to study the impact of the interbank market through examining the balance sheet of banks respectively during normal and crisis times. We show that in normal times the interbank market allows banks to cope with idiosyncratic liquidity shocks through the redistribution of the fixed amount of reserves holding within the banking system, therefore ensures the continuation of investments, and reduces the losses of banks enduring liquidity shocks, and thus improves the social welfare. We investigate three sources of liquidity risks with different implications for the role of the interbank market in spreading and amplifying the crisis in the banking system. First, a crisis could be triggered by the self-fulfilling bank-run where depositors’ attempt of premature withdrawal leads to a bank failure (Diamond and Dybvig, 1983). Second, a liquidity shock results from the revelation of
the asymmetric information about the balance sheet of the non-performing banks (Chari and Jagannathan, 1988; Acharya et al., 2012). We characterize the asymmetric information by introducing a gambling asset which delivers an extra profit to investors if the gambling is successful (Hellmann et al., 2000; Agliardi et al., 2009; and Hasman et al., 2013). Third, banks’ ex-ante safe assets could see their values suddenly depreciated. In practice, triple-A government bonds are hold by banks as liquidity reserves and they constitute the interbank market’s liquidity pool, rendering the banking system vulnerable to a sovereign debt crisis (Bolton and Jeanne, 2011; Reinhart and Rogoff, 2011).

One major finding of this paper is that the malfunctioning of the interbank market might not be avoided by banks’ optimal feasible risk reallocation, and it could be caused by unexpected shocks stemmed from within the banking system. This malfunctioning could be associated with banks’ ex-post inappropriate capital and/or reserve ratios in crisis times, although the latter are ex-ante in accordance with the interbank market participation constraint. Contrariwise, the interbank market, which facilitates the liquidity transfer among banks to deal with idiosyncratic shocks in normal times, may impair the stability of the banking system in crisis times. More precisely, a freezing of the interbank market will aggravate the financial position of borrowing banks.

Our results underline the importance of enhancing banks’ capital position so as to reinforce banks’ resilience to shocks. However, high opportunity cost of capital incites banks to keep their capital as low as possible. Our analysis supports thereby the regulatory reform targeted at imposing a higher capital ratio, as prescribed by the Basel III. Nevertheless, there exists a ceiling of the capital ratio beyond which banks’ capacity in raising deposits would be hampered. This is explained by the fact that the risk averse depositors have a threshold rate of return, a high capital ratio plummets the return for deposits and leads them to withdraw their deposits from banks, forcing the latter to become pure ‘equity banks’ and eliminating banks as intermediary between depositors and borrowers and thus problems associated with failures of such banks. The risk of banking crisis cannot be entirely ruled out by any forms of ex-ante regulation, implying that the government crisis responses are essential for avoiding the turmoil of the banking system during a crisis.

Another finding is that the interbank market can be a channel of contagion in the sense that may respond by an immediate lending suspension to shocks originated from
outside the banking system, which affect the aggregate liquidity condition. The role of the government’s crisis management becomes even more important in such a situation. Notwithstanding, the member-state governments of a monetary union have limited capabilities in banking bailout. Our model shows that the scope of the government’s involvement in the crisis management depends largely on its budgetary positions. The government’s bailout is credible if it does not lead to a risk of sovereign default. In other words, the inability of government to save in good times to build a war chest for bad times has often resulted in gut-wrenching twin financial and sovereign-debt crises. Consequently, the reform of the banking regulation should be accompanied by the fiscal reform.

The remainder of paper is organized as follows. The next section reviews the literature. Section 3 describes the basic model with an interbank market. Section 4 examines the functioning of the interbank market in the presence of liquidity risks due to a bank run, the asymmetric information, and the depreciation of ex ante risk-free assets. Section 5 studies the crisis management conducted by the fiscal authority and the relation between the fiscal bailout and the government’s budget position. The last section concludes.

2. Related literature

Rochet and Tirole (1996), Freixas and Parigi (1998) and Aghion et al. (2000) have underlined that the banking sector and the interbank market, as the most important components of the financial system, not only can contribute to spread shocks originating from outside the local financial system, but can also be original culprits of wide-spread crises. Diamond and Rajan (2005) show that due to the feedback interactions through the interbank market, the liquidity mismatch can induce the insolvency while the latter will aggravate the liquidity shortage and leads to a bank run. Heider et al. (2008) highlight the effects of banks’ asset risk, and private information about it, on functioning of a vital interbank market by showing that, depending on the level and distribution of risk in the banking sector, the liquidity trading may be smooth, impaired, or dry up completely, and massive liquidity injections by the central bank might not be sufficient to restore interbank activity. Tirole (2011) characterizes the recent crisis by massive illiquidity where transactions have been suspended in financial markets, leading financial institutions to struggle for
liquidity through restructuring prematurely assets at fire sale prices and panic investors to run on these financial institutions unless authorities implement substantial and credible rescuing package. In the spirit of these papers, we model the interactions between banks having a liquidity surplus and those enduring a liquidity shortage in the interbank market. The bank failure can result from the freezing of the interbank loans; likewise, the malfunction of the interbank lending can be the outcome of banks’ vulnerable balance sheets.

Our paper is most closely related to the seminal work of Allen et al. (2009), who show that in a framework where banks trade long-term assets in the interbank market to protect themselves against the liquidity mismatch, the interbank lending allows dealing with idiosyncratic liquidity shocks while reducing liquidity reserves of banks.

Ours setting is distinct from Allen et al., (2009) in three main aspects. First, rather than focusing uniquely on the uncertainty about deposit withdrawal, we consider also a shock arising from banks’ ongoing projects. More precisely, banks are submitted not only to the risk of non-performing assets that require refinancing at the intermediate date so as to avoid restructuring or the fire sale, but also to the risk of a self-fulfilling bank run. We show in our framework that these two risks could interact with each other. In this respect, our model is similar to Tirole (2011) in the sense that contagions arise from both assets and liabilities sides of the balance sheet, but different from Chen (1999), Allen and Gale (2004) and Diamond and Rajan (2011), who focus on shocks stemming from the liabilities side of the balance sheet.

Second, giving a role for the bank capital, our model is in line with Rochet and Tirole (1996), Aghion et al. (2000), and Allen and Gale (2000). To the difference of these earlier works, banks in our setting may have foreign shareholders and may choose a capital level in accordance with their optimal resources allocation. This reflects the situation in peripheral euro-zone countries, where commercial banks generally absorb deposits from domestic residents while raising capital from both domestic and foreign investors.

Third, we are interested in the government crisis management so as to illustrate the euro-zone crisis where assistances from the ECB are usually considered both deferred and insufficient, and the restoration of normal financial conditions largely depends on crisis managements conducted by national governments. In this respect, our paper is close to the model of sovereign debt crisis built by Bolton and Jeanne (2011) and Acharya et al. (2011). In contrast, most papers on financial contagion underline crisis
responses of the central bank (Freixas et al., 2000; Nier et al., 2007; Fahir and Tirole 2012).

3. The model

Our basic framework is built on Allen and Gale (2009) who extend the classic banking crisis model of Diamond and Dybvig (1983) by including a complete interbank market where banks purchase and sell long-term assets to hedge against liquidity shocks. The main difference with Allen and Gale (2009) is that we introduce the role of bank capital in ensuring the stability of the banking system and the well-functioning of the interbank market through which collateralized loans are made between banks.

3.1. Environment

The small open economy is populated by a large number of *ex-ante* identical residents of mass one. The economic activities are carried out in two periods marked by three dates denoted respectively by $t_0$ as the initial date, $t_1$ the intermediate date (or short-term) and $t_2$ the final date (or long-term). A single good is produced in this economy and is used for consumption and investment. Each resident has an endowment of $e$ units of the good at the planning date $t_0$. Domestic residents consume either at the intermediate date $t_1$ or at the final date $t_2$ according to their type (i.e., impatient or patient consumers). The information about the type of residents is only revealed at $t_1$. At date $t_0$, they only know the probability of being impatient ($\lambda$) and being patient ($1-\lambda$). Denoting by $x$ and $y$ the amount of good consumed at $t_1$ and at $t_2$ respectively. The expected utility of domestic residents at $t_0$ is:

$$\lambda u(x) + (1-\lambda)u(y),$$

where $u(.)$ is a CRRA instantaneous utility function defined by $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with $0 < \sigma < 1$. Since domestic residents do not consume at $t_0$, they will either invest by themselves their endowments or entrust the latter to banks depending on the rate of return offered respectively by these two options.

There are two types of investment vehicles in this economy: safe assets constituted of ‘risk-free’ domestic and foreign government bonds identical in terms of risk and
return, and a two-period long-term production technology. Government bonds are accessible to all agents, while the investment using the long-term technology requires a special human capital possessed exclusively by domestic banks to accompany the producer during all production processes to collect all the output.

The domestic government starts with a debt from precedent periods and issues bonds in domestic and international financial markets at date \( t_0 \). These bonds are quasi-liquidity and can be sold whenever necessary in the secondary market. Matured bonds will be redeemed at date \( t_2 \). For one unit of resource invested in this liquid asset at date \( t_0 \), investors can receive \( 1 + r_{01} \) if the bond is sold in the secondary market at date \( t_1 \), and \( 1 + r_{02} \) if bonds are held until date \( t_2 \), where \( r_{ij} \) denotes the interest rate on government bonds for the period from date \( i \) to date \( j \). For simplicity, we assume that in normal times both short-term and long-term interest rates attributed to these bonds, equal to corresponding international interest rates, are respectively \( r_{01} = 0 \) and \( r_{02} = r^* \). The domestic government raises a tax on banks’ investment income at a tax rate \( \tau \) and redeems matured bonds with fiscal revenues at the final date.

The long-term production technology is possessed by entrepreneurs with no endowment. To begin a project, each entrepreneur needs borrow one unit of good from one bank. There are a large number of entrepreneurs in the economy and only a fraction of projects can be funded by the resource available to banks. The insufficiency of long-term funding and the competition of entrepreneurs for financing their projects imply that banks can get all outputs of the projects that they have financed at date \( t_0 \). Long-term projects are risky in the sense that with a probability \( \pi \), a long-term asset turns out to be non-performing at the intermediate date \( t_1 \) and needs a refinancing equal to \( \phi \ll 1 \) units of good (fresh liquidity) so as to continue the production until its maturity at \( t_2 \). A matured long-term asset yields \((1 - \tau)R\) units of good at \( t_2 \) after tax regardless its position at \( t_1 \). However, if a bank fails to raise funds to fill the small liquidity gap \( \phi \) induced by the reinvestment at \( t_1 \), it yields nothing at \( t_2 \) and hence a big liquidity shock. Banks with an urgent need of liquidity cannot sell their long-terms assets at a normal market price defined by assets’ present value that a bank with abundant liquidity could wait for, and have to liquidate (or restructure) immature projects at lower price. For one unit of long-term asset liquidated through fire sale or restructured at the intermediary date, banks can obtain less than one unit of
good. More precisely, the fire sale of a performing asset at \( t_1 \) delivers \( (1 - \tau) r_i^p \) units of good after tax, while that of a non-performing asset yields \( (1 - \tau) r_i^{np} \) after tax, where

\[ R > 1 > r_i^p > r_i^{np}. \]  

(rl)

The condition (rl) indicates that the value from liquidating a performing asset is higher than that from a non-performing asset and the liquidation is costly for both types of banks. The relationship \( r_i^p > r_i^{np} \) is explained by the fact that the continuation of a non-performing project requires refunding

In spite of its riskiness, the long-term asset is much more appealing than the government bonds owing to its higher return. Such that even for a non-performing asset that requires a refunding of \( \phi \) units of good at \( t_1 \) besides the initial investment of one unit of good at \( t_0 \), its return is higher than the interest rate paid by government bonds, i.e.,

\[ (1 - \tau) R - (1 + \phi) > r^*, \]  

(return LT)

meaning that government bonds are dominated by long-term assets in terms of return.

To maximize their utility, domestic residents will entrust all endowments to banks that offer deposit contracts promising a fixed payment on the revelation of their type. In effect, the consumption of domestic residents is equal to the gross return of government bonds if they do not deposit their endowments in banks. Deposit contracts would be more attractive than the direct investment by depositors for two reasons: first, long-term assets with higher return are accessible only to banks; second, each resident bears the uncertainty regarding his type while for banks there is no aggregate uncertainty concerning the types of depositors according to the law of large number.

Commercial banks are ex-ante identical at the planning date \( t_0 \). They maximize the welfare of domestic residents by optimally allocating the resources in safe government bonds and risky long-term assets. At the intermediate date \( t_1 \), based on the quality of long-term assets, banks are divided into two types, i.e., ‘good’ banks and ‘bad’ banks. As long-term assets have a probability \( \pi \) of being non-performing at \( t_1 \), a proportion \( \pi \) of banks turn out to be ‘bad’, their assets need a reinvestment so as to deliver the normal return at \( t_2 \) and a proportion \( 1 - \pi \) of banks are ‘good’ in the sense that the return from their long-term investments is ensured at \( t_2 \) without requiring any refinancing at the intermediate date.
Assume that the soundness of banks is a public information and can be obtained by all agents without any cost at a time point just before \( t_1 \). Observing this information, ‘good’ banks make their decision about whether or not extending loans through the interbank market depending on their own liquidity position and on their expectation about the solvency of ‘bad’ banks.

Following Allen and Gale (2007) and Allen and Carletti (2006, 2008), we consider another type of investors with risk neutral preferences called ordinary shareholders who have an initial endowment \( a \) at \( t_0 \) and do not receive any endowment in future dates. They either consume or buy common shares of banks at the planning date. Being bank’s shareholders at \( t_0 \), they can claim dividends after the payments to banks’ creditors.

Denote by \( d_c \) dividends paid to shareholders at date \( t \). The utility function of shareholders is given by

\[
u(d_0, d_1, d_2) = R(1 - \tau)d_0 + d_1 + d_2.
\]

According to this linear utility function, shareholders can obtain a utility of \( R(1 - \tau)a \) from the immediate consumption of their endowment at \( t_0 \), and they are indifferent between the consumption at \( t_1 \) and \( t_2 \). Therefore, they have to be compensated by a gross return at least equal to \( R(1 - \tau) \) for each unit of consumption they renounced at the initial date \( t_0 \). Let \( K (\leq a) \) be the bank capital, i.e., the investment of shareholders in banks, then \( d_0 = a - K \) is the consumption of shareholders at \( t_0 \). The utility of an investor, provided that he buys common share, is then \((a - K)R(1 - \tau) + d_1 + d_2\).

For the investor as a shareholder, the utility from future dividends should not be less than that from the immediate consumption of all his endowment, or \((a - K)R(1 - \tau) + d_1 + d_2 \geq aR(1 - \tau)\). Thus, the incentive constraint for holding bank capital can be written as

\[
d_1 + d_2 \geq (1 - \tau)RK.
\]

Given this incentive constraint, banks should hold enough matured long-term assets. Consequently, the dividend can be distributed only at \( t_2 \), and the above incentive constraint for shareholders can be rewritten as follows

\[
d_2 \geq (1 - \tau)RK. \quad \text{(dvd)}
\]

Even that domestic banks can sell common shares to both domestic and foreign investors, we assume for simplicity that, at the aggregate level, domestic investors’
endowments are always large enough to meet banks’ capital needs, i.e., $a > K$. When the dividend for shareholders satisfies the condition (dvd), banks can always raise capital through issuing stocks.

The cost of capital is apparently higher than the expected return from the risky long-term asset. On the one hand, the bank capital harms the interest of depositors. The latter receive a fixed and non-negotiable payment conditional on the dates of deposit withdrawal, while shareholders’ dividends are not insured by any mandatory contract. The remuneration for shareholders is state contingent, depending on the financial situation of banks. On the other hand, when a bank is impacted by a negative shock, the bank capital works as a buffer to avoid or at least reduce the scale of the ‘fire sale’ of immature projects and shields depositors from losses. Therefore, the high compensation of shareholders in ‘normal circumstance’ is reasonable as long as it does not impair banks’ ability in absorbing deposits.

In a deposit market characterized by perfect competition, banks compete with each other in providing the best deposit contract they can so as to absorb as more as possible deposits from domestic residents. The rate of return to shareholders higher than that from the investment implies a subsidy from depositors to shareholders. Given that higher capital level results in lower remuneration to depositors, banks competing with each other for deposits tend to keep a capital stock as low as possible.

### 3.2. Interbank market

There exists a complete interbank market where banks with a liquidity surplus extend collateralized loans to banks with a liquidity shortage. Since payments to depositors are non-negotiable, the projects financed by deposits cannot be used as collateral. Consequently, the quantity of pledgeable assets is given by the amount of projects financed by bank capital and is equal to $K$. Provided that a refinanced project yields $(1 - \tau)R$ at $t_2$ and the international interest rate from $t_1$ to $t_2$ equals to $1 + r^*$, in equilibrium, the size of an interbank loan per unit of collateral is given by $\frac{(1-r)R}{1+r^*}$. Let $L$ be the amount of the interbank borrowing requested by a ‘bad’ bank, at the

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1The average expected net return from a matured long-term project is $(1 - \tau)R - [(1 - \pi) + \pi(1 + \phi)] = (1 - \tau)R - (1 + \pi\phi)$. 

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intermediate date $t_1$. Then the size of interbank loans limited by the value of bank capital used as collateral should be:

$$L \leq \frac{(1-\tau)R}{1+r^*} K. \quad (0)$$

The investment in long-term assets is risky. Yet this risk can be shared and thus reduced through a complete interbank market in ‘normal times’, if the size of interbank lending can fill the liquidity gap caused by non-performing production projects.

By using immature projects as collateral for interbank loans, the borrowing bank abandons the right of restructuring them. If the borrowing bank remains solvent, it has to deliver the full return from a collateral $(1-\tau)R$ at the final date $t_2$ to the lending bank to pay the loan made at $t_1$. In the event where the borrowing bank, after receiving the loan, goes into bankruptcy, the lending bank takes over the collateral and collects $(1-\tau)(R-\delta)$ units of good per collateral at $t_2$, with $R > \delta > 0$, where $\delta$ represents the effort cost of the lending bank in supervising a long-term project that it starts monitoring from the intermediate date.\(^2\) We assume that the payoff from taking over the collateral is low and hence unprofitable for lending banks. Thus, a bank with liquidity surplus will not make the interbank loan if it expects the borrowing bank might go into bankruptcy; whereas, if the loan has been already granted, the lending bank does not fire sale the collateral either since

$$(1-\tau)\eta_{i}^{mp} < (1-\tau)(R-\delta) < 1 + r_{02}. \quad (\text{delta})$$

The condition (delta) indicates that the return from the seized collateral $(1-\tau)(R-\delta)$ is lower than that from the government bonds $(1 + r_{02})$, while higher than the fire-sale price $(1-\tau)\eta_{i}^{mp}$.

During ‘normal times’, there is no other risk besides the idiosyncratic shocks concerning the quality of banks’ investments. Banks hold government bonds as liquidity reserves to pay early withdrawals of impatient depositors and the expected refinancing of non-performing long-term projects. ‘Good’ banks will not doubt about the solvency of ‘bad’ banks if the optimal resource allocation is implemented. At the planning date $t_0$, bankers know that, with probability $\pi$, they become a ‘bad’ one and

\(^2\) An alternative assumption is that a bank financing a project from the beginning can obtain $(1-\tau)R$ units of goods at the final date $t_2$, since it has a relatively complete information concerning the production process and the producer, while a lending bank starts monitoring the collateralized asset at the intermediate date has less information about the investment, thus has a limited capacity in collecting the return from it. Thereby, the maximum amount a lending bank can get is $(1-\tau)(R-\delta)$.  

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will need reinvest $\phi$ units of good at $t_1$ for each troubled long-term project. As banks are ex ante identical, each bank has the same probability ($\pi$) of being a ‘bad’ one. Let $A$ denote the investment in long-term assets. Without a complete interbank market, each bank should keep an amount of liquidity reserves for expected refinancing of non-preforming long-term projects (hereafter called refinancing reserves) up to $\phi A$ units of good at $t_1$ so as to ensure the expected reinvestment, given that the fire sale of long-term assets at $t_1$ is too costly.

However, if a bank with liquidity deficit can obtain an amount of interbank collateralized loan up to

$$L \equiv (1 - \pi)\phi A,$$  \hspace{1cm} (I)

it needs a refinancing reserve of only $\pi \phi A$ to achieve the expected reinvestment.\(^3\) The composite coefficient $\pi \phi$ is akin to a minimal refinancing reserve ratio that is required for a bank to be admitted to participate in the interbank market. Apparently, this refinancing reserve ratio is much lower than what is required in a banking system without interbank market. As each bank hoards a refinancing reserve $\pi \phi A$ for the expected reinvestment and ‘good’ banks account for a proportion $1 - \pi$ of all banks, the total liquidity surplus (or supply) in the interbank market at $t_1$ is $(1 - \pi)\pi \phi A$. Given that ‘bad’ banks stand for a proportion $\pi$ of all banks, the total liquidity shortage (or demand) of ‘bad’ banks at $t_1$ in the interbank market is $\pi (1 - \pi)\phi A$. If the interbank market functions well in reallocating the liquidity from surplus banks to shortage banks, the first-best risk reallocation can be implemented, allowing hence a higher investment in more lucrative long-term projects.

Combining constraints (0) and (I) leads to the interbank-market participation constraint:

$$(1 - \pi)\phi A \leq \frac{(1 - \tau)R}{1 + \tau^*} K, \hspace{1cm} \text{(ibm)}$$

The constraint (ibm) implies that, even without capital regulation, the interbank market could provide an efficient incentive to banks to keep their capital at a certain level. It results from (ibm) a minimum capital ratio $\kappa$ for participating banks, which is implicitly required by the interbank market:

$$\kappa^i \equiv \frac{K}{A} = \frac{(1 + \tau^*) (1 - \pi) \phi}{(1 - \tau) R}. \hspace{1cm} \text{(kal ratio)}$$

\(^3\)When the risk of long-term assets is hedged through the competitive interbank market, the optimal allocation (i.e., the investment scale in long-term projects and the payments to depositors) at $t_0$ is identical to the first-best allocation implemented by a social planner.
The minimal capital ratio $\kappa^i$ increases with the unit cost of refinancing a troubled project $\phi$. When this capital ratio is not satisfied, ‘bad’ banks will experience the illiquidity caused by the reinvestment scale, excessively large relative to the available collateral.

The capital ratio defined by the condition (kal ratio) is pro-cyclical in the sense that the interbank market demands a higher capital ratio $\kappa$ when the risk of assets $\pi$ is lower and vice versa. When the risk is low, the supply of liquidity reserve and the refinancing reserve ratio $\pi \phi$ required for participation in the interbank market are small, and banks need to keep a higher capital ratio to be able to invest when they are hit by adverse idiosyncratic shocks and vice versa.

When the interbank market functions well and banks hit by adverse idiosyncratic shocks can borrow the amount defined by (1), the expected return to bank capital in normal times can be expressed as follows:

$$E[d_2] = (1 - \tau) R \left[ K - \frac{(1 + \tau^*) (1 - \pi) \phi A}{(1 - \tau) R} \right] + (1 - \pi) \left( K + \frac{(1 + \tau^*) \pi \phi A}{(1 - \tau) R} \right) = (1 - \tau) RK. \quad (Edvd)$$

According to equation (Edvd), for shareholders of ‘bad’ banks accounting for a proportion $\pi$ of all banks, the total amount of dividend at $t_2$ is $(1 - \tau) R \left[ K - \frac{(1 + \tau^*) (1 - \pi) \phi A}{(1 - \tau) R} \right]$, where the term $\frac{(1 + \tau^*) (1 - \pi) \phi A}{(1 - \tau) R}$ stands for the return from long-term assets pledged to the repayment of interbank loans. For shareholders of ‘good’ banks representing a proportion $1 - \pi$ of all banks, the amount of the dividend is equal to $(1 - \tau) R \left( K + \frac{(1 + \tau^*) \pi \phi A}{(1 - \tau) R} \right)$, where $\frac{(1 + \tau^*) \pi \phi A}{(1 - \tau) R}$ corresponds to interbank loans lent by ‘good’ banks at $t_1$ that bring them $(1 + \tau^*) \pi \phi A$ units of goods at $t_2$ if ‘bad’ banks are solvable. Therefore, the expected amount of dividend for a risk neutral investor is $E[d_2] = (1 - \tau) RK$, implying that the constraint (dvd) is satisfied with equality.

It is straightforward to see that if banks do not hold the required capital amount, implying that the interbank market will be shut down, the expected rate of return for shareholders will be only $(1 - \pi)(1 - \tau)RK$ given that bad banks bear the risk of bankruptcy with the revenue from the fire sale of their assets insufficient to cover the withdrawal of depositors. Therefore, banks will respect voluntarily the capital requirement event it is not regulated by the government.

The sequence of events in normal times is described in Figure 1.
1) Banks issue deposits and common shares.
2) Banks invest in risky long-term projects and safe bonds.
3) Long-term assets mature.
4) Types of banks are revealed.
5) Good banks decide whether or not to grant loans.
6) Banks pay tax and clear all remaining claims.
7) The government redeems matured bonds.

Figure 1. The sequence of events in normal times.

3.3. The maximization problem of banks

To pool resources, banks compete for deposits by offering the highest return they can. The optimization problem of a representative bank at the planning date $t_0$ is

$$\max \{ E[\lambda u(x) + (1 - \lambda)u(y)] \},$$

subject to

$$K \geq \kappa^t A,$$  \hspace{1cm} (Ka)

$$\frac{B}{1 + r_{12}} + A \leq e + K,$$  \hspace{1cm} (2)

$$\lambda x + \pi \phi A \leq \frac{B}{1 + r_{12}} + (1 - \tau)\nu p l,$$  \hspace{1cm} (4)

$$(1 - \lambda)y + (1 - \tau)RK = (1 + r_{12})\left[\frac{B}{1 + r_{12}} + (1 - \tau)\nu p l - \lambda x - \pi \phi A \right] + (1 - \tau)R(A - l).$$  \hspace{1cm} (7)

The constraint (Ka) is the interbank market participation constraint. The constraint (2) is the resource constraint of the bank at the planning date $t_0$, which indicates that the bank’s total investment including $A$ units of long-term assets and $B$ units of government bonds cannot exceed its available resource $e + K$ at $t_0$. Government bonds are issued at discount per value, thus $\frac{B}{1 + r_{12}}$ in constraint (2) represents the value of $B$ units government bonds at $t_0$.

The constraint (4) is the bank’s feasibility or solvability condition at the intermediate date $t_1$. It indicates that the bank’s liquidity available at $t_1$ through selling government bonds $\frac{B}{1 + r_{12}}$ and liquidating $l(\leq A)$ units of long-term assets $(1 - \tau)\nu p l$ must at least meet the withdrawal by impatient depositors $\lambda x$ and its liquidity reserve $\pi \phi A$ imposed by the interbank market. The type of a bank will be revealed at a time point merely before $t_1$ and the interbank market provides only collateralized loans. All banks facing the same uncertainty of holding non-performing projects will set the value from liquidating one unit of long-term assets at $\nu p l$ in the event where they cannot borrow enough liquidity from the interbank market and have to fire sale them. When this constraint is satisfied, there will be no concern over the
solvability of the bank in the normal circumstance. Otherwise, the bank is insolvent, implying that it may need to liquidate entire holding of long-term assets.

Finally, the inequality (7) is bank’s feasibility condition at $t_2$. At the final date, the liquidity available to banks should be sufficient to clear all remaining claims by patient depositors, $(1 - \lambda)y$, and shareholders, $(1 - \tau)RK$. This constraint reflects the fact that, in perfectly competitive deposit market, banks realize no profit after the payment to depositors and shareholders.

There are not shocks affecting the aggregate liquidity at $t_1$. Banks in normal times consider only potential shocks impacting the liquidity needs due to refinancing non-performing assets.

### 3.4. The solutions of the optimization problem

Provided that the return from the restructuring is so low that any level of restructuring will lead to a loss to bank entrepreneurs, it is obvious that banks’ optimal allocation planned at $t_0$ will correspond to the case with no restructuring, or $l = 0$. It is straightforward to see that in optimum all constraints should be satisfied with equality so as to maximize the utility of domestic residents. Further, interest rates on riskless government bonds during normal times, $r_{12}$ and $r_{02}$, are equal to the international interest rate, $r^\ast$. As a result, in equilibrium, all banks will choose a capital stock satisfying the condition (kal ratio) as follows:

$$\tilde{R} = \kappa^iA,$$

where the tilde on the top of $K$ indicates optimal solutions. In effect, a high opportunity cost of capital and an environment of perfect competition incite banks to keep a capital level as low as possible. However, imposing a ratio lower than $\kappa^i$, banks cannot obtain enough interbank loans to answer to the potential risk and will thus suffer the liquidity shortage. On the contrary, keeping a capital ratio higher than $\kappa^i$ makes a bank uncompetitive in the deposits market if other banks keep a capital ratio of $\kappa^i$ and thus are able to offer more appealing deposit contracts.

The optimal allocation between $x$ and $y$ should satisfy the following social transformation curve obtained with the binding constraints of the optimization problem of banks:

$$\Phi \lambda x + (1 - \lambda)y \equiv \Phi e,$$
with \( \Phi \equiv \frac{(1-\kappa^i)(1-\tau)R}{1-\kappa^i+\pi\phi} > 1 \) standing for the total wealth for depositors at \( t_2 \) if all of them withdraw only at the final date. The right-hand side of (10) represents the expected value of total withdrawal when all depositors consume only at \( t_2 \). The composite coefficient \( \Phi \) represents the marginal rate of substitution between consumption at \( t_1 \) and that at \( t_2 \), meaning that if impatient depositors renounce to the consumption of \( \lambda x \) at \( t_1 \), they can obtain a consumption equal to \( \Phi \lambda x \) in \( t_2 \). It can be interpreted also as the expected return from the deposits withdrawn at \( t_2 \).

As described in the subsection 3.2, domestic residents will entrust resources to banks if its rate of return is no smaller than that on government bonds, such that the condition

\[ \Phi \geq 1 + r^* \]  

(constraint k)

should be satisfied. The condition (constraint k) is the incentive constraint. If it is satisfied, depositors will entrust their endowments to banks. The fact that \( \Phi \) decreases with \( \kappa^i \) implies that the welfare of domestic depositors declines with the capital ratio. This justifies the capital level in equilibrium given by the condition (K). Denote by \( \bar{\kappa} \) the maximum value of \( \kappa \) verifying the condition (constraint k). The constraint (constraint k) implies the minimal capital ratio \( \kappa^i \) must be such that

\[ \kappa^i \leq \bar{\kappa} \equiv \frac{(1-\tau)R-(1+\pi\phi)(1+r^*)}{(1-\tau)R-(1+r^*)}. \]

(kbar)

Given the value of \( \kappa^i \) defined by (kal ratio), the condition (kbar) is verified if the following condition is satisfied, i.e.:

\[ \phi \leq \frac{(1-\tau)R}{1+r^*} \left( 1 - \frac{\pi(1+r^*)}{(1-\tau)R-(1-\pi)(1+r^*)} \right). \]

(\phi0)

As both \( \phi \) and \( \pi \) are structural parameters of the economy, we may alternatively describe the condition (phi0) by \( \pi \leq 1 - \frac{(1-\tau)R}{(1+r^*)\phi} \left( \frac{(1-\tau)R}{1+r^*} \right) \). As a result, the interbank market grants the protection only for banks holding investments within a certain scope of riskiness measured by \( \pi\phi \). This implies that the interbank market discourages banks from taking too much risk. In the following, we focus on the case where the condition (phi0) always holds.

Using the social transformation curve defined by the condition (10) and the CRRA utility function of depositors, we obtain easily the following condition
\[
\left( \frac{x}{y} \right)^{-\sigma} = \Phi,
\]

The verification of (constraint k) means that \( \Phi^{\frac{1}{\sigma}} \geq 1 + r^* \), which ensures \( y > x(1 + r^*) \) implying that patient depositors will report honestly their type, and withdraw and consume at date \( t_2 \) in normal times. Thereby, this banking system, with the help of an efficient interbank market, is able to design an efficient deposit contract for each type of residents to attract deposits.

Combining (10) with (11) yields the best plan of revenue distribution between patient and impatient depositors as

\[
\bar{x} = \frac{\theta}{\lambda} e, \quad (x)
\]

\[
\bar{y} = \frac{1-\theta}{1-\lambda} \Phi e, \quad (y)
\]

where \( \bar{x} \) and \( \bar{y} \) stand for optimal payments to patient and impatient depositors respectively and \( \theta = \left( 1 + \frac{1-\lambda}{\lambda} \Phi^{-\sigma} \right)^{-1} \) taking its value within the unit interval is an important coefficient in determining the revenue distribution between impatient and patient depositors. The composite coefficient \( \theta \) decreases with \( \Phi \), meaning that the higher is \( \Phi \), the smaller the proportion of payoff to the impatient residents. It increases with \( \sigma \), indicating that the higher is the degree of risk aversion (measured by \( \sigma \)), the lower the depositors’ willingness to substitute consumption over time. An increase in \( \sigma \) induces a higher payment to impatient residents, implying that patient depositors will cross subsidy impatient ones.

Substituting the solutions of \( \bar{x} \) and \( \bar{y} \) given by (x) and (y) into binding constraints (2)-(7), we obtain the bank’s optimal investment plan \( \bar{A} \) and optimal holding of government bonds \( \bar{B} \) ensuring the best return to depositors as follows:

\[
\bar{A} = \frac{(1-\theta)}{1-\kappa+\pi\phi} e, \quad (A)
\]

\[
\bar{B} = \frac{(1+r^*)[\pi\phi+\theta(1-\kappa)]}{1-\kappa+\pi\phi} e. \quad (B)
\]

According to (A) and (B), the optimal investment in risky long-term assets is negatively related with \( \theta \), and the inverse is true for the optimal holding of government bonds. The scale of investment in both assets increases with the endowment of domestic agents, \( e \).

To put into evidence the role of the interbank market, we consider here a banking system with a perfect competitive deposit market without the interbank market. In this system, given that a minimal capital ratio is not anymore imposed, banks competing
for depositors will set the bank capital to zero so as to maximize the return for deposits. To deal with the expected refinancing of non-performing projects, they will keep a refinancing reserve up to $\phi A$. Accordingly, the social transformation curve is 

$$
\frac{(1-\tau)R}{1+\phi} \lambda x + (1 - \tau)\lambda y = \Phi' e', \quad \text{with} \quad \Phi' \equiv \frac{(1-\tau)R}{1+\phi}.
$$

Provided that $\phi < 1$, we have $\Phi > \Phi'$, i.e., the social wealth and hence the social welfare are higher in an economy with an efficient interbank market than in the one without it. In addition, the marginal rate of substitution is higher in the first than in the second. This implies that an early withdrawal by patient depositors is more costly when there is an interbank market.

The complete interbank market, by allowing banks to efficiently cope with idiosyncratic liquidity shocks during normal times, makes possible for banks to invest in a larger quantity of profitable long-term assets and thereby ensures a higher output level. In the meantime, it does not necessarily encourage an over risk-taking in the banking sector. The interbank market is auto-regulated in the sense that banks must implement a minimal capital ratio. By allowing the management of short-term liquidity gap through the interbank borrowing, it allows banks to reduce their liquidity reserve compared to the case where such a market does not exist.

Nevertheless, a banking system that implements the constrained optimal resources allocation described above is not immune to potential bank runs. Having exposed the bright side of the interbank market, we will investigate its dark side in the following section while putting accent on the role of the interbank market in amplifying and disseminating a banking crisis.

4. The crisis in the interbank market

In this section, we consider the functioning of the interbank market when individual banks are confronted to self-fulfilling run (confidence crisis), asymmetric information or a sudden depreciation of ex-ante safe assets. Our investigation is carried out in a context where the establishment of the interbank market makes the banking system more vulnerable to idiosyncratic or aggregate shocks given that the perspective of interbank lending leads banks to reduce their liquidity reserve.

4.1. Confidence crisis
The interbank market allows ex-ante well-capitalized banks to cope with the liquidity mismatch with lower liquidity reserves, implying that fewer funds are available in the adverse state and the risk of a bank run becomes higher. Our framework allows us to analyze bank runs localized respectively in two types of banks, and to examine their effects on the interbank market and show that there exists a feedback or an auto-reinforcement between the self-fulfilling run and the suspension of interbank lending.

We assume at this stage that there is no concern over the safety of government bonds and characterize the run equilibrium as a rare event, corresponding to an inefficient situation where the first-best allocation considered previously is not anymore feasible due to withdrawals by panicking depositors. In the present framework, a bank run is induced principally by a self-fulfilling loss of confidence.

With the revelation of the quality of long-term projects, banks are divided into ‘good’ and ‘bad’ banks. The confidence crisis could happen for both types of banks, while ‘bad’ banks burdened by non-performing assets are more vulnerable than ‘good’ banks with performing projects that yield a relatively higher liquidation value than non-performing ones. Furthermore, ‘good’ banks are subject only to the risk of premature withdrawal, whereas the solvency of ‘bad’ banks depends on expectations of both depositors and good banks.

Accordingly, the condition of existence of a run on ‘good’ banks is different from that on ‘bad’ ones. Thus, we consider separately these two types of run equilibrium in the following.

*The run equilibrium for ‘good’ banks*

A run on ‘good’ banks could happen simultaneously with a run on ‘bad’ banks, leading to a systemic banking crisis with the interbank market being frozen. The self-fulfilling crisis affecting a ‘good’ bank occurs if the latter cannot ensure the synchronicity between liquidity needs and liquidity inflows. More precisely, a run can occur if the liquidity needs exceed the available liquidity of the bank at the intermediate date $t_1$. Thus, a run on a ‘good’ bank is possible if

$$ z_p^+ \equiv \bar{x} - (1 - \tau)\eta_p^p \bar{A} - \frac{B}{1+r_*} > 0, $$

(Zp+)

The condition (Zp+) illustrates the situation whereas ‘good’ bank fails to honor withdrawals by depositors in the event of a run even after having depleted all liquidity
reserves $\frac{B}{1+r}$ and restructured all long-term asset $(1-\tau)\eta^P_t \tilde{A}$. If the condition (Zp+) holds, the bank will not able to get interbank loan to fill its temporary liquidity gap.

Using conditions (x), (y), (A), (B) and the definition of $\theta$, we express the condition (Zp+) in terms of structural parameters as follows:

$$\eta^P_t < \frac{r_1^+}{(1-r)} \quad \text{(Zp+1)}$$

where $r_1^+ \equiv \left(1 - \kappa^I + \pi \phi \right) \Phi^{(\sigma-1)/\sigma} - \pi \phi$.\footnote{Provided that $0 < \sigma < 1$, the illiquidity measure is always smaller than 1, i.e., $r_1^+ < 1$.} This is a kind of measure of illiquidity with a lower $r_1^+$ meaning less illiquidity. The term $\frac{r_1^+}{(1-r)}$ defines the lowest liquidation price that the performing asset must attain for eliminating a bank run on ‘good’ banks.

The value of $r_1^+$ depends on the structural parameters of the economy. It is straightforward to show that $\frac{\partial r_1^+}{\partial \sigma} > 0$ and $\frac{\partial r_1^+}{\partial \kappa^I} < 0$. This implies that banks are more vulnerable to a run if depositors have a high degree of risk aversion ($\sigma$) and when the capital ratio ($\kappa^I$) is low. For a given $r_1^+$, the ‘good’ bank is solvent if the fire sale price of immature assets $\eta^P_t$ is higher enough to fill the liquidity gap.

The verification of (Zp+1) indicates that ‘good’ banks fail in a run equilibrium. This results then in the depletion of the interbank market’s liquidity pool. For lack of funds required for refinancing, ‘bad’ banks can no more honor the payments to patient depositors at $t_2$, which triggers immediately a run on ‘bad’ banks. As a result, the failure of the interbank market has a “knock on” effect that spreads crisis from one bank to others and thus induces the systemic collapse.

While a systemic confidence crisis is possible in our model, we are more interested in less dramatic situations and in specifying the conditions under which the interbank market is resilient during a self-fulfilling crisis. From now on, we focus on the case where there is no risk of bank run on ‘good’ banks, and the solvability condition of ‘good’ banks during a banking crisis is such that

$$z_p^+ \leq 0, \quad \text{or} \quad \eta^P_t \geq \frac{r_1^+}{(1-r)}.$$

This assumption is justified by the fact that a bank with a well-managed balance sheet (i.e. promisingly profitable assets and thus no need for external funding) is generally rather resilient to liquidity shocks.
The run equilibrium for ‘bad’ banks

For ‘bad’ banks, the condition for the existence of a confidence crisis depends on the interactions between depositors of ‘bad’ banks and lending banks. Thereby, to analyze the solvability of ‘bad’ banks at $t_1$ during crisis times, we examine the conditions under which the interbank market is frozen.

We examine first the case where no interbank loan is granted. Proceeding as before, we obtain that a bank run on ‘bad’ banks is possible if the condition

$$z_{np}^+ \equiv \bar{x} - (1 - \tau) \eta_{np}^i A - \frac{\bar{B}}{1 + r_{12}} > 0$$

$$= z_p^+ + (1 - \tau) \bar{A}(r_{1p}^n - r_{np}^i) > 0 \quad (Znp+)$$

is satisfied. Given that a non-performing asset is less valuable than a performing one ($r_{1p}^n > r_{np}^i$), the condition for the existence of a run equilibrium is less restrictive for a ‘bad’ bank than for a ‘good’ one. It follows directly from (Znp+) that $z_{np}^+ > z_{p}^+$. In terms of structural parameters, (Znp+) can be expressed as:

$$r_{np}^i < \frac{r_{i}^p}{(1 - \tau)}. \quad (Znp+-1)$$

Comparing (Znp+-1) with (Zp+-1), given that $r_{np}^i < r_{i}^p$ and for a given illiquidity measure $r_{i}^+$, ‘good’ banks have a greater chance of surviving during a confidence crisis than ‘bad’ banks.

However, the verification of (Znp+) does not necessarily imply the failure of ‘bad’ banks that may survive if the interbank market is normally functioning. In effect, ‘good’ banks may offer loans to ‘bad’ banks even when (Znp+-1) is satisfied since they know that loans could improve the liquidity condition of ‘bad’ banks and render hence the latter solvent. Interbank loans will be offered if

$$z_{i}^+ \equiv \bar{x} - \frac{\bar{B}}{1 + r_{i}^*} - (1 - \tau) r_{np}^i A (1 - \kappa^i) - \frac{(1 - \tau) R}{1 + r_{i}^*} \kappa^i \bar{A}$$

$$\equiv z_{p}^+ + (1 - \tau) \bar{A}(r_{i}^p - r_{np}^i) - \left[ \frac{(1 - \tau) R}{1 + r_{i}^*} - (1 - \tau) r_{np}^i \right] \kappa^i \bar{A} < 0. \quad (Znpi)$$

The term $\frac{(1 - \tau) R}{1 + r_{i}^*} - (1 - \tau) r_{np}^i \kappa^i \bar{A}$ in condition (Znpi) represents the additional liquidity brought by interbank loans for $\kappa^i \bar{A}$ units of pledgeable non-performing assets compared to the liquidity delivered by restructuring them.

Comparing conditions (Zp+-1), (Znp+-1) and (Znpi), we can conclude that during a confidence crisis, ‘bad’ banks’ suffer larger liquidity pressures than ‘good’ ones and the
liquidity condition of ‘bad’ banks deteriorates when the interbank market is frozen, i.e., 

\[ z_p^+ < z_i^+ < z_{np}^+ . \]

The condition \( z_i^+ < z_{np}^+ \) shows that the liquidity position of ‘bad’ banks is vulnerable during the crisis time and hinges largely on the functioning of the interbank market. In the case where both \( (Znp+) \) and \( (Znpi) \) are satisfied, ‘bad’ banks survive in a run if the interbank lending is granted but fails if the latter is suspended. ‘Good’ banks immune to the risk of a bank run will extend collateralized loans, as the verification of \( (Znpi) \) means that there is no counterparty risk in the interbank market. Therefore, the interbank market enhances the stability of the banking system by allowing an optimal risk reallocation among banks.

On the contrary, if the condition \( (Znpi) \) is not verified, there results insolvency of ‘bad’ banks facing a run even though interbank loans are granted. In this case, the illiquidity of ‘bad’ banks is still measured as in \( (Znp+) \). When expecting the prevalence of the run equilibrium at the intermediate date, ‘good’ banks decide at a time point slightly before \( t_1 \) to suspend the loan to ‘bad’ banks. As we have described in the section 3, in a banking system without a role for the interbank market, each bank will keep a refinancing reserve up to \( \phi A \). For a banking system with the interbank market, the liquidity reserves of a bank is only \( \pi \phi A \) and the interbank market is expected to finance the gap equal to \( (1 - \pi) \phi A \). Consequently, when the interbank market fails, the liquidity position of a troubled bank is deteriorated compared to its position in a banking system without the interbank market.

The above analysis suggests that the failure of the interbank market plays an important role in triggering a self-fulfilling banking crisis. First, given that banks ignore in normal times the impact of premature liquidation, the interbank market could deteriorate the liquidity position of banks in a bank run if the fire sale price of immature projects is sufficiently low. Second, the interactions between the depositors of ‘bad’ banks and the lending banks could act as a catalyst for a confidence crisis if the condition \( (Znpi) \) is not verified. More precisely, lending banks make, at a time point slightly before \( t_1 \), their decision of interbank lending based on the expectations about the choice of borrowing banks’ depositors at \( t_1 \). The interbank market functions thus as a selective device in the sense that its freezing due to the pessimist expectations of lending banks will deliver a bad signal to borrowing banks’ depositors and incite them
to fulfill immediately the expectations of a run, although they may originally wish to withhold claims until \( t_2 \).

Proceeding as before, we can express the \((Znpi)\) with structural parameters as follows:

\[
\eta_{t}^{np} > \frac{r_2^+}{(1-\tau)}, \tag{znpi-1}
\]

where \( r_2^+ \equiv [\Phi^{\sigma-1}(1 - \kappa^i + \pi\phi) - \phi](1 - \kappa^i)^{-1} = \frac{r_2^+ - (1-\pi)\phi}{1-\kappa^i} \). Given that \( r_1^+ < 1 \) and \( \kappa^i < (1 - \pi)\phi \), we obtain \( r_2^+ < r_1^+ \) showing that ‘bad’ banks becomes less illiquid when the interbank market functions well. If \( \frac{r_2^+}{(1-\tau)} < \eta_{t}^{np} < \frac{r_1^+}{(1-\tau)} \), ‘bad’ banks survives only when they can borrow from the interbank market. These results confirm our previous analysis.

The minimal capital ratio required for borrowing from the interbank market, \( \kappa^i \), affects largely banks’ solvency condition in a run. The negative sensitivity of \( r_1^+ \) and \( r_2^+ \) to an increase in \( \kappa^i \), meaning that the minimal liquidation price compatible with absence of a run decreases with the minimal capital ratio and thus banks with a higher capital ratio are more resilient to a run. The capital ratio \( \kappa^i \), set at the planning date by taking only account of the risk of detaining non performing projects \( \pi \) and the unit cost of the refunding \( \phi \), may prove insufficient during crisis times such that \( \eta_{t}^{np} < \frac{r_2^+}{(1-\tau)} < \frac{r_1^+}{(1-\tau)} \). Consequently, the run equilibrium could be entirely ruled out if the government imposes at \( t_0 \) a regulatory capital ratio, \( \kappa^\theta \), sufficiently high such that the condition (znpi-1) is verified. Since the right-hand side of (znpi-1) is decreasing in capital ratio, we must have \( \kappa^\theta > \kappa^i \). The minimal regulatory capital ensuring the absence of bank run cannot be explicitly expressed in terms of structural parameters given the non-linearity in (znpi-1). For \( \sigma=0.5, \tau=0.1, \phi=0.2, \eta_{t}^{np} = 0.6, R = 2 \) and various values (from 0.1 to 0.7) of \( \pi \), numerical simulations show that \( \kappa^\theta \) could be too high such that at the run-free equilibrium, the role of banks as financial intermediary is ruled out (i.e., \( \kappa^\theta > \bar{\kappa} \)), implying that in our model the government cannot set a practicable regulatory capital ratio to eliminate completely the risk of bank runs.

4.2. Foreign debt crisis and the domestic interbank market

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In the above, domestic and foreign government bonds are assumed to be risk free in any circumstances. While this is the general perception among financial operators before the 2008-2009 global financial crisis, such an assumption is invalidated by the events in the recent euro-zone crisis. The latter has reminded the financial operators that sovereign debts might be subject to a risk of default. In a monetary union without a banking union, a banking crisis could lead to a sovereign debt crisis and vice versa, given that the national government has the obligation of supervising and bailing out the domestic banking system while the domestic banks hold a significant quantity of national debts. Due to the high degree of financial integration, such a twin crisis could have destructive impacts on the banking systems of other member states.

We examine in this subsection the effects of a financial shock originated outside the domestic country in a variant of the baseline model described in the section 3 to illustrate the situation in some euro-zone countries like France or Germany, whose domestic banking systems suffer the contagion of other countries’ sovereign debt crisis. We consider a case where the interest rate on foreign government bonds rises at the intermediate date $t_1$, reflecting the sudden discovery of the risk of sovereign default by market operators, such that:

$$r_{t_2}^f = r^* + \rho,$$

where $\rho$ is the risk premium.

The optimal allocation by the banking system made by the banking system at $t_0$ is based on the assumption that all government bonds are risk-free, and both domestic and foreign government bonds are indifferently hold by banks as liquidity reserves. The rise in the foreign country’s interest rate will affect the domestic banking system through the balance-sheet effect.

The composition of the portfolio of bonds at $t_1$ and just before the foreign financial shock is given by $B \equiv B^f + B^d$ with $B^f > 0$ referring to the amount of foreign bonds and $B^d > 0$ the amount of domestic bonds. Denote by $\eta \in (0,1)$ the share of foreign bonds in total bonds holdings. It follows:

$$B^d = (1 - \eta)B \quad \text{and} \quad B^f = \eta B. \quad \text{(BdBf)}$$

The depreciation of foreign government bonds erodes the liquidity position of domestic banks. Given constraints (4) and (BdBf), we obtain the aggregate capital loss of domestic banks directly caused by the foreign sovereign debt crisis as

$$l(\rho) = \frac{\eta \rho B}{(1 + r^* + \rho)(1 + r^* + \rho^*)}, \quad \text{(I)}$$

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Disequilibrium results in the interbank market because such losses reduce the available liquidity reserves of all types of banks, thus reducing the liquidity supply of banks with surplus reserves and increasing the liquidity demand of banks with liquidity shortage.

When foreign bonds depreciate, ‘good’ banks will divert part of their liquidity reserves invested in government bonds, initially destined to be lent through the interbank market, to ensure the payments to impatient depositors. Therefore, the total liquidity available in the interbank market is now

\[ I^s = (1 - \pi)[\pi \phi \bar{A} - l(\rho)]. \tag{laqibm} \]

Comparing (lqdibm) with (ibm), it is straightforward to see that \( I^s < (1 - \pi)I \), implying a credit crunch in the domestic interbank market during a foreign sovereign debt crisis.

The aggregate liquidity supply, \( I^s \), can be positive or negative. The case where \( I^s < 0 \) corresponds to a systemic liquidity crisis in the sense that ‘good’ banks themselves suffering liquidity shortage are forced to carry out a premature liquidation of their long-term projects. Therefore, the interbank market is entirely frozen and ‘bad’ banks, due to the lack of funds needed for reinvestment, fail immediately at \( t_1 \), given that the condition (zn/fp+) is verified.

The freezing of the interbank market could coincide with a systematic banking crisis if the condition for the existence of the run equilibrium is verified even for ‘good’ banks, i.e.,

\[ z_{pf}^+ \equiv z_p^+ + l(\rho) > 0. \tag{zf} \]

To examine the role of the interbank market during a foreign debt crisis, we consider an intermediate case where \( z_p^+ \) (and/or \( l(\rho) \)) is small enough such that (zf) does not hold and ‘good’ banks still have liquidity surplus to lend in the interbank market, i.e., \( I^s > 0 \). Consequently, the interbank market could remain functioning but becomes very strained as the liquidity offer decreases by \( (1 - \pi)l(\rho) \). The increase of ‘bad’ banks’ liquidity needs caused by the depreciation of foreign bonds is equal to \( \pi l(\rho) \). The liquidity shortage of ‘bad’ banks \( I^d \) cannot be satisfied by the liquidity supply from the interbank market \( I^s \), i.e.,

\[ I^d = \pi[(1 - \pi)\phi \bar{A} + l(\rho)] > I^s. \tag{ld} \]
As a result, ‘bad’ banks are compelled to liquidate a certain amount of long-term projects to meet their liquidity needs at $t_1$. Such liquidation implies, according to the solvency constraint (7), that ‘bad’ banks will be insolvent at the final date $t_2$.

The equilibrium depends on the decisions of patient depositors who use the state of the interbank market as a signal. This assumption reflects quite well the behavior of most depositors during a bank run given that they have less information than financial institutions participating in the interbank market and form their expectation based on reactions of financial institutions regarding their banks.

Assume that the foreign shock is such that, as long as the interbank market functions, the confidence crisis will not occur at $t_1$. In the case where collateralized interbank loans are granted, the run on ‘bad’ banks would be avoided at $t_1$ but could still occur before $t_2$. If interbank loans are suspended, depositors run immediately on ‘bad’ banks at $t_1$.

It is straightforward to see that the bankruptcy of ‘bad’ banks before the debt collection at $t_2$ will reduce ‘good’ banks’ return from interbank lending to $(1 - \tau)(R - \delta)$ according to (delta), given the effort cost $\delta$ of supervising the collateralized projects. However, ‘good’ banks could have interest to lend to these ‘bad’ banks if the latter are expected to fail only after having received payments from entrepreneurs of long-term projects.

After the arrival of foreign financial shocks, ‘good’ banks decide to lend by imposing less favorable terms on borrowing banks by lowering the price of a unit of collateral from $\frac{(1-\tau)R}{1+r^*}$ to $\frac{(1-\tau)(R-\delta)}{1+r^*}$, taking account of the effort cost. In fact, with a drop in collateral price at $t_1$, ‘good’ banks can make an extra revenue $\frac{(1-\tau)\delta}{1+r^*}$ from the lending to compensate its eventual effort cost, in the event where the insolvency of ‘bad’ bank is perceived by their depositors only until $t_2$.\footnote{This situation is different from that in 4.1 where during a confidence crisis an immediate run occurs definitely on ‘bad’ banks at $t_1$, if (znpi) holds. Therefore, there is no additional profit left to ‘good’ banks.} ‘Bad’ banks borrow with the reduced collateral price equal to $\frac{(1-\tau)(R-\delta)}{1+r^*}$, if the liquidity obtained from interbank lending is larger than that from premature liquidation, i.e., $\frac{(1-\tau)(R-\delta)}{1+r^*} > \eta^p_{n}$ and the maximal amount of the borrowing can be backed up by collateral is now reduced to $\mu \equiv \frac{\pi(1-\tau)(R-\delta)}{1+r^*} - \tilde{\alpha}$ for $\pi$ percent of all banks.
As a result, ‘good’ banks will grant collateralized loans to ‘bad’ banks if the condition for the existence of the run equilibrium at the intermediate date is not satisfied for ‘bad’ banks:

\[
Z_{\text{npf}}^+ = \begin{cases} 
Z_t^+ + l(\rho) + (1-\tau)\delta \tilde{\kappa} \bar{A} < 0, & \text{if } I^s > \mu, \\
Z_t^+ + l(\rho) + \frac{1}{\pi} \left( (1-\pi)l(\rho) - (\mu - I^s) \frac{(1+\tau)(1-\tau)r_t^{np}}{(1-\tau)(R-\delta)} \right) < 0, & \text{if } I^s \leq \mu.
\end{cases}
\]

When the condition (znpf) is verified, an immediate run at \( t_1 \) on ‘bad’ banks can be avoided.\(^6\) The liquidity position of a ‘bad’ banks is deteriorated by the foreign crisis such that \( Z_{\text{npf}}^+ > Z_t^+ \). Conditions (lqdbm), (ld) and the quantity of available collateral \( \kappa'\bar{A} \) fixed at \( t_0 \) imply that the extra liquidity shortage directly caused by the depreciation of foreign government bonds held by ‘bad’ banks \( \pi l(\rho) \) cannot be filled through the interbank market that suffers a liquidity crunch equal to \( (1-\pi)l(\rho) \), i.e., the depreciation of foreign bonds held by ‘good’ banks. In the meantime, the reduction of collateral price implies that the ‘bad’ banks’ borrowing capacity is deteriorated. The two alternative cases represented in (znpf) imply that the amount of interbank loans is limited by either lending banks’ liquidity surplus for \( I^s < \mu \) or the total value of collateral held by borrowing banks for \( I^s > \mu \). In the case where \( I^s > \mu \), the value of collateral \( \mu \) cannot support a loan equal to \( I^s \), and the total borrowing is equal to \( \mu \). The extra liquidity shortfall due to the depreciation of collateral is equal to \( \delta \kappa l \bar{A} \) and the total additional liquidity gap of a ‘bad’ bank is thus \( \gamma l(\rho) + \frac{(1-\tau)\delta \kappa l \bar{A}}{1+\tau} \).

In the case where \( I^s < \mu \), the value of collateral, despite its depreciation, exceeds the amount required for obtaining a loan equal to \( I^s \). ‘Bad’ banks will thus liquidate at the fire-sale price \( (1-\tau)r_t^{np} \) the amount of collateralized assets that is not anymore used as collateral, i.e., \( (\mu - I^s)(1+\tau r_t^{np})/(1-\tau)(R-\delta) \), and obtain \( (\mu - I^s)(1+\tau r_t^{np})/(1-\tau)(R-\delta) \). Thus, for ‘bad’ banks, given that the liquidity shortage caused by the interbank market credit

\(^6\) Note that as the solvability condition at \( t_2 \) is not satisfied, there will be a run during the period after \( t_1 \) until \( t_2 \) if depositors recognize that their banks are insolvent.

\(^7\) The additional liquidity shortage is obtained by calculating the difference between the amount of interbank loans granted in normal times \( (1-\pi)l(\rho) \) and that during a foreign debt crisis. The latter is defined by \( \min(\mu, I^s) \). Therefore, this additional liquidity shortfall is \( \delta \kappa l \bar{A} \) in the case where \( I^s > \mu \) and \( (1-\pi)l(\rho) \) in the contrary case. Besides, the direct liquidity shortfall caused by the depreciation of foreign bonds for each banks ‘good’ or ‘bad’ is equal to \( l(\rho) \).

\(^8\) The term \( \mu - I^s \) stands for the value of redundant collateral due to the credit crunch. As redundant collateral assets, \( \mu - I^s \) is measured in terms of actual price and it is divided by the actualized collateral price \( (1+\tau)(R-\delta)/(1+\tau^2) \) to obtain the quantity of collateral to be liquidated at the fire sale price \( (1-\tau)r_t^{np} \).
crunch in the is \((1 - \pi)l(\rho)\), the extra liquidity gap is equal to \(\pi l(\rho) + (1 - \pi)l(\rho) - (\mu - l^s) \left( \frac{(1+r^c)\varepsilon^{fp}}{R-\delta} \right)\). Thus, the extra liquidity gap for each bad bank is equal to \(l(\rho) + \frac{1}{\pi} \left[ (1 - \pi)l(\rho) - (\mu - l^s) \left( \frac{(1+r^c)(1-\tau)r^p}{(1-\tau)(R-\delta)} \right) \right]\). Due to the credit crunch and the fall in the collateral price in response to foreign sovereign debt crisis, the liquidity position of domestic banks is deteriorated and this could compromise the stability of the interbank market.

Here, the bank-run after \(t_1\) but before the achievement of long-term projects is not inefficient and is thus different from a self-fulfilling crisis at \(t_1\) where the panic-induced run reduces the utility of both depositors and banks’ shareholders. Such a delayed run results from the true insolvency of ‘bad’ banks, and it will increase depositors’ losses given that the condition \((znpf)\) holds but not the constraint \((7)\) compared to a self-fulfilling crisis. The insolvency of ‘bad’ banks may be avoided if they keep a capital level higher than that required by the interbank market and large enough to cover the direct and indirect losses caused by foreign debt crisis. However, increasing capital requirement is not always feasible. As in previous subsections, a capital ratio efficient in eliminating risks in the banking system could easily exceed the ceiling \(\bar{k}\) and thus deprives the role of banks as the financial intermediary.

### 4.3. Asymmetric information and interbank market

We examine now the impact of the interbank market on the stability of the banking system in a more complex and realistic environment characterized by asymmetric information between borrowing and lending banks at \(t_1\) due to the appearance of a gambling asset. Such a problem is perceived by banks only at a time point just before the intermediate date \(t_1\) and it is ignored by depositors so that it is not directly at the origin of a bank run. However, if banks gamble, this information will be revealed and obtained costlessly by all other agents at the final date \(t_2\). The bank run could be triggered when depositors perceived the gambling behavior of their banks.

A gambling asset is suddenly available for investment at the intermediate date \(t_1\). It yields \(\psi R > R\) at the final date \(t_2\) with the probability \(\vartheta < 1\) for each unit of goods invested at \(t_1\) and 0 otherwise. \((\psi - 1)R > 0\) is the unobservable excess return if the gambling asset succeeds. To capture the characteristics of a gambling asset, we
assume that its probability of success is low ($\theta$ is quite small), its unobservable rate of return is high ($\psi$ is large) and investing in this gambling asset is socially inefficient such that

$$\theta \psi R < R, \quad \text{(return g)}$$

The condition (return) indicates that the average expected return from a gambling asset is lower than that from the long-term project. Therefore, any risk-averse or risk-neutral investors will not fund the gambling asset with their own endowments. However, for ‘bad’ banks subject to limited liability, purchasing gambling asset could be a rational choice. More precisely, they have the incentive to divert funds from interbank loans and from the premature liquidation of long-term projects to invest in gambling asset, if

$$\theta(1 - \tau)R \left[ \psi(1 - \tau)(1 - \kappa^i)\bar{A}_t^{np} + \psi \left( \frac{(1-\tau)R}{1+r^s} \kappa^i \bar{A} - \kappa^i \bar{A} - \frac{(1-\lambda)\gamma}{(1-\tau)} \right) > (K - \kappa^i \bar{A})(1 - \tau)R \right. \quad \text{(incentive)}$$

The left hand side of (incentive) represents the expected gain for ‘bad’ banks’ shareholders, if they divert the funds obtained with the liquidation of a part of long-term projects equal to $(1 - \kappa^i)(1 - \tau)\bar{A}_t^{np}$, and the interbank loans $(\frac{(1-\tau)R}{1+r^s} - \kappa^i \bar{A})$ to purchase gambling assets, which are demanded by ‘bad’ banks according to (ibm). With the probability $\theta$, ‘bad’ banks may repay interbank loans $(1 - \tau)R \kappa^i \bar{A}$ and patient depositors $(1 - \lambda)\gamma$, their shareholders takes the remainder. The right hand side of the condition is the cost of investing in gambling assets. Given that banks must hold a bank capital equal to $\bar{K} = \kappa^i \bar{A}$, the right hand side of the condition is zero.

It is easy to see that the condition (incentive) breaks easily. A ‘bad’ bank will have no incentive to gamble if the condition (incentive) is not verified. In other words, it will not gamble if it keeps a capital ratio $\kappa^r$ imposed by the government’s regulation higher than the minimum level imposed by the interbank market, i.e.,

$$\kappa^r > \kappa^i + \frac{\theta}{1-\theta} \left[ \psi (1 - \tau) (1 - \kappa^i) r_t^{np} + \psi (1 - \pi) \phi - 1 \right]. \quad \text{(k/a)}$$

Therefore, the government regulation on capital requirement seems to be essential for avoiding gambling behaviors, as banks under perfect competition will not voluntarily adopt a capital ratio higher than what the interbank market requires. However, given the fact that $\psi$ is high and $\frac{\partial \kappa^r}{\partial \psi} > 0$, $\kappa^r$ could exceed the ceiling $\kappa$ defined by (kbar),

---

9 The condition (k/a) is obtained by inverting the inequality (incentive) while using (A), (y) and the definition of $\Phi$. 

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meaning that when the unobservable excess profit is sufficiently high, an *ex-ante* higher capital ratio introduced to avoid gambling could be incompatible with the existence of the equilibrium with banks as financial intermediaries.

Using $\tilde{A}$ given by (A) and $(1 - \lambda)y = A(1 - \kappa)(1 - \tau)R$ given by the constraint (7), the constraint (incentive) is equivalent to

$$r_l^{np} > \frac{r_3^+}{(1 - \tau)}, \quad (r3)$$

where $r_3^+ \equiv \frac{1 - (1 - \pi)\phi\psi}{(1 - \kappa^l)\psi} < 1$.\(^{10}\) When the condition (r3) is satisfied, ‘bad’ banks will invest all resources in the gambling asset since this is more profitable for their shareholders, even though it is socially inefficient. We obtain directly from the definition of $r_3^+$ that $\frac{\partial r_3^+}{\partial \kappa^l} < 0$, implying that the condition (r3) is more easily verified when the minimal capital ratio is higher. This result does not contradict (k/a). The condition (k/a) indicates that a bank capital higher than the minimal level $\kappa^l\tilde{A}$ required for obtaining interbank lending increases the cost of the premature liquidation (i.e., the term $(K - \kappa^lA)(1 - \tau)R$ at the right-hand side of (incentive)), and thus incites ‘bad’ banks to give up their gambling behavior. The definition of $r_3^+$ implies that a higher minimal capital ratio imposed by the interbank market leads to more liquidity that can be raised through the interbank market, generating thus more (and hence cheaper) funds for investing in the gambling asset.

The information revealed by the condition (r3) is costless for all banks but not available to depositors at $t_1$. Thus, the arrival of the gambling asset will not induce the run in the first place. ‘Good’ banks cannot directly monitor ‘bad’ banks’ gambling behavior. Their decision of lending depends on the verification or not of the condition (r3). If (r3) does not hold, ‘good’ banks will extend the interbank loans and the situation of the banking system remains the same as in the section 3. The presence of gambling asset has no influence on the equilibrium. In the contrary case, the gambling asset is more lucrative in terms of dividend than reinvestment for ‘bad’ banks’ shareholders, and there are two options left for ‘good’ banks. They may either refuse granting interbank loans to avoid the counterparty risk caused by the gambling or demand a higher return from the loans so as to compensate the additional risk.

\(^{10}\) Provided that $\kappa^l = \frac{(1 + r^*)(1 - \pi)\psi}{(1 - \tau)R}$, we directly obtain $\kappa^l < (1 - \pi)\phi$, which ensures that $r_3^+ < 1$.\(^{10}\)
Nevertheless, for risk neutral banks, the second option remains more appealing because it could yield higher dividends for their shareholders than the first option.

The initial safe return from the interbank loans is given by \((1 - \tau)R \kappa^iA\). With the presence of the gambling assets, lending banks may gamble with ‘bad’ banks by requiring a higher return from interbank loans equal to \(\frac{1}{\theta}(1 - \tau)R \kappa^iA\) so as to compensate the risk associated with the gambling behavior. Accordingly, ‘bad’ banks will still have the incentive in gambling with increased cost of interbank loans, if the condition

\[
\psi \left[ (1 - \tau)(1 - \kappa')A r^i_{t}^{np} + \frac{(1-\tau)R}{1+\tau} \kappa' A \right] - \frac{\kappa' A}{\theta} - \frac{(1-\lambda)y}{(1-\tau)R} > 0 \quad \text{(incentive')}
\]

is satisfied. The condition (incentive’) represents the case where the investment in the gambling asset remains the better solution for ‘bad’ banks when the cost of borrowing rises. If the interbank loan is not granted, ‘bad’ banks fail immediately at \(t_1\) and their shareholders receive no dividend. This consequence compels ‘bad’ banks to accept the less favorable loan terms if (incentive’) holds. ‘Bad’ banks’ shareholders may then recoup partially their investment with a probability \(\theta\) while their depositors potentially suffer a huge loss if the gambling fails.

The condition (incentive’) can be rewritten with the fundamental parameters as follows:

\[
r^i_{t}^{np} > \frac{r^+_4}{(1-\tau)}
\]

where \(r^+_4 \equiv \frac{\theta + \kappa(1-\theta) - \theta \psi(1-\tau)\phi}{\theta \psi(1-\kappa')}.\) When the condition (r4) holds, the interbank loans will be granted. Consequently, ‘bad’ banks carry out the premature liquidation of \((1 - \kappa^i)A\) units of long-term assets and invest the proceeds in the gambling asset.

A bank run on ‘bad’ banks happens if the interbank market freezes. This is possible if ‘bad’ banks cannot honor the repayment of the interbank loans borrowed at the risky rate \(\frac{(1-\tau)R}{\theta}\). Given that \(r^+_4 > r^+_3\), which follows immediately from comparing (r3) with (r4), there is a situation where (r3) holds but not (r4). In such a situation, a bank run on ‘bad’ banks occurs immediately at \(t_1\) since depositors acquire the information that, due to lack of refunding, their banks cannot ensure the payments at \(t_2\). As a result, neither the refinancing of long-term projects nor the investment in the gambling asset will be implemented. Thus, the presence of asymmetric information increases the chance of failure of ‘bad’ banks.
The interbank market functions poorly in answering to the unexpected shock to the banking system represented by the sudden appearance of the gambling asset at the intermediate date. Several observations could be made about the interplay between the interbank market and the gambling asset.

First, given the requirement of a minimal capital ratio for accessing to the interbank market, the interbank loans while ensuring optimal risk-sharing, could amplify the gambling activities of borrowers. As shown by the condition (incentive) and (incentive’), due to this minimal capital ratio, borrowing banks’ shareholders bear no additional cost for the failure of gambling activity while they may receive compensations if their banks win in the gambling. Consequently, such a minimal capital ratio is inefficient in the sense that it can neither ensure the liquidity transfer between banks nor prevent their over risk-taking when the economy is affected by the asymmetric information about the unexpected shock.

Second, the operating principal of the interbank market becomes incompatible with the presence of asymmetric information. Comparing (znpi-1) to (r4), we find that in normal circumstances, the interbank market protects banks with a relative liquid balance sheet (i.e., a higher $r_{l}^{np}$), whereas, in the presence of asymmetric information, the interbank market works if borrowing banks have a more illiquid balance sheet (i.e., a lower $r_{l}^{np}$). This paradoxical reaction of the interbank market in response to the unexpected shock destabilizes the banking system, in particular when banks have a relative sound balance sheet with more liquid assets.

Third, the interbank market facing an unexpected shock may encourage the over risk-taking of the banking system. As shown by (r3) and (r4), lending banks could have interest in permitting borrowing banks to gamble when the latter have a more liquid balance sheet. Their risk neutral shareholders prefer this risky activity, since it delivers an average expected return no less than that from the refunding of long-term projects. Nevertheless, risk-averse domestic residents suffer huge losses, in particular when the gambling banks fail. In this event, the production level falls sharply, since borrowing banks have to liquidate prematurely their long-term projects.

To enhance the stability of the banking system and the interbank market in the presence of asymmetric information, the government might stipulate a capital regulation that requires banks to hold a level of capital higher than the minimal capital imposed by the interbank lending. Nevertheless, as shown by the condition (k/a), a
regulation fully efficient in eliminating banking crisis could imply a capital ratio exceeding the capital ratio ceiling \( \bar{k} \), especially when the gambling is highly attractive (i.e., \( \theta \) and \( \psi \) are relatively high). As a result, government’s ex-post crisis management plays a crucial role in stabilizing the banking system and in restoring normal conditions of the interbank market during crisis times.

5. The government’s crisis response

The banking sector and the interbank market are the most important components of the financial system. In normal times, the existence of the interbank market reinforces the liquidity position of banks and boosts the economic performance, whereas dysfunction of the interbank market could aggravate the instability of the banking system during crisis times. More restrictive regulations, usually costly in terms of social welfare, are not always feasible due to the constraints imposed by the structural parameters of the economy. Consequently, the government’s crisis management is crucial to sustain the banking system and to restore the normal conditions of the interbank market during crisis times. This is confirmed by the experiences during recent banking crises across the world. However, lessons from several euro-peripheral countries such as Spain, Portugal or Ireland have revealed that the government’s ability of bailing out banks is largely limited by the institutional constraints introduced with the creation of the euro zone. Due to these constraints, the government of a member state might not be able to implement a credible bailout program if the latter’s execution impaired its budgetary position. The engagement in carrying out an infeasible bailout program could induce a twin sovereign debt and banking crises.

The domestic government in our model starts with an amount of debt \( D_0 \), which is financed by issuing at \( t_0 \) a quantity of long-term government bonds with a face value of \( B_{02}^g \) maturing at the end of period 2. They are sold at a discount of the par value, such that \( \frac{B_{02}^g}{1 + r_{02}} = D_0 \). During the period, the government collects taxes, \( T \), to finance public spending, \( G \). Without new bonds being issued after the initial date \( t_0 \), the amount of debt left at the end of \( t_2 \) is \( D_2 = B_{02}^g + (G - T) \). The debt \( D_2 \) is refinanced by issuing new bonds in the international financial market after \( t_2 \). International investors would accept to refinance the debt up to a certain level. To reflect this fact,
we assume that there exists a ceiling of the ratio of debt over GDP, $g_f$, above which the government’s risk of default on its debt in the future becomes significant, and that the cost of refinancing stays at normal levels if the ratio of debt over GDP does not exceed the exogenous ceiling $g_f$, i.e.

$$\frac{D}{\bar{Y}} \leq g_f.$$ (cgf)

In normal times, the level of taxes and of government spending is such that the ratio of debt over GDP remains at a constant level $\bar{g}$, i.e. $D_2 = D_0 = \bar{D}$, and all projects mature and the amount of taxes collected is therefore $\bar{T} = \tau R \bar{A} = \tau \bar{Y}$ with $\bar{Y}$ being total production in normal times. The amount of government spending consistent with a constant level of debt is $\bar{G} = \tau R \bar{A} - r^* \bar{D}$. The government debt lies within the credible level and the debt to GDP ratio in the sense that $\bar{g}$ does not surpass the ceiling $g_f$, such that the condition (cgf) holds.

If $g > g_f$, the government is exposed to the risk of sovereign default and will need to pay a risk premium $g^d$ increasing with the level of $g$ for its borrowing. The present value of government bonds will accordingly plummet. In the following, we first consider the case without gambling asset (subsections 5.1 and 5.2) then the case with it (subsection 5.3).

### 5.1. Bailout of a pure confidence crisis

To resolve a banking crisis resulted from a run by panic depositors on ‘bad’ banks, the government can try to rule out this run equilibrium by deciding, at $t_1$, to bail out banks through injecting liquidity into these banks. As shown by (znpi), the run equilibrium can be eliminated and the normal functioning of the interbank market can be restored, if the liquidity gap $z_i^+$ is filled. Given that ‘bad’ banks accounts for $\pi$ percent of all banks, the amount of liquidity injection required for eliminating the confidence crisis is $G_2^d = \pi z_i^+$. Without the seigniorage revenue, the government must issue new (short-term) bonds with face value $B_{12}^d$ sold at a discount of the par value. We take the case where the government’s solvency is not a concern, the interest rate applied to these bonds is $r_{12}^g = r^*$, so that:

$$G_2^d = \frac{B_{12}^d}{1 + r^*} = \pi z_i^+.$$ (g1)
In the meantime, the revenue of the government declines when ‘bad’ banks liquidate their investments for an amount of \( \Delta T^b = \bar{T} - (1 - \tau) (R - r_i^{np}) \pi \bar{A} \), where the superscript ‘b’ denotes the equilibrium with the run on ‘bad’ banks being avoided by the government bailout.

Assume that, in the crisis state, the government is able to commit to bail out ‘bad’ banks in a run. Then the condition

\[
g^b \equiv \frac{D^b}{Y^b} = \frac{D + G^b \Delta T^r}{Y - (R - r_i^{np}) \pi A} \leq g_f,
\]

should be satisfied. From the condition (G), we have \( g^b > \bar{g} \) indicating that, with taxes plummeting and public spending soaring, the fiscal bailout induced by a banking crisis endangers the sustainability of the government’s debt. The level of \( g^b \) depends largely on the scale of the crisis, which could be measured by the proportion of ‘bad’ banks \( \pi \), and the liquidity of the assets measured by the gap between \( r_i^b \) and \( r_i^{np} \).

In the case of \( g^b > g_f \), the government suffers a sovereign debt crisis due to the unsustainable debt level resulted from its engagement in the unviable bailout. Given the existence of the risk of default, the government can no more sell bonds without paying a risk premium over the international interest rate. This implies that the present value of domestic government bonds plummets at \( t_1 \). The liquidity gap of the domestic banks that hold massively these bonds on their balance sheet becomes larger. In consequence, the government must borrow more funds with higher costs to fill banks’ enlarged liquidity gap with sensitiveness of the liquidity gap to the risk premium given by \( \frac{\partial \pi^*}{\partial q} \equiv \frac{-(1 - \eta) B}{(1 + r^* + q)^2} \). In other words, if \( g^r > g_f \), the bailout is infeasible and its implementation will induce twin banking and sovereign debt crises.

In the case where \( g^r \leq g_f \), the confidence crisis can be easily ruled out once the government announces its bailout program \( G^b_1 \). The fact that a credible announce is sufficient to eliminate a bank run implies that no new debts need to be issued given that the interbank market resume its role in the liquidity transfer.

5.2. Bailout of a crisis resulted from foreign debt crisis

We consider now the government intervention during a liquidity crisis stemmed from the contagion of foreign sovereign debt crisis. There are two preeminent differences
between the confidence crisis and the crisis originated from depreciating assets. In terms of crisis origins, the first is due to liquidity mismatch in banks’ balance sheet induced by premature withdrawals, while the second is caused by the insolvency of ‘bad’ banks as a result of aggregate liquidity shortfall and the resulted plummeting collateral price. In terms of crisis management, the first could be ruled out with mere announcement of a credible bailout package while the second must be dealt with effective implementation of the bailout program destined to fill the liquidity gap.

To cope with a crisis induced by depreciation of foreign sovereign bonds, the government should inject sufficient liquidity to fill the aggregate liquidity gap \( l(q) \) so as to avoid the credit crunch and the drop of collateral price in the interbank market that could result into the failure of ‘bad’ banks. The total cost of the bailout package in this case is thus \( G_s^b = l(q) \). A government without monetary instrument, should issue at \( t_1 \) an amount of short-term bonds equal to \( \frac{B_{12}^s}{1 + r^*} = G_s^b \), if its solvency is not an concern of market participants. Consequently, this bailout program is feasible under the condition that\(^{31}\)

\[
g^s \equiv \frac{D^s}{\gamma^s} = \frac{D + G_s^b + \Delta T^b}{\gamma - (R - r_1^{NP})\pi A} \leq g_f. \tag{gs}
\]

Comparing \((\text{gbar})\) and \((G)\) directly yields that \( g^s > \bar{g} \), implying that government’s budgetary position could be largely deteriorated by the banking bailout. The value of \( g^s \) depends on the proportion of ‘bad’ banks \((\pi)\) in the banking system and their degree of illiquidity measured by the gap between \( r_2^+ \) and \( r_1^{NP} \), as well as on the connectedness between the domestic banking system and the foreign country measured by \( \eta \) and the magnitude of depreciation of foreign bonds \( q \). In the case where \( g^s > g_f \), the government is unable to rescue the banking sector contaminated by the foreign sovereign debt crisis. The implementation of incredible bailout policy, \( G_s^b \), will induce twin banking and sovereign debt crises in the domestic economy. ‘Good’ banks could also be involved in such crises if the depreciation of domestic and foreign government bonds is such that the condition \((zpf)\) is verified. To avoid such a scenario following the contagion of foreign sovereign debt crisis, the domestic government should initially keep sufficient room for policy maneuver during crisis times such that \( g^s < g_f \) holds even the bailout package \( l(q) \) is carried out.

\(^{31}\)In the condition \((gs)\), we consider as in \((G)\) the worst scenario where the run on ‘bad’ banks continues although banks survive with the intervention of the government. The latter’s bankruptcy decreases the tax revenue by an amount of \( \Delta T^b \) and the production by a quantity of \( \pi A \).
5.3. Preventive policy to avoid a crisis due to gambling behaviors

As described in the subsection 4.2, the gambling behaviors of ‘bad’ banks and the asymmetric information between banks at $t_1$ could induce the interbank market to either stop liquidity transfer from lending banks to borrowing banks or encourage the over risk-taking of the entire banking system. The capital regulation could be too costly or infeasible in ruling out the consequences of the sudden arrival of gambling asset at $t_1$ and the direct supervision over individual banks’ balance sheet will not be practical.

We assume that, while the domestic government does not receive more information about the soundness of banks’ balance sheet than financial markets’ participants, it has the capacity to verify at date $t_2$ whether banks gamble or not. To avoid the potential loss and destabilizing effects caused by the gambling behavior, the domestic government, as banking regulator, can announce before $t_1$ to raise a penalty tax to be collected on the gambling income at $t_2$. Let $\tau^P$ denote the penalty tax rate. This preventive policy is efficient if it can effectively destroy banks’ incentive for gambling such that

\[(1 - \tau - \tau^P)R \left[ \psi(1 - \tau)(1 - \kappa^i)\tilde{A}r^p_i + \psi \frac{(1-\tau)R\kappa^i\tilde{A}}{1+\tau} - [(1 - \tau)R\kappa^i\tilde{A} - (1 - \lambda)y] \right] < 0. \quad \text{(inc pt)}\]

The left hand side of the above condition stands for ‘bad’ banks’ gain from gambling, which is lower than that in the condition (incentive) due to the presence of penalty tax. The right hand side, identical to that in (incentive), represents ‘bad’ banks’ revenue loss due to abandon of viable long-term projects. Arranging the terms of the condition (inc pt), we obtain that, to prevent gambling behaviors, the government should set a penalty tax rate verifying the following condition:

\[\tau^P > (1 - \tau) \left\{ 1 - \frac{1}{\psi(1 - \tau)(1 - \kappa^i)r^p_i + (1 - \pi)\phi} \right\}. \quad \text{(tp)}\]

When the preventive policy, defined by a penalty tax rate $\tau^P$ satisfying (tp), is introduced, ‘bad’ banks have no interest to gamble as it yields a negative expected return. Therefore, at the final date $t_2$, no penalty tax will be actually collected.

Finally, the mere announcement of the credible preventive policy $\tau^P$ can completely eliminate gambling incentive. Thus, the introduction of such a policy protects efficiently the banking system from the destabilizing effects of gambling assets.
6. Conclusion

The model presented in this paper captures several features of recent banking crises characterized by the dysfunction of the interbank market. It is shown that, while the interbank market facilitates the liquidity transfer between banks and improves the social welfare during normal times, it could be a factor of instability by disseminating the effects of various shocks to the entire banking system during crisis times. We show that the interbank market could aggravate or induce the confidence crisis by making possible the self-reinforcing panic of both borrowing banks’ depositors and lending banks. It could increase banks’ risk-taking in the presence of gambling asset and aggravate the negative effects of the contagion from foreign sovereign debt crisis to domestic banking crisis.

The good functioning of the interbank market depends on the level of banks’ capital. Without a capital regulation, the minimal capital ratio required by the interbank market to achieve optimal risk-sharing in the banking system during normal times could be too low for being efficient during crisis times. Therefore, the capital regulation stipulated by the government should be considered as an essential instrument to enhance banks’ resilience during crises. However, a too restrictive regulation is impractical because, while stabilizing the banking system, it could hamper the role of banks as financial intermediaries. To minimize the gambling behaviors of banks, the government should introduce a penalty tax conditioned on the ex-post discovery of such behaviors.

Given that the banking crisis could not be eliminated by reasonably strict government regulations, the government’s crisis management become a key factor in stabilizing the banking sector during a crisis that could originate from self-fulfilling bank runs, contagion of foreign sovereign debt crisis, and gambling behaviors of banks. Notwithstanding, the government’s capacity of restoring the normal functioning of the banking system and the interbank market and of preventing the crisis contagion is constrained by its budgetary position. If its initial budgetary position makes the implementation of a bailout policy impossible in the sense that it increases the national public debt to an unsustainable level, an engagement in bailing out banks subject to run could result in twin banking and sovereign-debt crises,
particularly when the government does not have any monetary sovereignty as it is the case in the euro zone.

Reference:


