Liquidity Effect and Financial Shocks in a Monetary Model with Asymmetric Information and Time-Varying Risk

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Abstract

This paper develops a flexible-price monetary model with credit market imperfections and time-varying uncertainty. A monetary version of Carstrom and Fuerst (1997) framework, where money is introduced through a cash-in-advance constraint and, in addition to consumption goods, a fraction of the investment good is financed with cash. Time-varying risk is introduced through a time-varying uncertainty of entrepreneurial random idiosyncratic shock. Under a limited participation assumption, the model is able to generate a liquidity effect. This latter reaches its maximum when the investment is a "pure credit good". Asymmetric information in the credit market amplifies the models responses to expansionary monetary shocks and enhances the propagation mechanism, especially when the monetary policy is a Taylor rule. The response to a positive technology shock depends on the weight of output in the Taylor rule. When the central bank follows a Taylor rule with a large coefficient on output, the response of the economy to a technology shock is amplified and inflation decreases significantly. Furthermore, an positive uncertainty shock generates an increase of the loans' risk premium, so that investment, output and employment fall. The size of this negative impact is increasing with the shock's persistence.

Keywords: Asymmetric information, deterministic monitoring, risk premium, financial accelerator, liquidity effect, financial shocks, time-varying risk, inflation persistence

JEL Classification: E32, E41, E44, E58

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1 Introduction

“Plausible models of the monetary transmission mechanism should be consistent with at least the following facts about the effects of a contractionary monetary policy shock: the aggregate price level initially responds very little, aggregate output falls, interest rates initially rise, real wages decline by a modest amount, profits fall...”


A long standing feature of traditional macroeconomic analysis has been the liquidity effect produced through an expansionary open market operation. It refers to a fall in nominal interest rates following an exogenous persistent increase in narrow measures of the money supply. On the other hand the classical Fisher effect stipulates that an exogenous persistent increase in money is predicted to increase expected inflation and so increase nominal interest rates. Friedman (1968) argues that, in practice, both forces coexist: a persistent increase in the money supply both reduces nominal interest rates and raises expected inflation so that the real rate also declines. He believes that nominal and real rates may fall below their typical levels for up to a year, but, over time, rates will then tend to increase before tending to the levels consistent with the inflation generated by the original monetary impulse.

In order to capture Friedman’s intuition, Lucas (1991) modified the standard cash-in-advance (CIA) model by adding a simple timing assumption: households take their saving decision i.e. allocating their cash between goods market and assets market before observing the size of an open market operation. According to this new setting, an unexpected purchase of bonds will increase the cash available in the economy and exerts an upward pressure on bonds prices, pushing nominal interest rates to fall. This market segmentation assumption is integrated by Fuerst (1992) and Christiano and Eichenbaum (1991, 1992, 1995) into otherwise standard real business cycle models. Their key innovation was that each period firms are obliged to borrow money from the financial intermediary to pay their production bills. Therefore, firms are willing to absorb the increase in money balances, after a positive money supply shock, at a lower interest rate, which spurs labour demand and increases output.

Although these segmented markets models are consistent with the commonly held view that positive money shocks have a positive effect on output, their main limitation is that the liquidity effect is very transitory even when monetary shocks are persistent. Households can adjust their allocation of cash every period. Therefore, the liquidity effect is entirely driven by serially uncorrelated expectational errors in cash allocation. Many extensions have been presented in order to improve the ability of these flexible prices models to replicate the persistent response to monetary shocks recorded by real data. Christiano and Eichenbaum (1992) suggest that allowing for some costs of adjusting sectoral flows of funds in the model generates a long-lasting liquidity effect as well as a persistent increase in output. Another class of general equilibrium models
inspired by Baumol (1952) and Tobin’s (1956) inventory-theoretical analysis of money demand argues that a persistent and amplified liquidity effect is possible by splitting up households into two groups; a group that participates in asset markets and another one who does not. So that, because households do not acquire cash every period, they choose to spend their money holding slowly over time. Alvarez, Atkeson and Kehoe (2002) endogenize the fraction of households who participate in asset markets. They assume that households can participate if they pay a fixed cost. The model is able to generate a liquidity effect but it is still transitory. The quantitative insight provided by these limited participation models is questionable. To generate realistic persistent co-movements of money, interest and prices, calibrating these models of liquidity effect needs "large" asset market frictions. Christiano and Eichenbaum (1997) are suggesting to calibrate the model at an implausibly high labor supply elasticity (2%) and a high average markup (40 %). The most successful parameterization in their stochastic version, Alvarez, Atkeson and Edmond (2003) require the representative household to make withdrawals of money from an asset market account once every 24 – 36 months.

Although these limitations should not be interpreted as reasons for rejecting models of asset market segmentation which are making money stock (money demand) and its timing at the center of the analysis, the recent modeling developments have begun to minimize the role of the inverse relationship between the money supply and nominal interest rates in the monetary transmission mechanism and the monetary policy conduction. The increasing interest in using interest rate rules, most notably the Taylor rule, to characterize the monetary policy has contributed actively to place the money demand in the background. However, some recent papers proved that money aggregates, and hence money demand, still have an important role in improving models responses to shocks. For instance, Leeper and Roush (2003) demonstrate that the presence of money, based on a VAR model estimation results, alters significantly the economy’s response to a monetary policy shock. Moreover, Sims and Zha (2006) show that the introduction of money into an otherwise standard Taylor rule improves their empirical model’s coherence with the data. More recently, two theoretical examples were presented by Christiano, Mostagno, and Motto (2007) showing that money can help to anchor inflationary expectations and alleviate boom-bust cycles in financial markets.

With that motivation, I present a tractable limited participation general equilibrium model with financial and informational frictions, that is not subject to the previous criticism. An extensive literature proved recently that introducing financial frictions into RBC models and relaxing the assumptions of the Modigliani-Miller theorem offer a richer internal propagation and amplification mechanism and enhance the empirical performance of dynamic stochastic general equilibrium models. Bernanke and Gertler (1989) and Bernanke et al.(1999) introduced asymmetric information, and hence financial frictions, through some verification (monitoring) costs, stipulated by the loan contract, that the financial intermediary faces whenever the borrower defaults. This costly state verification framework, developed by Townsend (1979), is also studied in detail by Carlstrom and Fuerst (1997,1998) who confirmed Bernanke et al. findings. They showed the
importance of these costs in accounting for economic fluctuations and for replicating the observed hump-shaped responses of the main macroeconomic aggregates to the underlying shocks in the economy.

Using two different monetary policy rules, I find that the model is able to generate the liquidity effect and the responses to a monetary shock is significantly amplified and enhanced when asymmetric information in the credit market is considered, especially under a Taylor rule hypothesis. The liquidity effect reaches its maximum when investment is a "pure credit good" and variables’ responses to a positive technology shock depend on the weight of output in the nominal interest rate rule. When the central bank puts more weight on output in a Taylor rule, the response of real variables to a positive technology shock is amplified and inflation declines significantly. I also demonstrate that an increase of entrepreneurs risk induces a fall in real economic variables. The size of this negative impact is increasing with the level of persistence of the uncertainty shock. In addition, I find that the default rate is countercyclical contrary to Carlstrom and Fuerst (1997).

The remainder of the paper is organized as follows: the next section describes the economic environment of the model. Section 2 develops the optimal financial contract (and the related intuition) between the financial intermediary and and entrepreneurs as well as the agents optimal decision rules. Section 3 discusses calibration. Sections 4, 5 and 6 present simulation results, cyclical behavior and main results regarding the model’s response to different shocks. I conclude in section 7.

2 Model

I consider a closed economy, formulated in discrete time with an infinite horizon, populated by five types of agents: households, entrepreneurs, consumption-good-producing firms, a financial intermediary (later bank) and the central bank. The model is a monetary counterpart of the “investment model” proposed by Carlstrom and Fuerst (1997), with perfectly flexible prices and wages, so that money alters real behavior only via expected inflation effects. This work extends their original model by introducing money via a cash in advance constraint on households consumption and investment purchases, and allowing for a time-varying riskiness of entrepreneurs.

There is a continuum of identical, infinitely lived households, depicted by a representative one who consumes, supplies labor to firms, accumulates physical capital, saves and holds money balances. There is also a continuum of entrepreneurs, indexed by \( i \in (0,1) \) who consume, work and produce the investment good by transforming consumption goods into capital. The entrepreneurs also hold money balances, carried out from one period to another as a part of their net worth. Firms produce the consumption good using the labor supplied by households and entrepreneurs and the capital rent from households as inputs.
At each period, entrepreneurs are involved in the production of the investment good using a stochastic, constant returns-to-scale technology that contemporaneously transforms consumption goods into capital. The inputs’ acquisition is financed by the entrepreneurial net worth and loans, which are borrowed from the financial intermediary, using a contractual device. The net worth is made of two components: entrepreneurs’ cash-holdings at the beginning of the period and the income from supplying inelastically one unit of labor to firms. Agents are asymmetrically informed about the entrepreneur’s real return: while it is fully known by entrepreneurs, the financial intermediary can observe it only at a marginal positive cost, $\mu$.

The timing in this model is very important. Using the Lucas-Fuerst framework, the introduction of the “sluggish cash flow” assumption assumes that households cannot adjust their portfolio easily and immediately after a monetary shock. This assumption is intended to capture a set of institutional and financial constraints¹ that prevent households from reviewing their saving decisions. Consequently, an increase in the money supply will increase the money stock available for financial intermediaries. Entrepreneurs are willing to absorb all the extra cash through borrowing, only at a lower interest rate of loan. This mechanism is supposed to spur investment, output and employment. Therefore, in order to generate the liquidity effect, I assume that entrepreneurs and financial intermediaries negotiate the debt contact after observing aggregate shocks and prior to the realization of the entrepreneurial idiosyncratic productivity shock.

Meanwhile, firms hire labor from households and entrepreneurs, rent capital from households and produce the consumption good. Entrepreneurs buy this consumption good in order to transform it into capital that will be sold to households. At this level, each entrepreneur will decide either to reimburse the loan and purchases his own consumption with the remaining resources or to default and declare bankruptcy. In the latter case, the financial intermediary will monitor the project’s income and incurs a verification cost. At the end, firms and financial intermediaries distribute their profits to households. Entrepreneurs and households carry over their residual incomes to the next period as cash-holdings. Table 1 summarizes the chronological order of events in a given time period.

2.1 Entrepreneurs and Optimal financial contract

The contractual arrangement in this economic environment is designed to overcome the information asymmetry between entrepreneurs and the bank using a state verification cost. It is a typical principal-agent problem where the principal (the bank) is willing to incur a cost of verification in order to obtain a perfect or imperfect knowledge of the agent’s (the entrepreneur) private

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¹See G. Akerlof (1979) and Christiano and Eichenbaum (1992.a,b).
²Entrepreneurs are offering only one unit of labor to firms in order to guarantee a positive net worth at each period. The reason is that the optimal contract is not defined for $N = 0$. 

**Table 1. The sequence of events in a given time period**

1. Current aggregate productivity and uncertainty shocks are realized.

2. Households enter the current period with $K_t$ units of physical capital, and $M_h^t$ money holdings. They decide on their deposits $D_t$ for the current period and the remaining amount is kept on hand to finance total consumption purchases and a fraction $\varphi$ of the investment good.

3. Households and entrepreneurs sell labor and rent capital to firms, for which they will receive an income of $W_tL_t + P_tK_t$ for households and $W_e^t$ for entrepreneurs$^2$. Using these inputs, firms produce consumption goods.

4. The monetary shock is realized.

5. The bank negotiates the credit contract with entrepreneurs. Available funds are transferred in the form of loans to entrepreneurs who, along with their entire net worth, will place it into their capital-creation technology.

6. The entrepreneurial idiosyncratic technology shock is realized. This shock is privately observed by entrepreneurs.

7. Entrepreneurs purchase consumption goods from firms in order to transform them into capital goods using their one-to-one capital production technology.

8. Households purchase their consumption from firms also and investment goods from entrepreneurs for a total amount of $P_tC_t + \varphi Q_I^t$. The investment good will be part of the capital available at the beginning of the next period.

9. Entrepreneurs decide either to pay back their loans or to claim bankruptcy. If the loan is not repaid, the entrepreneur is monitored by the bank. Solvent entrepreneurs decide on their current consumption and money holding levels to carry over in $C_e^{t+1}$ and $M_e^{t+1}$.

10. Households will carry over the remaining cash in the next period, $M_h^{t+1}$.

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information, ex-post$^3$. In this framework, The role of the financial intermediary is well defined, and the banking activity emerges and becomes essential since it mitigates monitoring costs by performing a delegated monitoring activity$^4$ on behalf of funds’ owners.

I consider the optimal contract under a pre-committed, deterministic monitoring paradigm. The financial contract is negotiated at the beginning of the period $t$ and resolved by the end of the same period (quarter). Following Carlstrom and Fuerst (1997), I assume that there is enough anonymity in financial markets so that only one-period contracts can exist. Entrepreneurs with a net worth $N_t^5$, have access to a stochastic, constant returns-to-scale technology that transforms an amount $I_t$ of consumption goods into a random amount $\omega_tI_t$ of capital, where $\omega_t \in \Omega_t$ is the stochastic idiosyncratic disturbance to the entrepreneurs’ production. $\omega_t$ is i.i.d across entrepreneurs and time, with a distribution function $\Phi(\cdot)$ over a nonnegative support $\Omega_t$, a density function $\phi(\cdot)$, and a mean of unity and a standard deviation $\sigma_{\omega^t}$. Carlstrom and Fuerst

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$^3$This contracting approach has been developed by Townsend (1979) for the first time and extended by Gale and Hellwig (1985) to cover debt contracts.


$^5$The net worth $N_t$ is supposed to be sufficiently small so that the entrepreneur would like to borrow some external financing from the lender.
assume that entrepreneurs’ riskiness is constant. I relax this assumption, and I suppose time-varying uncertainty, i.e. the standard deviation being variable over time. A change in borrower’s riskiness, as a demand-side financial shock, impacts directly his credit worthiness and his optimal contract conditions\(^6\). Following Dorofeenko et al. (2008), I assume that the standard deviation \(\sigma_t^\omega\) is defined by the following first-order autoregressive process:

\[
\ln \sigma_t^\omega = (1 - \rho_\sigma) \ln \bar{\sigma}^\omega + \rho_\sigma \ln \sigma_{t-1} + \varepsilon_t^\sigma,
\]

(1)

where \(\rho_\sigma \in (0, 1)\), \(\bar{\sigma}^\omega\) is the unconditional mean of \(\sigma_t^\omega\) and \(\varepsilon_t^\sigma \sim i.i.d.\) with a mean of unity.

In order to run this risky project, entrepreneurs need to rely on the bank for external finance; both of them are supposed to be risk-neutral and are interacting in a perfectly competitive capital market. The borrowed amount is \(P_t(I_t - N_t)\), where \(P_t\) is the price of the consumption good. As mentioned earlier, the entrepreneurs’ net worth is the sum of their labor income after supplying one unit of work to the firms at a nominal wage \(W^e_t\), and their nominal money holdings carried over the previous period, \(M^e_t\), then it is given by:

\[
P_t N_t = M^e_t + W^e_t
\]

(2)

The underlying contract stipulates that entrepreneurs reimburse an amount of \(R^l_t P_t(I_t - N_t)\) to the financial intermediary at the end of the same period and after the realization of the idiosyncratic productivity shock \(\omega_t\), where \(R^l_t\) is the gross interest rate of loan. If an entrepreneur defaults, the financial intermediary expends an amount of \(\mu Q_t I_t\) in order to monitor the project returns, assuming that entrepreneurs may misreport the true value of their production. Therefore, the optimal contract is designed in such a way that entrepreneurs have no incentive to deviate, and they truthfully report their realizations. They will default if and only if their income from selling the capital produced \(Q_t \omega_t I_t\), where \(Q_t\) is the price of capital good, is less than the loan’s repayment i.e. \(R^l_t P_t(I_t - N_t)\), or equivalently, when the realization of \(\omega_t\) is not high enough to reach the threshold value \(\bar{\omega}_t\) defined by

\[
\bar{\omega}_t = \frac{R^l_t P_t I_t - N_t}{Q_t I_t}.
\]

Entrepreneurs’ decision rule is given by:

- If \(\omega_t \in \Omega_t^* (= (0, \bar{\omega}_t))\), the entrepreneur cannot honor his contractual commitments and thus declares bankruptcy. In this situation, the lending intermediary pays the auditing cost \(\mu\) and then confiscates all the returns from the project. That is, the intermediary net receipts are \((\omega_t - \mu)Q_t I_t\) and entrepreneurs receive nothing.

- If \(\omega_t \in \Omega_t^* (= [\bar{\omega}_t, \infty))\), i.e. the entrepreneurs’ revenue from selling their produced capital

\[\text{The impact of borrowers’ risk on investment is extensively studied in theoretical models. See Bloom et al (2007) and Justiniano and Primiceri (2006) for macroeconomic approach.}\]
is more than the repayment, they reimburse the promised amount \(R_t P_t (I_t - N_t)\) and keep the difference that equals to \(Q_t \omega_t I_t - R_t P_t (I_t - N_t)\), for their own consumption purpose.

Using the definition of \(\bar{\omega}_t\), one can derive both entrepreneurial and bank’s expected payoffs:

\[
E^e(\bar{\omega}_t, \sigma_t^w, I_t) = Q_t I_t f(\bar{\omega}_t, \sigma_t^w)
\]

\[
E^l(\bar{\omega}_t, \sigma_t^w, I_t) = Q_t I_t g(\bar{\omega}_t, \sigma_t^w).
\]

Where

\[
f(\bar{\omega}_t, \sigma_t^w) = \int_{\bar{\omega}_t}^{\infty} \omega \phi(\omega, \sigma_t^w) d\omega - \bar{\omega}_t [1 - \Phi(\bar{\omega}_t, \sigma_t^w)].
\]

and,

\[
g(\bar{\omega}_t, \sigma_t^w) = \int_{0}^{\bar{\omega}_t} \omega \phi(\omega, \sigma_t^w) d\omega - \mu \Phi(\omega, \sigma_t^w) + \bar{\omega}_t [1 - \Phi(\bar{\omega}_t, \sigma_t^w)].
\]

\(f(\bar{\omega}_t, \sigma_t^w)\) and \(g(\bar{\omega}_t, \sigma_t^w)\) are interpreted as the fractions of the expected net capital output received by entrepreneurs and the bank, respectively. Note that the cutoff value \(\bar{\omega}_t\) determines the sharing of the expected gross income \(Q_t I_t\) between entrepreneurs and the bank since

\[
f(\bar{\omega}_t, \sigma_t^w) + g(\bar{\omega}_t, \sigma_t^w) = 1 - \mu \Phi(\omega, \sigma_t^w)
\]

So that on average, an amount of \(\mu \Phi(\omega, \sigma_t^w)\) of the produced capital is destroyed by auditing.

The optimal financial contract is given by the pair \((\bar{\omega}_t, I_t)\) which is an interior solution\(^7\) to the following principal-agent problem:

\[
\max_{(\bar{\omega}_t, I_t)} E^e(\bar{\omega}_t, \sigma_t^w, I_t)
\]

subject to

\[
E^l(\bar{\omega}_t, \sigma_t^w, I_t) \geq R_t P_t (I_t - N_t) \quad \text{(i)}
\]

\[
E^e(\bar{\omega}_t, \sigma_t^w, I_t) \geq N_t \quad \text{(ii)}
\]

The solution to the problem is summarized by the following two equations:

\[
1 - \mu \Phi(\omega, \sigma_t^w) + \mu \phi(\omega, \sigma_t^w) \frac{f(\bar{\omega}_t, \sigma_t^w)}{f'(\bar{\omega}_t, \sigma_t^w)} = \frac{P_t R_t}{Q_t}
\]

Equation (3) defines an implicit function \(\bar{\omega}_t(Q_t/P_t, R_t, \sigma_t)\), such that

\[
\bar{\omega}_t, Q/P > 0, \quad \bar{\omega}_t, R < 0, \quad \bar{\omega}_t, \sigma < 0
\]

\(^7\)I suppose that \(\bar{\omega}_t h(\bar{\omega}_t, \sigma_t^w)\) is increasing in \(\bar{\omega}_t\), where \(h(\bar{\omega}_t, \sigma_t^w) = \phi(\bar{\omega}_t, \sigma_t^w) / (1 - \Phi(\bar{\omega}_t, \sigma_t^w))\) is the hazard rate. This condition is satisfied for most of the conventional distributions. In addition, I assume that \(Q_t / (P_t R_t) < 1 / g(\bar{\omega}_t, \sigma_t^w)\). Under these assumptions, the lender’s expected payoff reaches a global maximum in the interior of the support of \(\bar{\omega}_t\).
And,

\[ I_t = \frac{P_tR_t}{P_tR_t - Q_tg(\bar{\omega}_t, \sigma^\omega_t)}N_t \]

\[ \equiv \psi(\bar{\omega}_t, \sigma^\omega_t)N_t \] \hspace{1cm} (4)

With \( \psi_{Q/P} > 0, \quad \psi_R < 0, \quad \psi_{\sigma^\omega} < 0 \)

The entrepreneurial investment is a linear function of the net worth \( N_t \) with a proportionality factor of \( \psi(Q_t/P_t, R_t, \sigma^\omega_t) \) that exceeds 1. In addition, the investment level, is a decreasing function of the entrepreneurs’ riskiness, controlled by the level of their productivity uncertainty \( \sigma^\omega \). The effect of an increase in the productivity uncertainty on investment, as it is shown by equation (4), is an accumulation of two parallel effects. The first one is, with a mean-preserving spread in the distribution of \( \bar{\omega}_t \), the increase of capital price as shown in equation (4) due to the increase of total default costs, pushed up by high levels of \( \sigma^\omega \). The second is due to the increase of the bank’s share of return \( g(\bar{\omega}_t, \sigma^\omega_t) \) further to entrepreneurs’ riskiness augmentation.

According to the equation (4), the investment level decreases when the level of uncertainty increases.

\( \bar{\omega}_t \) defines the amount of consumption goods that entrepreneurs, with a net worth \( N_t \), place into the capital technology. The net aggregate investment (new-capital) supply function, \( I^* \), is simply the entrepreneurial capital output given by equation (4), minus the total amount of capital destroyed by monitoring. Then,

\[ I^*(Q_t/P_t, N_t, R_t, \sigma^\omega_t) \equiv I_t[1 - \mu \Phi(\omega, \sigma^\omega_t)], \]

with \( I^*_{Q/P} > 0, \quad I^*_N > 0, \quad I^*_R < 0, \quad I^*_{\sigma^\omega} < 0. \)

In addition to their investment decision, solvent entrepreneurs need to decide on their consumption level \( C^*_e \), and on their net worth accumulation taking the form of a strictly positive cash holding \( M^e_t \) carried over into the next period. While, cash holding is equal to 0 for faulty entrepreneurs since all their production is confiscated by the financial intermediary. Formally, they maximize their inter-temporal utility

\[ E_0 \sum_{t=0}^{\infty} \gamma^t C^e_t \]

where \( \gamma \in (0, 1) \) is the discount factor, subject to the following budget constraint

\[ M^e_{t+1} = I_tQ_tf(\bar{\omega}_t, \sigma^\omega_t) - P_tC^e_t, \quad \text{if} \quad \omega_t \geq \bar{\omega}_t; \]

\[ M^e_{t+1} = 0 \quad \text{and} \quad C^e_t = 0, \quad \text{if} \quad \omega_t < \bar{\omega}_t. \] \hspace{1cm} (5)

Assuming an interior solution, solving the entrepreneurs’ problem implies the followings Euler

Footnote: Many similar multipliers to \( \psi(\cdot) \) can be found in the literature and in many other models with agency costs. Holmstrom and Tirole (1998) found the same multiplier in their model, called equity multiplier.
equation:
\[ E \left\{ 1 - \gamma \frac{Q_{t+1}}{P_{t+1}} \frac{P_t R_{t+1} f(\omega_{t+1})}{P_{t+1} R_{t+1} - Q_{t+1} g(\omega_{t+1})} \right\} = 0. \] (6)

2.2 Households

The objective of the household is to maximize
\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - L_t) \]

Where \( E_0 \) denotes the expectation operator conditional on time-0 information; \( C_t \) is time-\( t \) consumption; \( L_t \) is time-\( t \) labor and the leisure endowment is normalized to unity, so that \( L_t \in (0, 1) \). \( 0 < \beta < 1 \) is a constant discount factor such that \( \beta < \gamma \): entrepreneurs discount the future more heavily than do households. This impatience is intended to prevent entrepreneurial sector from accumulating wealth and being fully self-financed\(^9\). As mentioned above, the household enters period \( t \) with a nominal money balance equals to \( M^h_t \), carried over from the previous period. \( M^h_t \) is split into deposits \( D_t \), consumption's spending \( P_t C_t \) and investment's spending \( \varphi Q_t I_t \). Therefore, the CIA constraint is given by:
\[ P_t C_t + \varphi Q_t I_t \leq M^h_t - D_t. \] (7)

The capital accumulation process obeys the following law of motion:
\[ I_t = \frac{K_{t+1} - (1 - \delta)K_t}{1 - \mu \Phi(\omega, \sigma_t)} \] (8)

where \( \delta \in (0, 1) \) is the depreciation rate of capital. In the course of each period, the household receives its labor income \( W_t L_t \), the nominal capital income \( r_t P_t K_{t-1} \) and his intra-period deposits’ repayment, including the earned interest, \( R_t D_t \). Since, the household owns the firms and the bank, then it receives cash dividend payment from both of them. Hence, the representative household’s inter-temporal budget constraint is given by:
\[ M^h_{t+1} = W_t L_t + r_t P_t K_t + R_t D_t + \Pi_t^F + \Pi_t^B + M^h_t - (P_t C_t + \varphi Q_t I_t + D_t), \]

where \( r_t \) is the real rental rate of capital.

Formally, the representative household maximizes its utility function subject to the budget and CIA constraints by choosing sequences of consumption, \( \{C_t\}_{0}^{\infty} \), labor and capital supply, \( \{L_t\}_{0}^{\infty} \) and \( \{K_{t+1}\}_{0}^{\infty} \), money demand \( \{M^h_{t+1}\}_{0}^{\infty} \) and deposits, \( \{D_t\}_{0}^{\infty} \), yielding the following first-order

\(^9\)Bernanke et al (1999) used the assumption of finite horizons for entrepreneurs (exponential death) to preclude the possibility of being fully self-financed.
conditions:

\[ U_{C,t} = \lambda_t + \chi_t, \quad (9) \]

\[ U_{L,t} + \lambda_t w_t = 0, \quad (10) \]

\[ \varphi q_t (\lambda_t + \chi_t) = \beta E_t [\lambda_{t+1} r_{t+1} + \varphi(1 - \delta) q_{t+1} (\lambda_{t+1} + \chi_{t+1})], \quad (11) \]

\[ \beta E_t (\lambda_{t+1} + \chi_{t+1}) = \lambda_t E_t (\pi_{t+1}), \quad (12) \]

\[ \lambda_t R_t = \lambda_t + \chi_t, \quad (13) \]

where \( q_t = Q_t / P_t \) is the relative price of capital, \( w_t \) is the households’ real wage and \( \pi_t = P_t / P_{t-1} \) is the inflation rate for the current period. \( \lambda_t \) and \( \chi_t \) denote the Lagrange multipliers corresponding to the budget and the CIA constraints, respectively.

### 2.3 Firms

The firms are engaged in the production of the homogeneous consumption good in a competitive market, using a constant-returns-to-scale production technology. In order to facilitate aggregation, I assume that all firms in the economy are represented by one price-taker firm, maximizing its profit and producing the aggregate output \( Y_t \) using the following technology:

\[ Y_t = A_t F(K_t, H_t, H^e_t), \quad (14) \]

\( K_t \) and \( H_t \) are households’ capital and labor, \( H^e_t \) is entrepreneurs’ labor and \( A_t \) is the aggregate productivity shock. \( A_t \) evolves according to the following law of motion

\[ \ln A_t = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon^A_t, \quad (15) \]

where \( \varepsilon^A_t \sim i.i.d. (0, \sigma^2_A) \), \( \rho_A \in (0, 1) \) is the autocorrelation coefficient and \( \bar{A} \) is the non-stochastic steady state value of \( A_t \).

The optimality conditions for the representative firm are given by

\[ w_t = A_t F_H(K_t, H_t, H^e_t), \quad (16) \]

\[ w^e_t = A_t F^e_H(K_t, H_t, H^e_t), \quad (17) \]

\[ r_t = A_t F_K(K_t, H_t, H^e_t), \quad (18) \]

where \( w^e_t \) is the entrepreneurs’ real wage. The goods and labor markets market-clearing-conditions
are:
\[ Y_t = C_t + C_t^e + I_t, \]
\[ H_t = L_t, \]
\[ H_t^e = 1. \]

2.4 Financial intermediary

The financial intermediary copes with financial operations, controlling all borrowing-lending activities in the economy. At the beginning of each period, its balance is augmented with a new money \((M_t - M_{t-1})\), coming from the monetary authority. In addition, the bank receives deposits \(D_t\) from households. The accumulated cash is loaned to entrepreneurs using the optimal debt contract derived earlier. The financial intermediary has a well defined role because of coordinating lending from risk-averse households to risk-neutral entrepreneurs\(^{10}\) and mitigating the total verification cost by eliminating the duplication of monitoring activity which would emerge in case of direct lending\(^{11}\). The financial market clears when accumulated funds are fully borrowed by entrepreneurs:
\[ M_t - M_{t-1} + D_t = P_t(I_t - N_t), \]

2.5 Monetary policy

The monetary policy is conducted by the central bank through two ways. In the first case, I assume that money is transferred to the banking sector according to a first-order autoregressive process of money growth, given by:
\[ \Delta \ln M_t = (1 - \gamma) \ln \bar{\tau} + \gamma \Delta \ln M_{t-1} + u_t, \]
where \(u_t\) is an i.i.d random variable with \(u_t \sim N(0, \sigma_u^2)\) and \(\gamma \in (0, 1)\) is the autocorrelation coefficient. \(\bar{\tau}\) is the steady-state gross money growth rate.

In the second case, I suppose that the central bank conducts its monetary policy using a Taylor rule. Following Dittmar et al. (2005), I use a backward looking interest rate rule, which is a reaction function to previous inflation rate and output gap. This choice was motivated by many

\(^{10}\)Since they are risk neutral, entrepreneurs will care only about expected returns and bear all the risk. Loans are insured by the financial intermediary which is supposed to be risk-neutral (because of its diversified loan portfolio), then without loss of generality, households can be considered effectively in this context as risk-neutral.

\(^{11}\)Diamond (1984) and Williamson (1987) showed that an equilibrium allocation where financial intermediation activity is performed Pareto dominates the direct lending allocation, under the assumption of "large scale investment project".
considerations. First, most of the empirical models studying inflation persistence use backward-looking Taylor rules and it showed its efficiency in capturing such stylized fact. This will facilitate considerably the comparative analysis. Second, the model’s calibration, with a backward looking nominal interest rate rule and investment subject to CIA constraint, ensures real and nominal determinacy. According to Carlstrom and Fuerst (2001), under this parametrization, the backward-looking rule is necessary for the model’s stability and sunspot equilibria elimination, which are welfare-reducing.

Formally, the nominal interest rate \( R_t \) is settled as follows:

\[
R_t - \bar{R} = \psi_y (\ln Y_t - \ln \bar{Y}) + \psi_{\pi} (\pi_t - \bar{\pi}) + \nu_t, \tag{24}
\]

where:

\[
\nu_t = \rho_R \nu_{t-1} + \varepsilon^R_t, \quad \varepsilon^R_t \sim i.i.d. (0, \sigma^2_R). \tag{25}
\]

\( \bar{R}, \bar{Y} \) and \( \bar{\pi} \) are the steady-state values of nominal interest rate, output level and inflation rate respectively. Money market clears when money supply equals its demand, so that:

\[
M_t = M^h_t + M^e_t. \tag{26}
\]

### 3 Calibration

The model is parametrized to match empirical counterparts of U.S. data. The calibration exercise is standard and based on main empirical estimates used in the existing literature on business cycle fluctuations. Following Carlstrom and Fuerst (1997), the household’s utility function is specified by \( U(C_t, L_t) = \ln C_t + \eta(1 - L_t) \). The constant \( \eta \) is chosen such that the ratio \( L/(1 - L) \), matches its empirical value 0.3, so that \( \eta \) is set equal to 2.78. The annual real interest rate is 4%, implying a quarterly discount rate, \( \beta \), of 0.99. The quarterly depreciation rate of capital \( \delta \) is assigned an estimate calculated by Christiano (1991), which is 0.0212.

The production of the consumption good is supposed to be Cobb-Douglas, with a capital share of 0.36 and household labor share of 0.6399. Thus the entrepreneurial labor share is 0.0001. It is selected arbitrary small so that the labor income plays a minor role in determining the entrepreneurial net worth. Being in line with Prescott (1986), the persistence of the productivity shock \( \rho_A \) is assumed to be 0.95 and its standard deviation, \( \sigma_A \), is equal to 0.0075. The value of \( \bar{A} \) is normalized to unity.

Turning to the monetary policy, there is no consensus in the literature on the central bank’s reaction to inflation nor to its reaction to output. Therefore, we use the value proposed by Taylor for the weight of inflation \( \psi_{\pi} \), to be 1.5. This value guarantees, as mentioned previously...
the model’s nominal and real determinacy as suggested by Carlstrom and Fuerst (2001). However, the weight of output $\psi_Y$ is fixed to 0.19. In the sensitive analysis below, I examine also the case of an output weight equals to zero. The persistence of the monetary shock is calibrated to 0.5 and its standard deviation to 0.125\textsuperscript{12}. Following Christiano and Eichenbaum (1995), the steady-state monetary growth rate is fixed at 1.0119. Concerning the autocorrelation coefficients $\gamma$, it is set to be equal to 0.32.

Some non-standard parameters appear in the financial sector. The distribution of the entrepreneurial productivity, $\Phi(.)$ is assumed to be lognormal with a mean of unity and time-varying standard deviation, $\sigma_\omega$. The steady-state value $\bar{\sigma}_\omega$ is calibrated to its expected value in Carlstrom and Fuerst (1997) which is 0.207. I set the persistence of the borrowers’ riskiness, $\rho_\sigma$, to 0.9. This parameter will be examined below through a sensitivity analysis in order to evaluate the impact of uncertainty’s persistence on the model’s cyclical behavior. I take the annual business failure rate, $\Phi(\bar{\omega})$, to be 2.8%, the approximate value in the data. The entrepreneurs’ rate of discount $\gamma$, is selected to imply an annualized external finance premium of 300 basis points, the average spread between the Moody’s seasoned Baa corporate bond yield and the 3-month treasury bill for the period between 1934 and 2008. Matching these values, $\gamma$ is set equal to 0.958. The bankruptcy rate $\mu$ is calibrated to 0.25 as in Carlstrom and Fuerst (1997).

Taking into consideration these parameter values, the equilibrium of the economy is fully described by the equations (??) – (??), constituting a set of nonlinear-rational-expectation conditions that need to be log-linearized around the non-stochastic steady state. The numerical solution is obtained using the method of undetermined coefficients.

4 Monetary shock and liquidity effect

The first experiment consists of simulating the model for three different cases corresponding to three different values of $\varphi$. The two extreme cases i.e. $\varphi = 0$ and $\varphi = 1$ match the situations where the investment is considered as a credit good and as a cash good, respectively. The third case, $\varphi = 0.5$ is an intermediary situation where a half of households’ investment goods purchases is financed using their cash holdings. The model succeeded to generate the liquidity effect in all cases regardless the value of $\varphi$. An expansionary monetary policy corresponding to a shock of 1 percentage point to money supply, induces a decrease in the nominal interest rate and an increase of investment, output and employment. The intuition is that an increase in money supply was absorbed mainly by entrepreneurs through loan contracts since households are enable to review their saving decisions. The only way to channel the additional money to real economy is to decrease nominal interest in order to enable entrepreneurs to borrow more.

\textsuperscript{12}The same values are used in Dittmar et al. (2005) in a model using a backward-looking Taylor rule.
Figure 1. and figure 2. show that the maximum liquidity effect that can be obtained with the model is when $\varphi = 0$, i.e., when investment is a pure credit good. This condition makes the cash-in-advance constraint more loose and keeps the response of investment to an increase of money supply more flexible. The rest of the experiment was conducted using the version corresponding to investment as a "pure credit good".

I was interested in the impact of asymmetric information in the credit market on the liquidity effect and how it may alter the propagation mechanism of an expansionary monetary shock. I compare in figure 3. and figure 4. the responses of two economies: with and without asymmetric information to a single one-time shock on money growth. Since the total amount of newly injected money is totally absorbed by entrepreneurs because of the incapacity of households to review their portfolio decision, the quantity of capital created by entrepreneurs increases and consequently, the investment increases. This transfer is possible only if the nominal interest rate decreases, a feature that the model was able to replicate. The capital expansion induces a decrease of the relative price of capital, which encourages households to increase their labor supply in order to buy more capital goods. Even though the level of consumption in the first period is fixed by the CIA constraint, households increase their consumption during the subsequent periods because of their income increase. The revenue effect dominates the substitution effect: an increase of the price of the consumption-good stimulated the households' consumption because the increase of the labor income could overcome the negative impact of the expected inflation effect which is a common feature of standard monetary business cycle models. The inflation effect stipulates that, the persistence of the money supply shock over time forces households to take expected inflation into consideration when deciding about their consumption-investment spendings. Therefore, households would shift from consumption (the cash good) to investment (the credit good) and potentially reduce their labor supply. Obviously, it is not the case for the model above.

Fachat (2000) mentioned the existence of a "labor demand effect" following a positive shock to the growth rate of money supply. This effect is summarized by an increase of the labor demand by firms at the steady-state. The increase of investment raises the level of aggregate capital accumulated by households and available for firms at the steady-state, inciting firms to produce more. The model above could replicate this effect also since labor demand increased by around 0.05% after the shock.

The comparative figures show that a model with asymmetric information in the credit market does not generate a more persistent liquidity effect comparing to a model without any asymmetric information, when the monetary policy is conducted using a monetary growth rule. The liquidity effect is transitory for both models. A results that has been shown in most of the previous studies of liquidity effect in a monetary model with flexible prices. The nominal interest rate decrease lasts between 2 and 4 quarters, whether there are credit market imperfections or not. On the other hand, one can see that a monetary model with credit market imperfections experiences a
more important decrease in the nominal interest rate following the positive monetary shock. This result does not allow to draw any final conclusions about the contribution of the credit market informational imperfections to the propagation of the monetary shock, when the monetary policy is a money supply rule, since no clear amplification is observed in the real economy. For this reason, I consider an economy where the monetary policy is conducted using an interest rate rule and I keep working on the model with \( \varphi = 0 \).

The simulation results are reported in figures 5 and 6. The experiment is to realize an expansionary monetary policy by shocking \( \nu_t \) in equation (??) by \(-1\) percentage point.

The results are enhanced and show clearly the importance of the credit market imperfections in amplifying the monetary shocks as well as in improving the liquidity effect persistence. The comparative figures brought to light the role that asymmetric information plays in the propagation of a monetary shock. The liquidity effect lasts more than 30 quarters and the size of the nominal interest rate response as well as the real economy is significantly higher than the responses of a model without asymmetric information in credit market. The intuition is that a fall in the nominal interest rate raises money demand in the economy, so that households and entrepreneurs increase their money holdings. Although their consumption is fixed by the CIA constraint in the first period, households increase it during the subsequent periods. On the other hand, entrepreneurs’ money holding increases their net worth, inducing an increase of the investment and a decrease of the default rate in the economy. The aggregate capital expansion spurs the labor demand and output. Note that the model’s response to a negative shock on nominal interest rate is consistent with the conventional wisdom regarding the liquidity effect and the role of credit market imperfections in monetary shocks’ amplification and persistence is quite obvious.

5 Technology shock

The model’s response to a positive technology shock is quite familiar. A productivity shock generates a spike in investment, employment and output followed by a return to normal as productivity starts declining back to its steady state.

The most important feature of the asymmetric information model’s response to the technology shock is the hump-shaped response functions for investment, which is not the case for output: the model is subject to Cogley-Nason criticism. As Cogley and Nason (1995) demonstrate, the dynamics of investment, employment and output are all inherited from the autocorrelation structure of the technology shock. Their study documents that hump-shaped response of output to a transient shock is consistent with U.S. times series and that standard RBC models are unable to deliver this hump-shaped behavior because of their weak internal propagation mechanism.
As shown by figures 7 and 8, the model with asymmetric information in the credit market is able to replicate the hump-shaped investment. In fact, investment increases for several quarters after the shock and reaches its maximum after 4 quarters. I consider two different cases: one is the baseline case where the output weight in the Taylor rule is 0.19 and the other case is where I assume that output weight is null.

In the case where the output weight is equal to zero, the distinctive feature of U.S. data, the so-called mean-reversion, is delivered by our model through the following mechanism: the positive technology shock increases the households' investment demand, which drives down their consumption, raises their labour supply and pushes up the relative price of capital. Since the productivity shock decreases default rate and increases the funds returns and in order to take advantage of the prices? increase (relative price of capital and entrepreneurial real wage), the risk-neutral entrepreneurs increase their consumption, reduce their net worth and increase capital supply.

Due to the high autocorrelation of the technology shock, output, investment and labour demand move slowly back to normal.

when the weight of output in the Taylor rule turned to be positive, the model's response to the technology shock is different and it converges to the opposite response for some variables. The responses of real variables: output, investment, employment and consumption and aggregate capital are amplified. On the other hand, putting more weight on output generates a negative response of nominal interest rate to a technology shock. This result was found by Kydland et al (2005). The intuition is that the decline of the expected inflation dominates the output increase after a technology shock. I find here also the same result of Kydland et al (2005) when a positive technology shock is followed by a significant reduction in inflation rate when the monetary authority follows a Taylor rule with a high coefficient on output.

6 Financial shock

The increase of uncertainty in the economy is generated by an increase of the standard deviation of the entrepreneurial idiosyncratic shock. The uncertainty is supposed to be time-varying and it is presented by the law of motion given by equation (??). The results of an increase of $\varepsilon_t$ by a 1 percentage point are presented in figures 9. and 10.

A greater uncertainty induces an increase of the bankruptcy rate in the economy following an increase of bankruptcy (agency) costs. Therefore, the investment projects' rate of return decreases, forcing entrepreneurs to reduce their investment and consequently, their consumption. Entrepreneurs' reaction causes output to fall significantly.
The experiment is realized for three different levels of the uncertainty shock persistence. Note that all variables' responses, except for the relative price of capital, are increasing in the degree of persistence. Because of the important increase of inflation, the relative price of capital response is altered. Note that high persistence has a dramatic qualitative effect on entrepreneurial net worth. This result is due to the interaction between entrepreneurial consumption, investment and its net worth accumulation. When the uncertainty increase is persistent, financial intermediaries have no incentive to expand their lending activity since the default rate increases significantly in the economy. Therefore, external funding becomes more expensive and entrepreneurs are forced to decrease their capital production, which explains the increase of capital price. The decrease of entrepreneurial expected income reduces their money holdings and their net worth.

The countercyclical characteristic of the bankruptcy rate which has been found to be procyclical in Carlstrom and Fuerst (1997) is noteworthy. Their intuition was based on the idea that a positive technology shock increases output and demand for capital, which increases lending activity and hence, increases the default rate. Dorofeenko et al (2008) found the same result of the current model, which implies that second moment effect plays an important role and may alter correlations over the business cycle.

7 Conclusion

In this paper I developed a flexible prices, monetary business cycle model with dynamic asymmetric information in the credit market. The ex-post moral hazard in lending created by the costly state verification framework allowed for more persistence in the liquidity effect and improved the propagation mechanism, especially under a Taylor rule. Another important feature of the model is that its response to a technology shock replicated the hump-shaped investment observed in U.S. data, as well as the negative response of inflation when the central bank puts more weight on output in the Taylor rule. While monitoring costs produce amplification, a stronger propagation and a significant persistence; and the role of the financial intermediary is well justified, the assumption that the costly state verification covers all public capital ownership in the model is unrealistic. This assumption does not distinguish between equity and debt investments since they are both subject to state verification. Therefore, the costly state verification framework adopted in the actual model and in previous studies seems to lead to ?wrong? conclusions, since it presents a stark contrast to actual experience. A typical capital investment involves very complex set of contingent obligations and payments, all absent from the standard loan contract adopted in the current context. An interesting extension of the actual work is to assume that households are more optimistic than the financial intermediary. In other words, agents should exhibit diverse probability beliefs in the financial market about entrepreneurial productivity. This new assumption is supported by a vast empirical evidence. In fact, if potential entrepreneurs have different beliefs about potential investment projects, one would expect that investors who
are pessimistic about the outcomes of potential projects would generally choose not to invest at all. Moreover, this new case of divergence in probability beliefs is a relatively simple setting where an optimal contract can be derived and solved numerically.

References


Fig. 1. Impulse responses to an expansionary monetary shock (1)-Money growth rule
Fig. 2. Impulse responses to an expansionary monetary shock (2)-Money growth rule
Fig. 3. Impulse responses to an expansionary monetary shock (1) - Money growth rule, $\varphi = 0$
Fig. 4. Impulse responses to an expansionary monetary shock (2)-Money growth rule, \( \varphi = 0 \)
Fig. 5. Impulse responses to an expansionary monetary shock (1)-Taylor rule
Fig. 6. Impulse responses to an expansionary monetary shock (2)-Taylor rule
Fig. 7. Impulse responses to a productivity shock in two economies with different Taylor rule specifications (different output weights): $\psi_Y = 0$ and $\psi_Y = 0.19$. 
Fig. 8. Impulse responses to a productivity shock in two economies with different Taylor rule specifications (different output weights): $\psi_Y = 0$ and $\psi_Y = 0.19$. 

28
Fig. 9. Impulse responses to an uncertainty shock in high ($\rho_\sigma = 0.93$), medium ($\rho_\sigma = 0.5$) and low ($\rho_\sigma = 0.1$) persistence economies (1)
Fig. 10. Impulse responses to an uncertainty shock in high ($\rho_\sigma = 0.93$), medium ($\rho_\sigma = 0.5$) and low ($\rho_\sigma = 0.1$) persistence economies (2)