Optimal Monetary Provisions in Plural Forms Franchise Systems*

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Abstract

In this paper, we study the optimal monetary provision of dual distribution systems in franchise contracts. In a double moral hazard environment, we show that optimal monetary provisions are interdependent, and the relationship between the royalty and commission rate is negative. Moreover, the existence of correlation between market events makes the optimal provision for both units closer to efficiency, and this gives a rational for the usage of dual distribution system. Finally, most of our theoretical predictions are supported by the empirical findings in terms of risk aversion, effectiveness of effort and demand characteristics.

Keywords:
Franchising, dual distribution, royalty rate, commission rate, moral hazard.

JEL codes:
L14, D82

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1 Introduction

In this paper we study the contractual design in franchising networks. Following the definition of Blair and Lafontaine (2005), franchising is based on a contractual relationship between two independent firms, by which the franchisor allows the franchisee to use his brand name and his business format or know-how, in exchange of an economic compensation in the form of a royalty rate and/or an up-front fee. This form of organization is present in many economic sectors and it is internationally widespread. Thus, Brazilian franchised sector increased sales by 20% between 2009 and 2010 (Brazilian Franchise Association, 2010), in United States franchising represents 40.9% of the retailing sector (International Franchising Association, 2011). In situations where the franchise contract exists, this is characterized mostly by a dual or mixed distribution. This means that a franchisor will seldom organize his activity with only independent units (franchised), but will complement his activity with units which are company owned. In table below, we represent the organizational structure of the franchise networks in different countries and we see that the coexistence of both types of retailers is the most frequent organizational form. The theoretical explanations for a mixed system abound, and it comprises theories from incomplete contracts to signaling and coping with market externalities.

This paper builds upon Bhattacharyya and Lafontaine (1995). They study a situation of double moral hazard between franchisor and franchisee, in which it is shown the optimality of uniform contract. The lower monetary provisions (royalty rate) are associated to high-powered

<table>
<thead>
<tr>
<th>Country</th>
<th>Number of Networks</th>
<th>Only Franchised Units</th>
<th>Only Owned Units</th>
<th>Dual Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil (2012)</td>
<td>202</td>
<td>17%</td>
<td>6%</td>
<td>77%</td>
</tr>
<tr>
<td>France (2007)</td>
<td>307</td>
<td>30%</td>
<td>5%</td>
<td>65%</td>
</tr>
<tr>
<td>Venezuela (2012)</td>
<td>217</td>
<td>13%</td>
<td>20%</td>
<td>67%</td>
</tr>
<tr>
<td>United States (2012)</td>
<td>94</td>
<td>16%</td>
<td>10%</td>
<td>74%</td>
</tr>
</tbody>
</table>

Table 1: Type of distribution in Franchise Contracts. From: Brazilian Franchise Association (2012), INSEE (2007), Front Consulting Group and Cámara Venezolana de Franquicias-Profranquicias (2012) and Bond’s guide (2012).
incentives in order to get all the potential of local market knowledge of the franchisee. So, the principal could use the monetary provisions as incentive in order incentive the agents (to avoid moral hazard problem), as establish for agency theory. The main difference between the models is that in our model we introduce the assumption of mixed network as the model of Bao C. and Tao Z. (2000) and starting from that, we try to understand how to encourage both agents to achieved the best non-observable effort, in a risk environment associated to the variation of the sales in the unit.

In this paper we do not aim at providing an exhaustive theory of the dual system but we assume it as given. We consider a situation where a franchisor has developed a new idea and before delegating part of the business, it has developed his activity through an owned outlet. Once the product has matured in the market, the franchisor has delegated some of its sales to an independent outlet, and hence, in a certain point in time the sales of the structure is undertaken by both owned and independent units. With a dual structure, we are interested in determining how the optimal monetary provisions of this dual system look like. At this regard, we make use of the agency theory and we work in an environment where the volume of sales of each unit is affected by a non-verifiable effort and a random component. Since the effort cannot be made part of the contract, the equilibrium monetary provisions will aim at giving incentives to the retailing units to exert valuable effort.

We show that the equilibrium monetary provisions for the independent and the owned unit - royalty and commission rate - are interdependent and a negative relationship between them exists. As a result, in a situation where optimality requires that the commission rate of the owned unit is high, would entail “ceteris paribus” a lower royalty rate for the franchisee. Moreover, we show that in situations where the different markets are affected by similar shocks such that there exist a correlation between the sales in both markets, the equilibrium provisions are closer to optimality. Hence, it is cheaper for the franchisor to provide incentives for effort exertion in the system. This last result gives us a rationale for the existence of a dual system in situations where the variability of sales in the market with the owned unit gives us more information about the variability of sales in the other market. Hence, agency theory per se can explain the existence of a dual system.

The remaining of the paper is organized as follows. After a short literature review, we present the formal model in section (3). We then proceed to the formal analysis in section (4).
In order to study the interdependencies between the rates and how the equilibrium monetary provisions differs from optimality, we begin by introducing a benchmark where the provision of effort can be part of the contract designed by the franchisor. We respectively analyze situations where the market events from the different units are either interrelated or not. In section (5), we conclude and give suggestions for further research. All the proofs are relegated to the appendix.

2 Literature review

A wide empirical and theoretical literature has analyzed the presence of mixed networks in franchising\(^1\). Among the reasons that explain dual distribution we can mentioned: the cost of control, the signaling by means of company outlets, the superiority of it over the forms of franchising governance structure -the networks predominantly franchised (whollyfranchised) and predominantly company owned-units (whollyowned)- and the complementary between the two types of units.

An essential determinant of the extent of franchising undoubtedly is the cost of monitoring and maintaining control over a system (Rubin 1978). The managers (company owned) have to be closely monitored, because his incentive is related, but not strongly, to the interests of the franchisors; while the franchisee has a stronger incentive to align his interests with the franchisor, that would reduce shirking and other forms of opportunistic behavior by the employees. When monitoring becomes more difficult, costs increase, which in turn tends to lower the number of franchisee-owned units. Specifically, control and monitoring costs relate to the behavior of system members, including individual marketing activities and the cleanliness of an outlet (Brickley and Dark 1987). Control costs rise disproportionately with the increasing growth of the system, especially as the franchisor penetrates into more distant territories. However, other research interprets control costs differently in relation to the remuneration of salespeople (Anderson and Schmittlein 1984), which may apply to franchising as follows: if sales data provide poor indicators of the branch operators efforts, because of stochastic influences, the need for monitoring and control increases, and the associated

\(^1\)Mixed networks, plural form organization or dual distribution network refers to the simultaneous presence of both franchised and company-owned outlets in the same network.
costs rise. In this case, the system may trend toward using branch operators, who must be monitored but are subject to a comparatively direct influence within the framework of an employer-employee relationship. Conversely, if sales data offer good approximations of sales effort, the system should include more franchisee-owned units.

Even if franchising reduce the monitory cost, the franchisor should be able to manage the lost of control of the chain. Dual distribution is a way that allows to the franchisor reduce the monitory cost, having still the control of some units (company owned-units). Behind that, the question of what is the ideal proportion of each type of units appears. Burkle and Posselt (2008), concerned with this issue, developed a theoretical model base on risk based theory. They demonstrated formally that the proportion of franchisees depends on cost of control and risk. At the optimal the proportion of franchisees would reduce with a low costs of control, which favoring the presence of company owned-units.

Other interesting explanation to existence of dual distribution is that proposed by Gallini and Lutz (1992), who assuming that there is two type of franchisors (good and bad), demonstrate formally that good type franchisors can signal their type, and therefore provide relevant information to the future franchisees. The signals device are organizational and contractual forms that make the franchisor’s revenue highly dependent upon the performance of the business concept. These signaling devices are: the presence of company-owned outlets in the network in addition with the franchised units, in other words “dual distribution”, and a high level of royalty rate. Dual distribution is analyzed as a complementary signaling mechanism, the franchisor being directly involved with the presence in the network of company-owned retail units. Lafontaine (1993) using US data and the predictions deriving from Gallini and Lutz (1992)’s seminal theoretical model of signal in franchising demonstrated that signaling theory is not quite appropriate to study franchising.

Later, Hendrikse and Jiang (2011) based on incomplete contracting theory develop a property rights model, in which they proposed the superiority of dual distribution over the others forms of governance. At this regard Perrigot et al. (2009) found evidence that in hotel system chains, the use dual distributions as governance form, is more efficient than the others. This efficiency is due to dual distribution offers a dynamic of learning into the chain, Sorenson O. and Sorensen B. (2001), which allow the franchisor adapts faster to the constant change of the market. In addition, the franchisors can implement economies of scale depending on the
economic sector. The importance of historical dynamics in a chain is supported empirically also by Mitsubishi *et al.* (2008).

In line with this, Bao C. and Tao Z. (2000) study dual distribution adapting the Holmstrom and Milgrom (1991)’s multitasks model. They draw attention to the complementarities between company-owned units and independent retailers coexisting in the same network. In this model the agent can do both tasks available, the sales and the promotion of brand name. While the first task contributes solely to the retailing unit, the second contribute to the network. So, the task in the model are complements and the franchisor need to encourage their agents to ensure that activities are done. Thus, the franchisee would prefer to put his effort in sales, because his revenue depends on it, while the manager of owned-unit would prefer to do the second task, because his salary is fixed. Therefore, with dual distribution, each type of downstream unit is devoted to a specific task; owned units being more involved in the promotion of the common brand while franchised units would be more involved in sales efforts.

Dual distribution allows to the franchisor take advantage of special features of each type of unit. Sorenson O. and Sorensen (2001) support the idea of complementary into the network. Routines and practices will be transferred from the franchisor to franchisee, hence, managers of company owned-units implement and standardize the practices. The franchisee based on these routines, brings his own experience and knowledge about the market, developing new routines and adapts those existent, which allows to franchisor to have a better knowledge about the market. In others words, the franchisors have the opportunity to have a feedback of learning that allows to develop new techniques and improves the existent ones.

As we have seen the idea of complementary is well present in the empirical and the theoretical literature. This literature allows to better understand the presence of both types of units in the same chain. In our paper we assume that the dual system is given, and starting to this point we study how the optimal monetary provisions are determined.

The existing literature about monetary provisions has only considered the pure franchise systems. At this regard, the existing theory has developed from the seminal article of Mathewson and Winter (1985) devoted to the interest of profit-sharing contracts in distribution. This notion was augmented by Lal (1990), Bhattacharyya and Lafontaine (1995), Brickley (2002). This literature shows that the royalty rate can be used as an incentive mechanism
in a context of bilateral moral hazard in a two-sided incentive problem. The royalty rate is an ongoing payment variable usually expressed as a percentage of the franchisee’s turnover. The assumption of two-sided incentive problems seems to be sufficient to theoretically justify the practice of using royalty rate in franchising. The share parameter (the royalty rate) determines ongoing payments to the franchisor which motivate its efforts to promote the brand and the network.

As said above our paper targets at concealing both strands of the literature and it aim at studying optimal monetary provisions of a dual distribution context. We analyze how those provisions are determined and the interdependencies among them.

3 Model

We consider a double moral hazard model with an exogenous market structure composed by a franchisee, denoted by (f), and a company owned unit that we denote by (m). Each unit has territorial exclusivity and hence there is no direct interaction in the market place. We further assume that the retailing units are price takers and they can only influence demand or the volume of sales through a non verifiable effort. The structure of the organization is represented in the figure and our objective is to establish the optimal monetary provisions of the system, that are the royalty and commission rate.

![Diagram]

Franchise
Chain

Operated by a
franchisor (p)

Franchise-
Unit

Operated by a
franchisee (f)

Incentives depend on:
· Royalty rate (r)
· Fee (F)

Owned-
Unit

Operated by a
manager (m)

Incentives depend on:
· Commission Rate (y)
· Wage (w)

We have a static game that takes place in four stages. It stage 1, the franchisor designs the contract for each of the units and each one of them decides whether to accept or reject the contract. The game ends if none of the retailers accepts. In stage two, all the players
undertake non-verifiable effort which affects the volume of sales in the market. After the
effort is provided, the nature determines the state of the world, which as it is described below
the nature represent random shocks that affect the demand realization in any of the markets
independently. In the last stage of the game, the outcome is realized and the payoffs are
executed according to the terms of the contract. Because the effort in our game is not
observable and due to the the random realization of demand the effort cannot be part of the
contract. Hence, the objective of the franchisor is to design a contract that gives incentives
to the agents to undertake effort. The timeline of the game is summarized below.

<table>
<thead>
<tr>
<th>The franchisor</th>
<th>The franchisee</th>
<th>The agents</th>
<th>Nature</th>
<th>Outcome and payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>designs the contract</td>
<td>and the manager</td>
<td>and the principal</td>
<td>determines the state of the world</td>
<td>and payoffs</td>
</tr>
<tr>
<td>· Monetary clauses that encourage the agents’ effort</td>
<td>· accept</td>
<td>· supply non-verifiable effort</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>· reject</td>
<td></td>
<td>· Double moral hazard</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>· End if both agents do not accept the contract</td>
</tr>
</tbody>
</table>

3.1 General description

The principal and agents in our game are risk-averse with constant risk aversion. Their utility
is expressed by $u(I) = -\exp(-\rho I)$, where $I$ stands for the income and $\rho$ is the coefficient
of absolute risk aversion. With risk aversion, the objective of each player is to maximize the
certainty-equivalent income

$$CE = E(I) - R,$$

where $E(\cdot)$ is the expectation operator and $R$ stands for the risk premium. Because all players
have constant absolute risk aversion, the risk premium equals

$$R = \frac{\rho}{2} Var(I),$$

and this accounts for the coefficient of absolute risk aversion $\rho$ and the variability of income $Var(I)$.

To generate income for the system each retailer sells the product of the franchisor, who
produces at no cost. We assume that the selling price is normalized to 1 and to generate
sales in their outlet each retailer unit needs to undertake costly effort $e \geq 0$. The franchisor is also able to increase the sales of both outlets by undertaking effort. Apart from the effort of each player, the volume of sales is affected by a random component $\epsilon$, which stands for a random demand shock that none of the players is able to control. This variability precludes the franchisor from inferring the effort form the observed volume of sales and gives scope to a moral hazard problem. The number of sales of each units is then:

\[
S_f (e_f, e_p) = \alpha e_f + \frac{\delta e_p}{2} + \epsilon_f, \tag{3.1}
\]

\[
S_m (e_m, e_p) = \beta e_m + \frac{\delta e_p}{2} + \epsilon_m, \tag{3.2}
\]

for the franchisee and the manager respectively, where $\alpha, \beta$ and $\delta$ are exogenous demand parameters which represent the sensitiveness of effort to the volume of sales. The sales of each outlet is only affected by his own effort and by the effort of the franchisor which equally benefits both outlets. Hence, there are no effort spillovers, and the effort undertaken by the franchisor works as a public good for the system and it is undertaken to promote the network. The random variable $\epsilon_j$ follows a normal distribution $\epsilon_j \sim N(0, \theta^2_j)$ for $j = f, m$, and we assume that the cost of effort is increasing and convex and it is the same for all the players.

\[C(e) = \frac{e^2}{2}.\]

Introducing this cost of effort to the utility, we obtain that the certain equivalent for each player is

\[CE_i = E(I_i) - \frac{\rho_i}{2} Var(I_i) - \frac{\epsilon_i^2}{2} \quad \text{for } i = p, m, j.\]

Regarding the parameters of the model, we make the following assumptions that will be used later on for the analysis of the model

**Assumption 1.** The parameter of lost aversion is such that $\frac{\rho_f}{\rho_p} \geq \frac{\alpha^2(\delta^2 + 4\beta^2)}{2\delta^2 \beta^2}$. 

**Assumption 2.** The variability of sales of the manager with respect to the franchisee is

\[\frac{\theta_m^2}{\theta_f^2} \geq \frac{4(\rho_m + \rho_p) - \rho_p \delta^2 (\rho_m + 2\rho_p)}{\rho_p \rho_f \delta^2}.\]

**Assumption 3.** The influence of the effort by the franchisor on the total sales $\delta^2 \leq \frac{4(\rho_m + \rho_p)}{\rho_p (\rho_m + 2\rho_p)}$. 

9
The aim of the franchisor is then the design of a contract to promote incentives for the different units to undertake costly effort such that the certain equivalent of the former is maximized. We now proceed with the formal analysis of the model.

4 Analysis

As shown by Bhattacharyya and Lafontaine (1995), without loss of generality, we can restrict our analysis to linear contracts. Hence, we introduce the following assumption.

**Assumption 4. We consider lineal contracts of the form \{F, x\}. Where F is a fixed payment and x is an output rate royalty scheme.**

Therefore, by using linear simple contracts, the risk premium for the parties is:

\[ R_f = \frac{\rho_f}{2} \text{Var}((1 - r)\theta_f), \] (4.1)

\[ R_m = \frac{\rho_m}{2} \text{Var}(y\theta_m), \] (4.2)

\[ R_p = \frac{\rho_p}{2} \text{Var}(r\theta_f + (1 - y)\theta_m). \] (4.3)

Hence, the risk premium associated with the franchisee is the percentage of revenues he receives \((1 - r)\), and the variance in sales represented by \(\theta_f\). The manager’s risk premium captures the percentage of revenues he receives \((y)\) and the variance in sales \(\theta_m\). Lastly, the franchisor’s risk premium \((r, (1 - y))\) uses the two percentages that he receives and the variance in sales of both units. By introducing this expressions, we obtain that the certain equivalent for the franchisor is

\[ CE_p = r \left( \alpha e_f + \frac{\delta e_p}{2} \right) + (1 - y) \left( \beta e_m + \frac{\delta e_p}{2} \right) + F - \frac{e_p^2}{2} - w \]

\[ -\frac{\rho_p}{2} \left( r^2 \theta_f^2 + (1 - y)^2 \theta_m^2 + 2 \text{Cov}(\theta_f, \theta_m(1 - y)) \right). \] (4.4)

The first and the second part of the expression represents the income provided by the franchisee and the manager and depends on the effort provided by both respectively. The last

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2The evidence concerning the popularity of linear payment rules is rather extensive. Several authors, including Arrow (1985), McAfee and McMillan (1987), Holstom and Milgrom (1987), and Milgrom and Roberts (1992), have noted this tendency toward relatively simple, and often linear, rules.
component is the cost of the franchisor and it is composed by the risk premium and the cost of effort.

The certain equivalent by the franchisee is

\[
CE_f = (1 - r) \left( \alpha e_f + \frac{\delta e_p}{2} \right) - F - \frac{e_f^2}{2} - \frac{\rho_f}{2} (1 - r)^2 \theta_f^2,
\]

(4.5)

and this depends negatively on the royalties \( (r) \) and the fixed fee \( (F) \) that he has to pay to the franchisor. He is also negatively affected by the risk premium and the cost of effort.

Finally, the certain equivalent of the manager

\[
CE_m = w + y \left( \beta e_m + \frac{\delta e_p}{2} \right) - \frac{e_m^2}{2} - \frac{\rho_m}{2} y^2 \theta_m^2,
\]

(4.6)

depends positively on both the wage \( (w) \) and the commission rate \( (y) \) received from the franchisor, and he is negatively affected by the risk premium and the cost of effort.

We proceed by presenting the franchisor’s problem. In any franchise systems, the franchisor is typically responsible for promoting and advertising the chain. The franchisee and manager, are responsible for managing the outlets on a day-to-day basis. The effort of all the parties involved directly affects the performance of the outlet. However, the intensity of the effort is not easily monitored. Therefore, the franchisor’s problem equals to

\[
Max_{[r,y,w,F,e_p]} CE_p = \left( \alpha e_f + \frac{\delta e_p}{2} \right) + F + (1 - y) \left( \beta e_m + \frac{\delta e_p}{2} \right) - w - \frac{e^2_{e_p}}{2}
\]

\[
\underbrace{\text{Rev. from franchised-unit}} \quad \underbrace{\text{Rev. from owned-unit}} \quad \underbrace{\text{Cost}}
\]

\[\underbrace{- \frac{\rho_p}{2} \left( r^2 \theta_f^2 + (1 - y)^2 \theta_m^2 + 2Cov(\theta_f, \theta_m(1 - y)) \right)} \quad \text{Franchisor risk aversion}
\]
subject to:

\[ (i) \quad CE_m = w + y \left( \beta e_m + \frac{\delta e_p}{2} \right) - \frac{e_m^2}{2} - \frac{\rho_m}{2} y^2 \theta_m^2 \geq 0 \quad (IR_m) \]

\[ (ii) \quad CE_f = (1 - r) \left( \alpha e_f + \frac{\delta e_p}{2} \right) - \frac{e_f^2}{2} - \frac{\rho_f}{2} (1 - r)^2 \theta_f^2 \geq 0 \quad (IR_f) \]

\[ (iii) \quad \frac{d(CEp)}{de_p} = r \frac{\delta}{2} + (1 - y) \frac{\delta}{2} - e_p = 0 \quad (IC_p) \]

\[ (iv) \quad \frac{d(CEF)}{de_f} = (1 - r) \alpha - e_f = 0 \quad (IC_f) \]

\[ (v) \quad \frac{d(CEm)}{de_m} = y \beta - e_m = 0 \quad (IC_m). \]

Conditions (i) and (ii) are the participation constraints and need to be satisfied to induce the manager and the franchisee to accept the contract. We normalize to zero the outside option of both agents. The last three conditions stand for the incentive constraints which state the amount of effort that will be supplied in equilibrium. Since our variable effort is continuous, we have taken the first-order approach. In what follows, we analyze some different cases regarding the information structure and the relationship of the market events.

### 4.1 Case 1: Symmetric information

We present the case where efforts are observable and they can be part of the contract. In this case, the optimal monetary provision only depends on the relative risk aversion and it is introduced in the following proposition.

**Proposition 1.** Under complete information, optimal monetary provisions only depend on the degree of risk aversion of the players.

\[ r^* = \frac{\rho_f}{\rho_p + \rho_f}; \quad (1 - y^*) = \frac{\rho_m}{\rho_p + \rho_m} \]

Because effort is now part of the contract of the franchisor, the distribution of risk between the players only depend on the relative absolute risk aversion of the party accepting the contract and the franchisor. Then, optimality entails that the lower is the risk aversion of one of the parties with respect to the other, the more risk he will bear. Since there is no need to provide incentives to induce effort, the franchisor’s maximization problem only considers
the participation constraints for both agents and the calculations to obtain the monetary provisions can be found in the appendix.

The royalty and commission rates act as insurance devices for the agents and principal. Suppose that the franchisor’s risk aversion increases. In this situation, the franchisor prefers a safer device to extract the surplus of the franchised unit. Therefore, the franchisor sets a higher up-front fee rather than a higher royalty rate, as the second device depends on sales. In addition, to extract income from the owned-unit, the franchisor would prefer to pay a low wage with a higher commission rate to force the manager take on the risk. Now suppose that the franchisee’s risk aversion increases. To obtain an insurance mechanism, the franchisee prefers to pay a higher royalty rate and a low up-front fee, as this is a way to share the sales risk with the franchisor. Furthermore, when the manager’s risk aversion’s increases, he will prefer to be compensated with a higher fixed wage and lower commission, as the final wage would depend little on the sales in the downstream market.

4.2 Case 2: Asymmetric information and no relation of market events

We present the case where efforts are not observable and they cannot be part of the contract. In this case our game is subject to moral hazard and the contract provided by the franchisor have to provide incentives in order to promote effort. Moreover, the correlation between market events is zero. In this case, the optimal monetary provision not only depends on the relative risk aversion but they have to vary according to the output obtained. The monetary provision of the contract are stated in the following proposition

Proposition 2. With non contractible effort and uncorrelated events \( \text{Cov}(\theta_f, \theta_m) = 0 \), the monetary provisions are interdependent and are equal to.

\[
 r^{AU} = \frac{4\theta_f^2 \rho_f + \delta^2 (1 + y^{AU})}{4\theta_f^2 (\rho_p + \rho_f) + 4\alpha^2 + \delta^2},
\]

\[
 (1 - y^{AU}) = \frac{4\theta_m^2 \rho_m + \delta^2 (2 - r^{AU})}{4\theta_m^2 (\rho_p + \rho_m) + 4\beta^2 + \delta^2}.
\]

Moreover,

i) they are negatively related, and

ii) they are lower than the situation where the effort is contractible, i.e. \( r^* \geq r^{AU} \) and
Now effort is not contractible and cannot be part of the contract offered by the franchisor. Therefore, the franchisor’s maximization problem now considers the participation constraints and the incentive constraints for both agents. The calculations can be found in the appendix.

The royalty and commission rate depend on the relative absolute risk aversion of the party accepting the contract and the franchisor as in the previous case. In addition, they are depending on the efforts provided for the agents and the franchisor. Thus, suppose that the manager is friendlier to customers, which could generate more sales; in other words, the effect of the manager’s effort on downstream sales $\beta$ increases, and hence principal should incentivize him through an increase in salary (commission rate). The same is true for the franchisee, except that he will be incentivized through a reduction in the royalty rate (share parameter). Now suppose that the franchisor’s advertising effort is more effective in attracting customers, which would result in increased sales. In this case, franchisor would attempt to receive the most compensation, which is why the royalty rate increases and the commission rate decreases. What is interesting in this case is that the monetary provisions are interrelated even if the market shocks are not correlated and this comes from the fact that the effort of the principal works as a public good for both retailing units. We also find the standard result in the literature that by making the effort not contractible the principal offers more incentive related contracts by making both the royalty the commission rate to decrease. This generates an inefficient distribution of risk but allows for promoting incentives to induce effort by the agents.

4.3 Case 3: Asymmetric information and correlation of market events

In addition to asymmetric information, we assume now that the market events are correlated. The covariance is equal to

$$\text{Cov}(\theta_m(1 - y), (\theta_f r)) = \theta_f \theta_m r (1 - y),$$

and it is positive if only the monetary provisions have a positive royalty and commission rate. If it was the case that the payments by one of the agents was invariant in the volume of sales the covariance would be zero. By introducing this covariance to the problem, we obtain that

$$(1 - y^*) \geq (1 - y^{AU}).$$
the results are introduced in the following proposition.

**Proposition 3.** With moral hazard and correlated events $\text{Cov}(\theta_f, \theta_m) \neq 0$, the optimal provisions are interdependent and are higher than with no correlation.

\[
\begin{align*}
    r^{AC} &= r^{AU}(y^{AC}) - \frac{4\rho_p \theta_f \theta_m (1 - y^{AC})}{4\theta_f^2 (\rho_p + \rho_f) + 4\alpha^2 + \delta^2} \\
    (1 - y^{AC}) &= (1 - y^{AU}(r^{AU})) - \frac{4\rho_p \theta_f \theta_m r^{AC}}{4\theta_m^2 (\rho_p + \rho_m) + 4\beta^2 + \delta^2}
\end{align*}
\]

The maximization problem is the same as the one before but now we introduce the covariance to the problem and we get interdependent and are higher than with no correlation between the monetary provisions.

When the franchisor sets the royalty rate at the optimal level, which is in part determined by the commission rate, there is a direct relationship between the two rates. That is, there is an inverse relationship between the commission rate that the franchisor receives from the owned-unit and the royalty rate he receives from the franchised unit. A similar result is obtained when the franchisor sets the commission rate at the optimal level, which is in part determined by the royalty rate. If the optimal $y^*$ is substituted into $r^*$, the commission rate will also depend on the impact of the franchisee’s effort and vice-versa. Thus, a relationship exists between the two rates, as they are related to the same network. This result suggests that the units are complementary, which is consistent with Bai and Tao (2000), Sorenson O. and Sorensen (2001)\(^3\).

\(^3\)In addition we use the Coase Theorem (Transaction cost theory), where the value maximization principle is an allocation among a group of people whose preferences display no wealth effects is efficient only if it maximizes total value affected parties. Applying this theorem, the function to be maximized is the certain equivalent of the network, instead of the franchisor. The results are identical to the case 3. The maximization of the franchisor’s certain equivalent is in consequence maximizing equivalent of the network to the maximization of certain equivalent of the network. The explanation is that in the franchisor problem (under agency theory vision), we add the incentive constraints ((iii), (iv) and (v) in the franchisor’s maximization problem); and in the network problem each agent and principal maximize his own certain equivalent function, which is maximizing. Therefore, under agency theory or transaction cost theory the results for royalty rate and commission rate, remain still the same.
5 Conclusion

The existing literature on franchising have extensively studied the existence of mixed distribution networks and have established different theories explaining its existence. In the current paper we have indirectly established a rational for the existence of a mixed network through the determination of the optimal monetary provisions. In our model, the interdependencies among monetary provisions makes that whenever the markets events are correlated, it is cheaper for the principal or the franchisor to provide incentives to the agents to induce valuable effort. In other words, the monetary provisions are closer to a situation where the effort can be part of the contract.

Moreover, to the best of our knowledge we are the first to characterize the optimal monetary provisions in a mixed distribution, and to show that interdependence between the monetary rates is inversely related, and this exists even when outcomes are not correlated between markets. In general, empirical results support the theoretical results obtained in our paper regarding the evolution of the rates to the fundamentals of the economy such as demand parameters and degree of risk aversion form the players. The table below summarize the results for the different cases.

Table 2: Comparative statics for different fundaments of the economy.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symmetric Information</th>
<th>Asymmetric Information</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Independent Rates</td>
<td>Not independent Rates</td>
</tr>
<tr>
<td>(\rho_f \uparrow)</td>
<td>(r \uparrow)</td>
<td>(r \uparrow)</td>
</tr>
<tr>
<td>(\rho_p \uparrow)</td>
<td>(r \downarrow)</td>
<td>(r \downarrow)</td>
</tr>
<tr>
<td>(\rho_m \uparrow)</td>
<td>((1 - y) \uparrow)</td>
<td>((1 - y) \uparrow)</td>
</tr>
<tr>
<td>(\rho_p \uparrow)</td>
<td>((1 - y) \downarrow)</td>
<td>((1 - y) \downarrow)</td>
</tr>
<tr>
<td>(\delta \uparrow)</td>
<td>(r \uparrow)</td>
<td>(r \uparrow)</td>
</tr>
<tr>
<td>(\delta \uparrow)</td>
<td>((1 - y) \uparrow)</td>
<td>((1 - y) \uparrow)</td>
</tr>
<tr>
<td>(\alpha \uparrow)</td>
<td>(r \downarrow)</td>
<td>(r \downarrow)</td>
</tr>
<tr>
<td>(\beta \uparrow)</td>
<td>((1 - y) \downarrow)</td>
<td>((1 - y) \downarrow)</td>
</tr>
<tr>
<td>(r \uparrow)</td>
<td>((1 - y) \downarrow)</td>
<td>((1 - y) \downarrow)</td>
</tr>
<tr>
<td>((1 - y) \uparrow)</td>
<td>(r \downarrow)</td>
<td>(r \downarrow)</td>
</tr>
</tbody>
</table>

However, we are aware of the limitations of our paper and future versions will aim at tackling the following issues. First, we have considered the structure of the market exogenous
and the franchisor in our model can not decide on the market structure that will be used to market its product. In our model we have obtained an indirect rationale on why the mixed distribution should be implemented but we need to compare it with other possible structures. We also wonder how the results might change if we introduced demand interdependency not coming from possible demand shocks but through the effort provided by the different distributing units. At this regard, we are considering that at some extend there are effort spillovers.

References


6 Appendices

6.1 Appendix A

**Proof of proposition (1):**  here we consider the case where effort is observable and can be part of the contract offered by the franchisor. Therefore, the reduced problem is

\[
Max_{[r,y,F,e_f,e_m]} CE_p = r \times \left( \alpha e_f + \frac{\delta e_p}{2} \right) + F + (1-y) \times \left( \beta e_m + \frac{\delta e_p}{2} \right) - w - \frac{e_p^2}{2} \text{ Rev. from franchised-unit} \\
- \frac{\rho_p}{2} \times \left( r^2 \theta_f^2 + (1-y)^2 \theta_m^2 + 2Cov(\theta_f r, \theta_m(1-y)) \right) \text{ Franchisor risk aversion} \\
\text{Cost}
\]

subject to:

\[
(i) \quad CE_m = w + y \left( \beta e_m + \frac{\delta e_p}{2} \right) - \frac{e_m^2}{2} - \frac{\rho_m}{2} y^2 \theta_m^2 \geq 0 \quad (IR_m)
\]

\[
(ii) \quad CE_f = (1-r) \left( \alpha e_f + \frac{\delta e_p}{2} \right) - F - \frac{e_f^2}{2} - \frac{\rho_f}{2} (1-r)^2 \theta_f^2 \geq 0 \quad (IR_f)
\]

The Lagrangian of the problem is then

\[
\lambda = r \times \left( \alpha e_f + \frac{\delta e_p}{2} \right) + (1-y) \times \left( \beta e_m + \frac{\delta e_p}{2} \right) + F - \frac{e_p^2}{2} - w - \frac{\rho_p}{2} \times \left( r^2 \theta_f^2 + (1-y)^2 \theta_m^2 \right) \\
- \lambda_1 \times \left( \frac{e_m^2}{2} + \frac{\rho_m}{2} y^2 \theta_m^2 - w - y \left( \beta e_m + \frac{\delta e_p}{2} \right) \right) \\
- \lambda_2 \times \left( F + \frac{e_f^2}{2} + \frac{\rho_f}{2} (1-r)^2 \theta_f^2 - (1-r) \left( \alpha e_f + \frac{\delta e_p}{2} \right) \right).
\]

We then proceed to calculate the first order conditions.\(^4\) With the first order conditions of the Lagrangian with respect to wages and the up-front fee we obtain:

\[
\lambda_1 = 1, \quad (6.1)
\]

\[
\lambda_2 = 1 \quad (6.2)
\]

\(^4\)We assume that is sufficiently concave and that are sufficiently convex to ensure that the second-order conditions hold (Bhattacharya S. and Lafontaine F., 1995).
respectively. And with respect to both Lagrange multipliers we get conditions

\[ w + y \left( \beta e_m + \frac{e_{p}}{2} \right) - \frac{e_{m}^{2}}{2} - \frac{\rho_{m} y^{2} \theta_{m}^{2}}{2} = 0 \]  
(6.3)

\[ (1 - r) \left( \alpha e_f + \frac{\delta e_{p}}{2} \right) - F - \frac{e_{f}^{2}}{2} - \frac{\rho_{f} (1 - r) \theta_{f}^{2}}{2} = 0 \]  
(6.4)

Therefore, the franchisee’s and manager’s individual rationality constraint must be binding, which implies that there is no rent left downstream. By calculating the first-order condition of the Lagrangian with respect to the effort of the principal we obtain

\[ r \delta \left( \frac{\delta}{2} + (1 - y) \frac{\delta}{2} - e_p - \lambda_1 \left( -\frac{\delta}{2} \right) - \lambda_2 \left( -(1 - r) \frac{\delta}{2} \right) = 0, \]  
(6.5)

and substituting by equations (6.1) and (6.2) into (6.5), we get

\[ r \frac{\delta}{2} + \frac{\delta}{2} - \frac{\delta}{2} - e_p + \frac{\delta}{2} + \frac{\delta}{2} - \frac{\delta}{2} r = 0 \rightarrow e_p = \delta. \]

This expression indicates that the effort of the franchisor is such that the marginal costs equals the marginal benefit, and this last effect is represented by the effectiveness that the effort has on the demand to both units. By the same procedure we obtain that the effort for the manager and the franchisee are equal to

\[ e_m = \beta; \quad e_f = \alpha \]

The first order condition with respect to the royalty rate we obtain that

\[ \alpha e_f + \frac{\delta e_{p}}{2} - \rho_{p} \theta_{j}^{2} r - \lambda_2 \left( -\rho_{f} \theta_{f}^{2} r + \alpha e_{f} + \frac{\delta e_{p}}{2} \right) = 0, \]  
(6.6)

and substituting with the expressions that we have obtained we get\(^5\)

\[ - \rho_{p} \theta_{j}^{2} r + \rho_{f} \theta_{j}^{2} r - \rho_{f} \theta_{j}^{2} r = 0 \rightarrow r^{*} = \frac{\rho_{f}}{\rho_{p} + \rho_{f}} \]  
(6.7)

\(^5\)The set of restrictions is convex, suggesting a maximum, which is verified using Matlab.
The same calculations are applied for the commission rate and we get

\[-\beta e_m - \frac{\delta e_p}{2} + \rho_p \theta_m^2 - \rho_p \theta_p^2 y - \lambda_1 \left( \rho_m \theta_m^2 y - \beta e_m - \frac{\delta e_p}{2} \right) = 0 \]

\[\rho_p \theta_m^2 - \rho_p \theta_p^2 y - \rho_m \theta_m^2 y = 0 \rightarrow y^* = \frac{\rho_p}{\rho_p + \rho_m} \rightarrow (1 - y^*) = \frac{\rho_m}{\rho_p + \rho_m} \]

(6.8)

**Proof of proposition (2):** Here we consider the case where efforts are not observable and they cannot be part of the contract. The reduced problem is the same as case 1 but now we introduce the incentive constraints as well which are given by:

\[(iii) \quad \frac{d(CEp)}{de_p} = r \delta + (1 - y) \delta - e_p = 0 \quad (IC_p)\]

\[(iv) \quad \frac{d(CEf)}{de_f} = (1 - r) \alpha - e_f = 0 \quad (IC_f)\]

\[(v) \quad \frac{d(CEm)}{de_m} = y \beta - e_m = 0 \quad (IC_m)\]

We are first going to characterize the monetary provisions and then we prove point i) and ii) of the proposition.

The Lagrangian is equal to

\[\iota = r \times \left( \alpha e_f + \frac{\delta e_p}{2} \right) + (1 - y) \times \left( \beta e_m + \frac{\delta e_p}{2} \right) + F - \frac{e_p^2}{2} - w - \frac{\rho_p}{2} \times \left( r^2 \theta_f^2 + (1 - y)^2 \theta_m^2 \right) - \lambda_1 \times \left( \frac{e_p^2}{2} + \rho_m \theta_m^2 y^2 - w - y(\beta e_m + \frac{\delta e_p}{2}) \right) - \lambda_2 \times \left( F + \frac{e_p^2}{2} + \frac{\rho_f}{2} (1 - r)^2 \theta_f^2 - (1 - r)(\alpha e_f + \frac{\delta e_p}{2}) \right) - \lambda_3 \times \left( (e_p - r \delta - (1 - y) \frac{\delta}{2}) - \lambda_4 \times (e_f - (1 - r) \alpha) - \lambda_5 (e_m - y \beta) \right).\]

We then proceed to calculate the first order conditions.\(^6\) With the first order conditions of the Lagrangian with respect to wages and the up-front fee we obtain:

\[\lambda_1 = 1, \quad (6.9)\]

\[\lambda_2 = 1 \quad (6.10)\]

\(^6\)We assume that is sufficiently concave and that are sufficiently convex to ensure that the second-order conditions hold (Bhattacharya S. and Lafontaine F., 1995).
respectively. And with respect to all Lagrange multipliers we get conditions:

\[ w + y(\beta m + \frac{\delta e_p}{2}) - \frac{e_m^2}{2} - \frac{\rho_m}{2} y^2 \theta_m^2 = 0 \]  

(6.11)

\[ (1 - r)(\alpha f + \frac{\delta e_p}{2}) - F - \frac{e_f^2}{2} - \frac{\rho_f}{2} (1 - r)^2 \theta_f^2 = 0 \]  

(6.12)

Therefore, the franchisee’s and manager’s individual rationality constraint must be binding, which implies that there is no rent left downstream.

\[ (r + (1 - y))\frac{\delta e_p}{2} = e_p \]  

(6.13)

\[ (1 - r)\alpha = e_f \]  

(6.14)

\[ y\beta = e_m \]  

(6.15)

The effort provided by each agent and principal is equal to their incentives and the impact of their effort in determining downstream sales. By calculating the first-order condition of the Lagrangian with respect to the effort of the principal we obtain

\[ r \frac{\delta}{2} + (1 - y) \frac{\delta}{2} - e_p - \lambda_1 \left( -\frac{\delta}{2y} \right) - \lambda_2 \left( -(1 - r) \frac{\delta}{2} \right) - \lambda_3 = 0 \]  

(6.16)

Substituting by equations (6.9) and (6.10) and (iii) into (6.16), we have

\[ \lambda_3 = \frac{\delta}{2} (y - r + 1) \]  

(6.17)

The Lagrangian with respect to the manager’s effort indicates that

\[ \beta - y\beta - \lambda_1 (e_m - y\beta) - \lambda_5 = 0 \]  

(6.18)

Substituting equation (6.9) into (6.18), we get
\[ \lambda_5 = \beta(1 - y) \] (6.19)

The Lagrangian with respect to the franchisee’s effort indicates we get

\[ r\alpha - \lambda_2 (e_f - (1 - r)\alpha) - \lambda_4 = 0 \] (6.20)

and substituting equation (6.10) and (iv) into (6.20), we have

\[ \lambda_4 = r\alpha \] (6.21)

The first order condition with respect to the royalty rate we obtain that

\[ \alpha e_f + \delta e_p^2 - \rho_p \theta_f^2 r - \lambda_2 (-\rho_f \theta_f^2 + \rho_f \theta_f^2 r + \alpha e_f + \frac{\delta e_p^2}{2}) + \lambda_3 \frac{\delta e_p^2}{2} - \lambda_4 \alpha, \] (6.22)

and substituting equation (6.10), (6.17) and (6.21) into (6.22), we obtain \(^7\)

\[ -4\rho_p \theta_f^2 r + 4\rho_f \theta_f^2 + \delta^2 (y - r + 1) - 4r^2 = \] \[ \rightarrow r^{AU} = \frac{4\rho_f \theta_f^2 + \delta^2 y^{AU} + \delta^2}{4\rho_p \theta_f^2 + 4\rho_f \theta_f^2 + 4\alpha^2 + \delta^2} = \frac{4\theta_f^2 \rho_f + \delta^2 (1 + y^{AU})}{4\theta_f^2 (\rho_p + \rho_f) + 4\alpha^2 + \delta^2} \] (6.23)

The same calculations are applied for the commission rate and we get

\[ 4\rho_p \theta_m^2 - 4\rho_p \theta_m^2 y - 4\rho_m \theta_m^2 y - \delta^2 (y - r + 1) + 4(1 - y)\beta^2 = 0 \]

\[ \rightarrow y^{AU} = \frac{4\rho_p \theta_m^2 + r^{AU} \delta^2 - \delta^2 + 4\beta^2}{4\rho_p \theta_m^2 + 4\rho_m \theta_m^2 + \delta^2 + 4\beta^2} \]

\[ \rightarrow (1 - y^{AU}) = \frac{4\theta_m^2 \rho_m + \delta^2 (2 - r^{AU})}{4\theta_m^2 (\rho_p + \rho_m) + 4\beta^2 + \delta^2} \] (6.24)

From the previous expressions, we see that point i) is obtained directly. In order to show point ii) we solve both expression to obtain only a functions of the parameters. Hence, we

\(^7\)The set of restrictions is convex, suggesting a maximum, which is verified using Matlab.
obtain

\[
\rho_f \alpha^2 \left( \delta^2 + 4(\theta_m^2 (\rho_m + \rho_p) + \beta^2) \right) \geq \rho_p \delta^2 (\rho_f \theta_f^2 + \theta_m^2 (\rho_m + 2 \rho_p) + 2 \beta^2)
\]

\[
\rightarrow \quad \rho_f \alpha^2 (\delta^2 + 4 \beta^2) - \rho_p \delta^2 \beta^2 + \theta_m^2 (4 (\rho_m + \rho_p) - \rho_p \delta^2 (\rho_m + 2 \rho_p)) - \rho_p \rho_f \delta^2 \theta_f^2 \geq 0.
\]

By the assumptions considered about the parameters of the model, the above is always true and we obtain that \( r^* > r^{AC} \). A similar procedure can be used to show that this is also the case for the commission rate of the owned unit.

\[
y^{AU} = \frac{\theta_f^2 (\rho_f + \rho_p)(4 \rho_p \theta_m^2 + \delta^2 + 4 \beta^2) + (4 \alpha^2 + \delta^2)(\rho_p \theta_m^2 + 4 \beta^2) + \delta^2(\theta_f^2 \rho_f - \alpha^2)}{\theta_m^2 (\rho_m + \rho_p)(4 \theta_f^2 \rho_f + 4 \theta_f^2 \rho_p + 4 \alpha^2 + \delta^2) + (\delta^2 + 4 \beta^2)(\theta_f^2 \rho_f + \theta_f^2 \rho_p + \alpha^2) + \beta^2 \delta^2}
\]

And we obtain that \( y^* < y^{AU8} \) if

\[
4 \alpha^2 \theta_m^2 \rho_m \rho_p + \delta^2 \theta_f^2 \rho_f^2 + 2 \alpha^2 \delta^2 \rho_p \leq \theta_f^2 (\rho_p + \rho_f)(\rho_p \delta^2 + \delta^2 \rho_m \theta_f^2 + 4 \beta^2 \theta_f^2 \rho_m) + \rho_m (4 \rho_p \theta_m^2 + 4 \beta^2 \alpha^2 + \beta^2 \delta^2 + \theta_f^2 \rho_f \delta^2)
\]

\[
\rightarrow \quad -\theta_f^2 \rho_m (\rho_p + \rho_f)(\delta^2 - 4 \beta^2) + \delta^2 \alpha^2 (\rho_m + 2 \rho_p) - \theta_f^2 \rho_f \delta^2 (\rho_m + \rho_p) - \beta^2 \rho_m (4 \alpha^2 + \delta^2) \leq 0
\]

By the assumptions considered about the parameters of the model, the above is always true and we obtain that \( y^* < y^{AU} \iff (1 - y^*) > (1 - y^{AU}) \)

**Proof of proposition (3):** now we consider the case where the market events are correlated and the covariance. Following Lafontaine and Blair (2005) the covariance is equal to

\[
\frac{\rho_f}{2} Var(I_f) = \frac{\rho_f}{2} Var((1 - r) \theta_f) = \frac{\rho_f \theta_f^2}{2} (1 - r)^2
\]

\(^8(1 - y^*) > (1 - y^{AU}) \iff y^* < y^{AU}\)
However, the variance definition indicates that

\[ \text{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2, \]

where \( X \) is a random variable with average \( \mu = E(X) \). As in Lafontaine and Blair (2005) we assume that this variation is close to 0, and this is why we have set it to \((1 - r)^2\).

The covariance of two random variables \( X \) and \( Y \) is

\[ \text{Cov}(XY) = E[(X - E(X))(Y - E(Y))], \]

and we easily obtain that the covariance in our model equals

\[ \text{Cov}((\theta_m(1 - y)), (\theta_f r)) = \theta_f \theta_m r (1 - y) \]

Following the same reasoning of the previous cases we obtain that the Lagrangian is equal to

\[
\begin{align*}
t &= r \times \left( \alpha e_f + \frac{\delta e_p}{2} \right) + (1 - y) \times \left( \beta e_m + \frac{\delta e_p}{2} \right) + F - \frac{e_f^2}{2} - w - \frac{\rho_p}{2} \times \left( r^2 \theta_f^2 + (1 - y)^2 \theta_m^2 + 2 \theta_f \theta_m r (1 - y) \right) \\
&\quad - \lambda_1 \times \left( \frac{e_m^2}{2} + \frac{\alpha e_m + \delta e_p}{2} - w - y (\beta e_m + \delta e_p) \right) \\
&\quad - \lambda_2 \times \left( F + \frac{e_f^2}{2} + \frac{\rho_f}{2} (1 - r)^2 \theta_f^2 - (1 - r) (\alpha e_f + \delta e_p) \right) \\
&\quad - \lambda_3 \times \left( e_p - r \frac{\delta}{2} - (1 - y) \frac{\delta}{2} \right) - \lambda_4 \times \left( e_f - (1 - r) \alpha \right) - \lambda_5 \times (e_m - y \beta).
\end{align*}
\]

We then proceed to calculate the first order conditions.\(^9\) With the first order conditions of the Lagrangians remain identical to those in the case without the covariance between \( r \) and \( y \).

\[
\begin{align*}
\lambda_1 &= 1 \\
\lambda_2 &= 1
\end{align*}
\]  

\(^9\)We assume that is sufficiently concave and that are sufficiently convex to ensure that the second-order conditions hold (Bhattacharya S. and Lafontaine F., 1995).
\[ \lambda_3 = \frac{\delta}{2} \left( y - r + 1 \right) \] (6.27)

\[ \lambda_4 = r\alpha \] (6.28)

\[ \lambda_5 = \beta(1 - y) \] (6.29)

The first order condition with respect to the royalty rate we obtain that

\[ \alpha e_f + \frac{\delta e_p}{2} - r_p \theta_f^2 r - r_p \theta_f \theta_m (1 - y) - \lambda_2 (\theta_f^2 - r_p \theta_f + \alpha e_f + \frac{\delta e_p}{2}) + \lambda_3 \delta - \lambda_4 \alpha = 0 \] (6.30)

Substituting equation (6.26), (6.27) and (6.28) into (6.30), we have\(^{10}\)

\[ -4 \lambda_p \theta_f^2 y - 4 \lambda_p \theta_f \theta_m (1 - y) + 4 \lambda_p \theta_f^2 \theta_m + 4 \lambda_p \theta_f \theta_m y + 4 \lambda_p \theta_f \theta_m r - \delta^2 (y - r + 1) + 4 \lambda_p \theta_f \theta_m + 4 \lambda_p \theta_f \theta_m y + 4 \lambda_p \theta_f \theta_m r - \delta^2 (y - r + 1) + 4 \lambda_p \theta_f \theta_m y = 0 \]

\[ \rightarrow \quad r^{AC} = \frac{4 \lambda_p \theta_f^2 + \delta^2 \gamma^AC + \delta^2 - 4 \lambda_p \theta_f \theta_m + 4 \lambda_p \theta_f \theta_m y + 4 \lambda_p \theta_f \theta_m r - \delta^2 (y - r + 1) + 4 \lambda_p \theta_f \theta_m}{4 \lambda_p \theta_f^2 + 4 \lambda_p \theta_f^2 + 4 \alpha^2 + \delta^2} \]

\[ \rightarrow \quad r^{AC} = r^{AU} (y^{AC}) - \frac{4 \lambda_p \theta_f \theta_m (1 - y^{AC})}{4 \lambda_p \theta_f^2 + 4 \lambda_p \theta_f^2 + 4 \alpha^2 + \delta^2} \]

The same calculations are applied for the commission rate and we get

\[ 4 \lambda_p \theta_m^2 - 4 \lambda_p \theta_m^2 y + 4 \lambda_p \theta_m^2 y + 4 \lambda_p \theta_m r - \delta^2 (y - r + 1) + 4 (1 - y) \beta^2 = 0 \]

\[ \rightarrow \quad y^{AC} = \frac{4 \lambda_p \theta_m^2 + 4 \lambda_p \theta_m r^{AC} + r \delta^2 - \delta^2 + 4 \beta^2}{4 \lambda_p \theta_m^2 + 4 \lambda_p \theta_m^2 + \delta^2 + 4 \beta^2} \]

\[ \rightarrow \quad (1 - y^{AC}) = (1 - y^{AU} (r^{AU})) - \frac{4 \lambda_p \theta_f \theta_m r^{AC}}{4 \theta_m^2 (\rho_p + \rho_m) + 4 \beta^2 + \delta^2} \]

In order to obtain only a functions of the parameters, we solve both expression for both. Hence, we obtain

\[ r^{AC} = \frac{4 \lambda_p \theta_f^2 (\rho_p + \rho_m) + 6 \lambda_p \theta_f (4 \beta^2 + \delta^2) + 2 \theta^2 \rho_p \theta_f \theta_m (\beta^2 - 2 \theta^2 \rho_p \rho_m) + \theta^2 \theta_m^2 \rho_m - 2 \theta \rho_p + 2 \beta \delta^2}{-2 \delta^2 \rho_p \theta_f \theta_m + 4 \theta^2 \theta_m^2 \rho_p \theta_f + \rho_p \rho_m + \theta^2 (\rho_p + \rho_m) (\delta^2 + 4 \alpha^2) + \theta \theta_m (\rho_p + \rho_f) (4 \beta^2 + \delta^2) + \beta \theta^2 (\delta^2 + \alpha^2) + \delta^2 \alpha^2} \]

and for commission rate we obtain:

\(^{10}\)The set of restrictions is convex, suggesting a maximum, which is verified using Matlab.
\[ y^{AC} = \frac{4(\rho_p + \rho_f)(\rho_p \theta^2 \theta_m^2 + \beta^2 \theta_f^2) + (4\alpha^2 + \delta^2)(\rho_p \theta^2_m + \beta^2) - \delta^2 \rho_p \theta^2_f - \delta^2 \alpha^2}{(\rho_p + \rho_f)(4\theta^2_m \delta^2 + \theta_f^2 \delta^2 + 4\theta_f \beta^2) + \rho_m(4\alpha^2 + \delta^2) + 4\rho_f \rho_p \theta^2_m \theta_m^2 + \theta_m^4(\rho_p + \delta^2(\alpha^2 - 2\rho_p \theta_f \theta_f) + \beta^2(\alpha^2 + \delta^2))} \]