Equilibrium Models of the Marriage Market

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- Testable restrictions (beyond income pooling)
  - price effects: SNR1 (BC 2002)
  - no price effect: distribution factors proportionality (BBC 2000)
  - labor supply (Chiappori 1988, 92)
- Identification: role of:
  - exclusion restrictions (CE 2008)
  - distribution factors (BBC 2000, CE 2008)

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Empirical applications:

Consumption, labor supply, savings,…

Reference	Outcome	Country	Df's
Anderson & Baland (2002)	Women's participation in a	Kenya	1
	rosca, saving		
Aronsson et al (2001)	Leisure demand	Sweden	2,3,4,5,6
Attanasio & Lechene (2002)	Commodity demands;	Mexico	1
	influence on various decisions		
Barmby & Smith (2001)	Labour supplies	Denmark, UK	2
Bayudan (2006)	Female labour supply	Philippines	2, 11
Bourguignon et al (1993)	Commodity demands	France	1
Browning (1995)	Saving	Canada	1
Browning & Bonke (2006)	Commodity demands	Denmark	1,7,8,10
Browning & Gørtz (2006)	Commodity demands, leisures	Denmark	2,4,7
Browning et al (1994)	Demand for clothing	Canada	1,4,7
Browning & Chiappori (1998)	Commodity demands	Canada	1,4
Chiappori et al (2002)	Labour supplies	US	6
Couprie (2007)	Labour supply and leisure	UK	2,3
Donni (2007)	Labour supplies, demands	France	1,7
Duflo (2003)	Child health	South Africa	1

Ermisch & Pronzato (2006)	Child support payments	UK	1
Fortin & Lacroix (1997)	Joint labour supply	Canada	1,2
Haddad & Hoddinott (1994)	Child health	Cote D'Ivoire	1
Hoddinott & Haddad (1995)	Food, alcohol and tobacco	Cote D'Ivoire	1
Lundberg, et al (1997)	Clothing demands	UK	3
Phipps & Burton (1998)	Commodity demands	Canada	1
Schultz (1990)	Labour supplies and fertility	Thailand	3
Thomas (1990)	Child health	Brazil	3
Udry (1996)	Farm production	Burkina Faso	9
Vermeulen (2005)	Labour supplies	Netherlands	3,4,12

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Importance of *transfers*

('Gale – Shapley vs Becker – Shapley – Shubik')

Transferable Utility (TU)

Definition

A group satisfies TU if there exists monotone transformations of individual utilities such that the Pareto frontier is an hyperplane for all values of prices and income.

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In practice:

- Quasi Linear (QL) preferences (but highly unrealistic)
- ‘Generalized Quasi Linear (GQL, Bergstrom and Cornes 1981):

$$u_s(x_s, X) = F_s[A_s(x_s^2, \ldots, x_s^n, X) + x_s^1 b_s(X)]$$

with $b_s(X) = b(X)$ for all s

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Note:

- Ordinal property
- Restrictions on heterogeneity
- But 'acceptably' realistic (Chiappori JET forthcoming)

Properties of TU frameworks

- **Unanimity regarding group's decisions**

  $\longrightarrow$ clear distinction between aggregate behavior and intragroup allocation of power/resources/welfare

  $\longrightarrow$ here: concentrate of 'power' issues

- **Matching models: stable allocations maximize total surplus**

Properties of TU frameworks

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- **Matching models: stable allocations maximize total surplus**

  ⟶ **mathematical structure: optimal transportation models**

- **Formally:**
  - Separable metric spaces X, Y with measures F and G
  - Surplus s(x,y) upper semicontinuous

Stable matches under TU

Match:

- Assignment of spouses (x marries $y = f(x)$)
- Allocation of surplus: $u(x) + v[f(x)] = s(x, f(x))$

Stability

$$u(x) + v(y) \geq s(x,y) \quad \text{for all } x,y$$

$$u(x) + v[f(x)] = s(x, f(x))$$

Basic property:

(f,u,v) is stable if and only if f maximizes total surplus

Intuition: duality

Basic Duality (finite case)

Primal problem

$$\max_a \sum_{ik} a_{ik} s_{ik}$$

$$\sum_i a_{ik} \leq 1$$

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Dual problem

$$\min_{u,v} \sum_i u_i + \sum_k v_k$$
$$u_i + v_k \geq s_{ik}$$

Supermodularity

Definition: surplus $s(x,y)$ is supermodular iff when $x > x'$ and $y > y'$ then

$$s(x,y) + s(x',y') > s(x,y') + s(x',y)$$

Note: if differentiable, then the cross derivative is > 0

Basic property:

If s supermodular, then the only stable match exhibits assortative matching

Intuition: surplus maximization

Application: marriage market

Structure:

- Men and women, respective income distributions F and G
- TU; surplus s(x,y), derived from a collective model
- Assume s(x,y) supermodular

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- TU; surplus $s(x,y)$, derived from a collective model
- Assume $s(x,y)$ supermodular
- Ex. (CIW 07): $u_m = Q\ (1+q_m)$, $u_f = Q\ (a+q_f)$, then

  $s(x,y) = (x+y+a+1)^2/4$ and $D^2_{xy}\, s = 1/2 > 0$

Application: marriage market

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- Men and women, respective income distributions F and G
- TU; surplus $s(x,y)$, derived from a collective model
- Assume $s(x,y)$ supermodular
- In general: $s(x,y) = h(x + y)$ and public goods require h to be supermodular

Application: marriage market

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- TU; surplus s(x,y), derived from a collective model
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Then:

- Assortative matching:

$$x = \phi(y) = F^{-1}[G(y)] \quad \text{or} \quad y = \psi(x) = G^{-1}[F(x)]$$

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- Individual utilities can be derived

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therefore:

$$u(x) = k + \int_0^x \frac{\partial s(t,\psi(t))}{\partial x} dt \quad \text{and} \quad v(y) = k' + \int_0^y \frac{\partial s(\phi(t),t)}{\partial y} dt$$

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→ Endogenize the Pareto weight!

Example: shifting female income distribution

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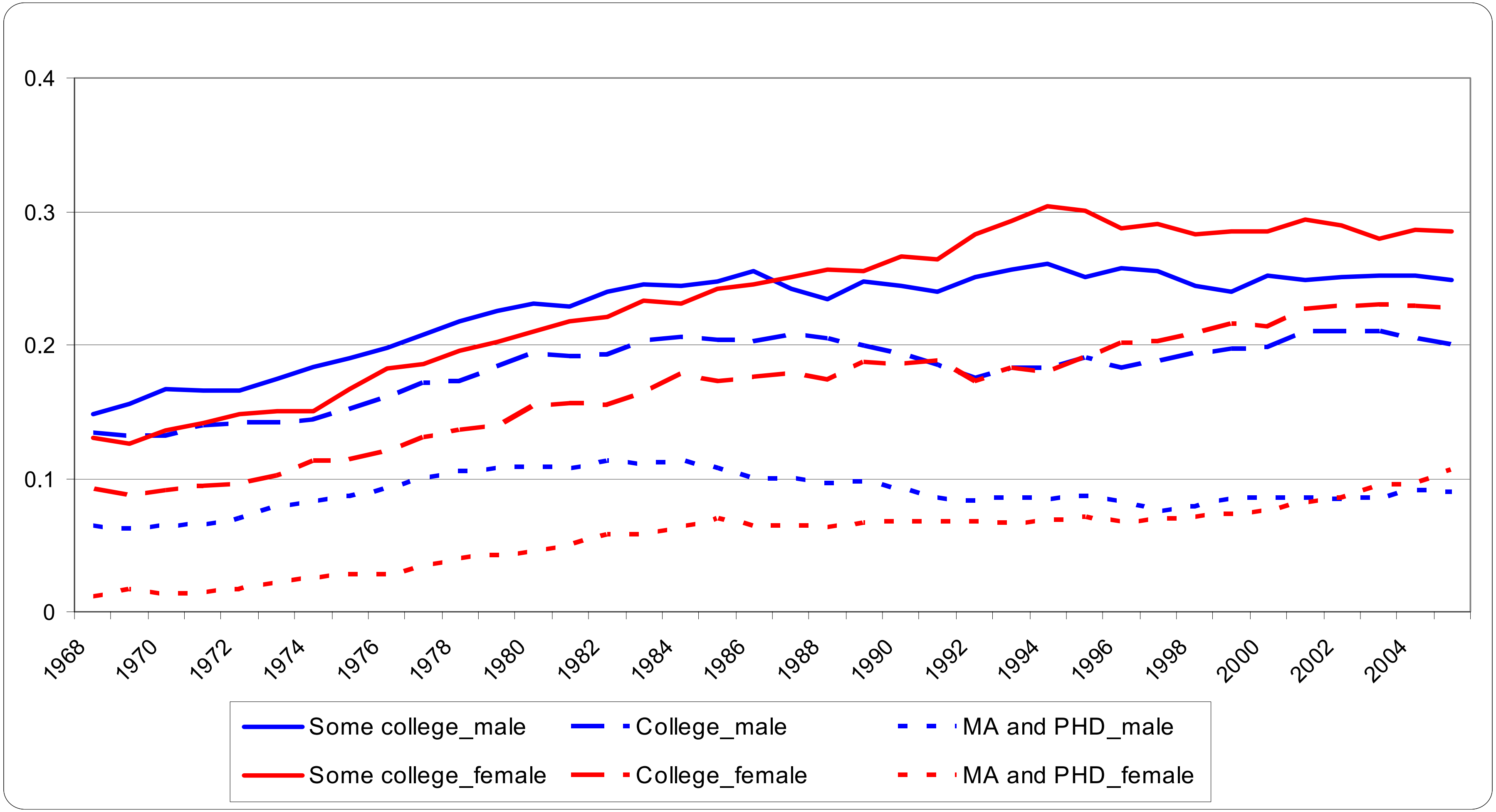
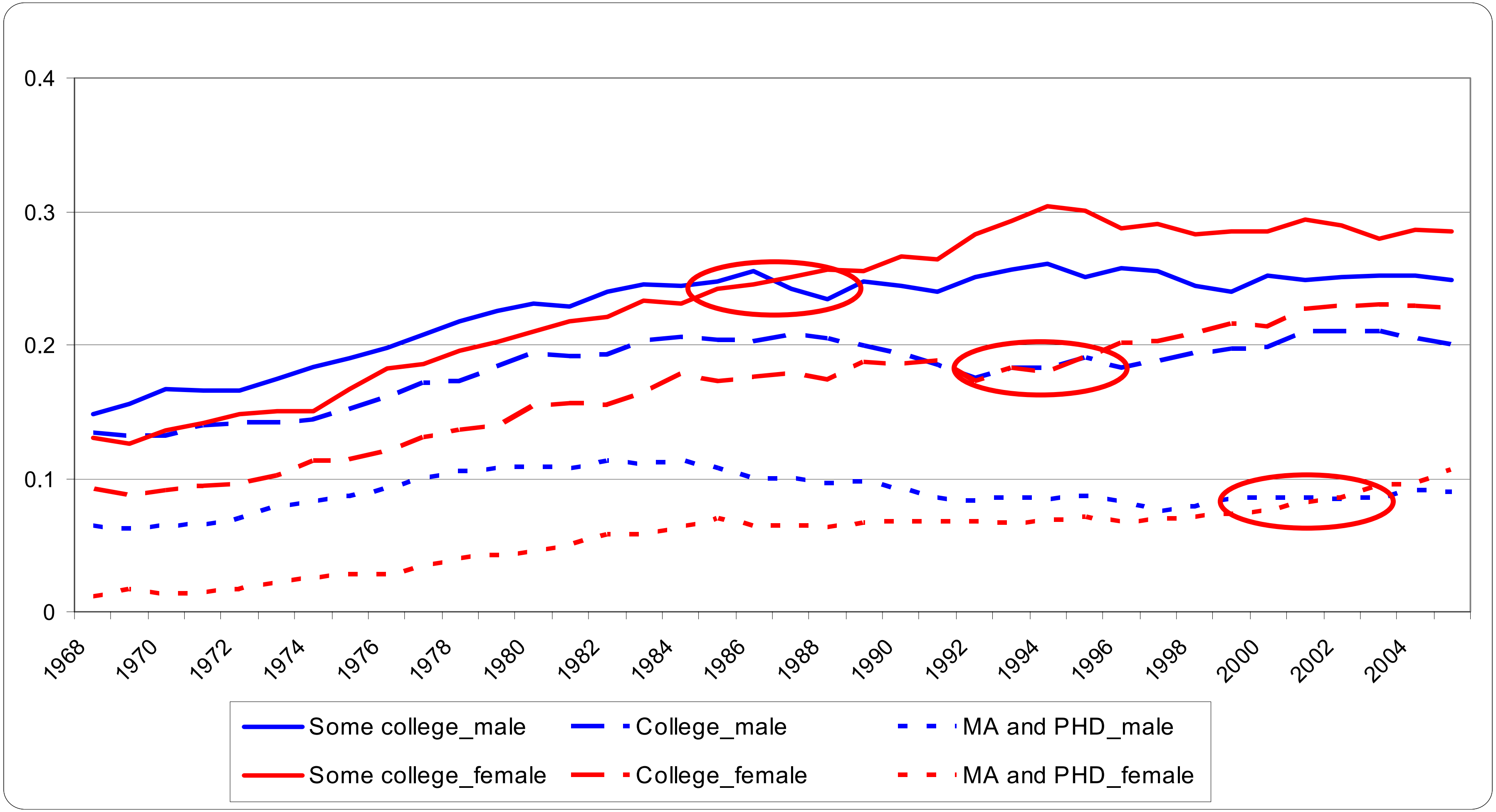
Motivation: remarkable increase in female education, labor supply, incomes during the last decades.



Proportion of some college, college and advanced degrees, by sex, age 30-40



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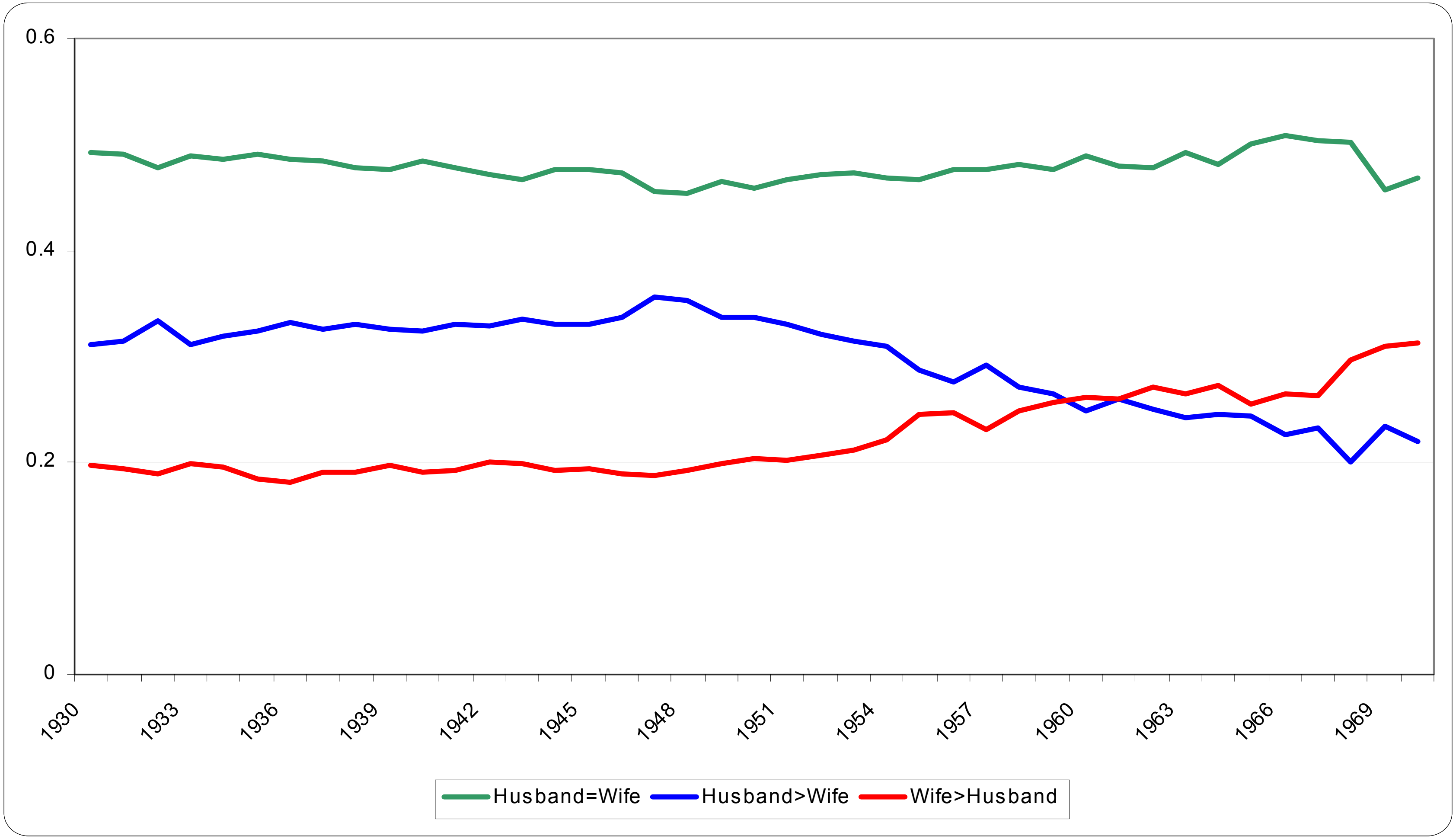

Education of Spouses, by Husband's Year of Birth, US

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Question: impact on intrahousehold allocation?

Modeling shifts in female income distribution

Assume

- $F(t) = G(\lambda t)$ for $\lambda < 1$ (then $\varphi(y) = y/\lambda$ and $\psi(x) = \lambda x$)

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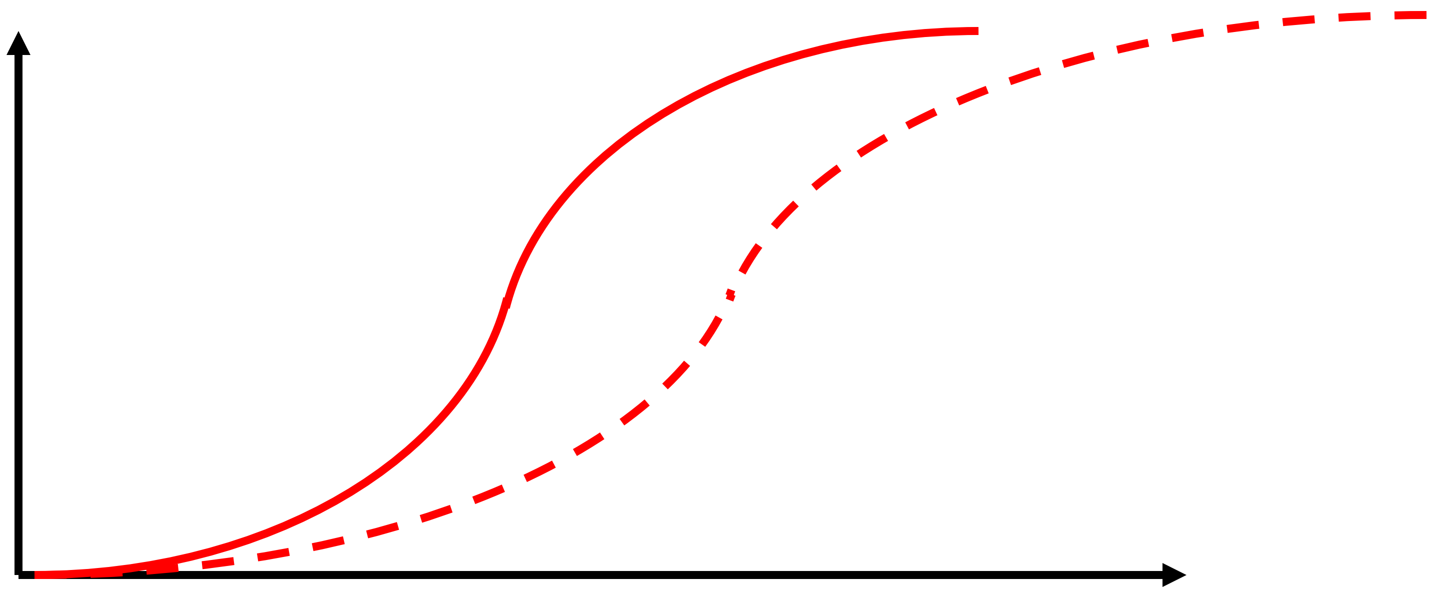

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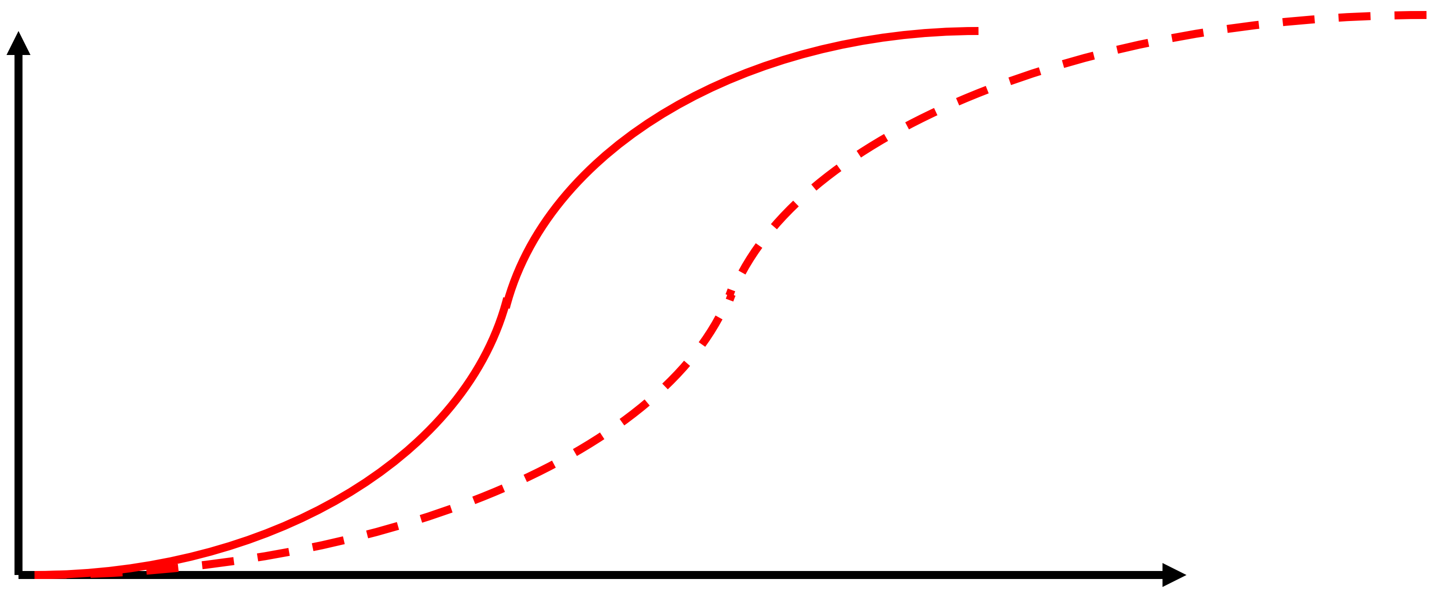
- F(t) = G(λt) for λ < 1 (then φ(y) = y/λ and ψ(x) = λx)

Note: for instance, LogNormal distributions with different μ but same σ.



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and

- $s(x, y) = H(x+y)$, $H(0) = 0$

Example: shifting female income distribution

Then

$v(y) = \frac{\lambda}{\lambda+1} H(\phi(y) + y)$ and $u(x) = \frac{1}{\lambda+1} H(x + \psi(x))$

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Upward shift in female incomes: y becomes ay, a > 1

Then in the neighborhood of a = 1

$$\frac{\partial v}{\partial a} = \frac{\lambda}{(\lambda+1)^2} H(x+y) + \frac{\lambda y}{\lambda+1} H'(x+y)$$

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Upward shift in female incomes: y becomes ay, a > 1

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$$\frac{\partial v}{\partial a} = \underbrace{\frac{\lambda}{(\lambda+1)^2} H(x+y)}_{\text{redistribution}} + \underbrace{\frac{\lambda y}{\lambda+1} H'(x+y)}_{\text{additional surplus}}$$

Some applications

Application 1: matching on preferences (CO JPE 06)

- Continuum of men and women; one private commodity → intrahousehold allocation of consumption an issue; children
- Men all identical; quasi linear utility $U_H(a_H, k) = a_H + u_H.k$ if married; zero utility of children if single

  Or: heterogeneous males: u_H distributed over [-A, B]
- Women: quasi linear utility $U(a, k) = a + uk$ where u belongs to [0,U], density f; note that utility is *transferable.*
- Income: men Y, women y without children, $z < y$ with children
- Unwanted pregnancies, probability p
- Frictionless marriage market (matching model); surplus generated by children

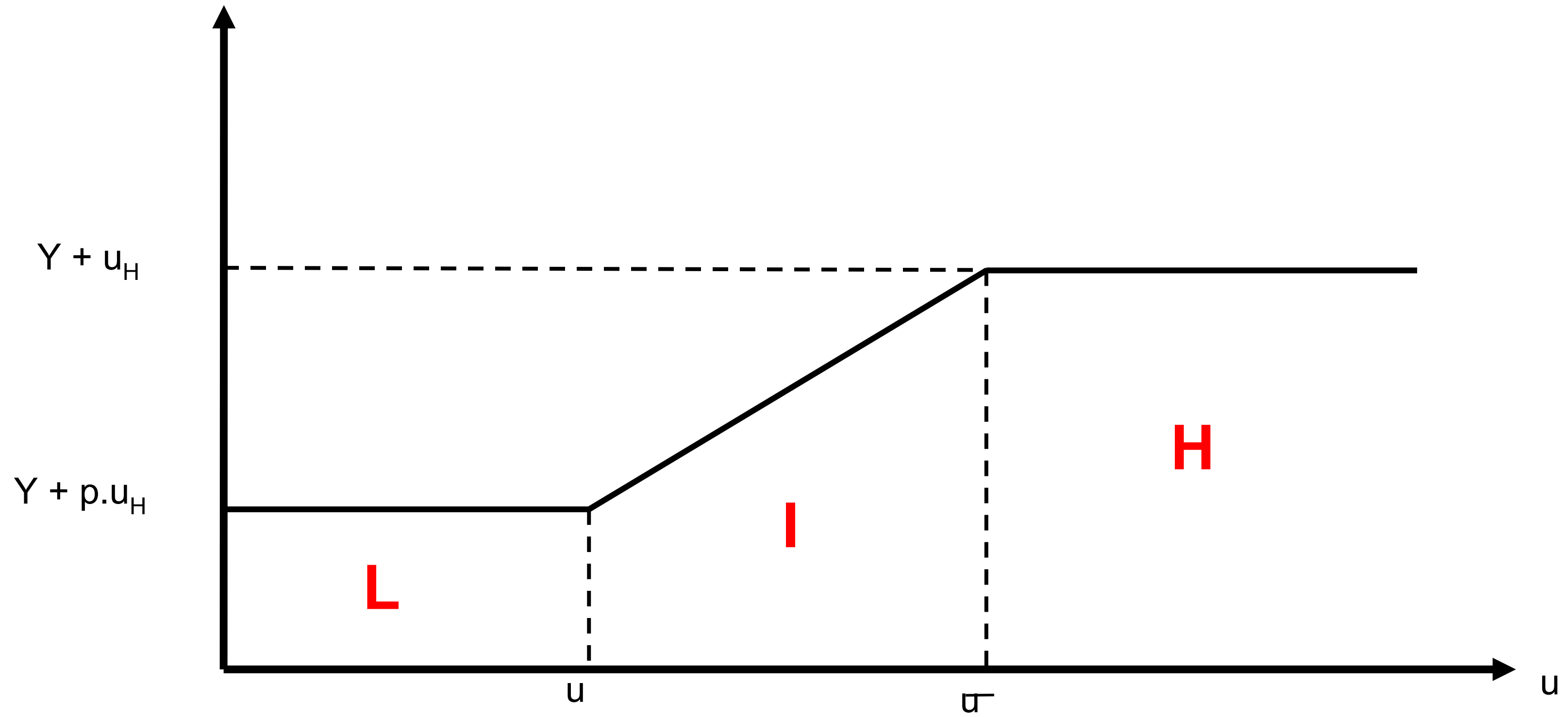
  → equilibria as stable matches
- Mass 1 of women, M of men

Fertility decisions

- Single women
  - If $u < y - z = \bar{u}$: no children
  - Otherwise: children
- Couples
  - Efficiency: children if maximizes total surplus
  - Hence: children if $u > y - z - u_H = \underline{u}$;
- Hence three types of women (depending on preferences):
  - 'low': $u < \underline{u}$ **never** want a child
  - 'intermediate': $\underline{u} < u < \bar{u}$ want a child **only** when married
  - 'high': $u > \bar{u}$ **always** want a child
- Heterogeneous men: same, but $\underline{u}$ is match specific

Stable match: excess supply of women

Basic graph: husband's *maximal* utility (as a function of u)



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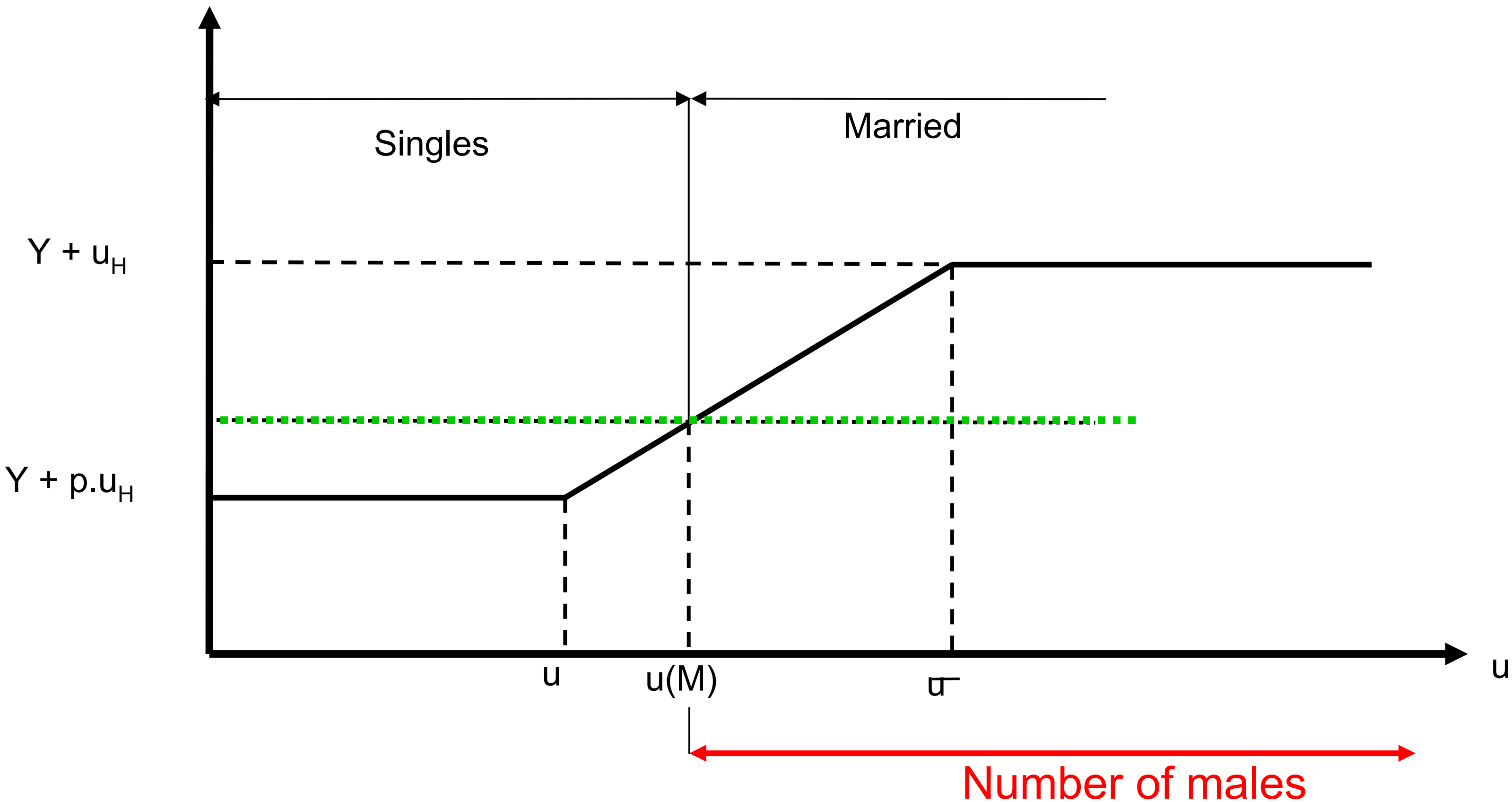
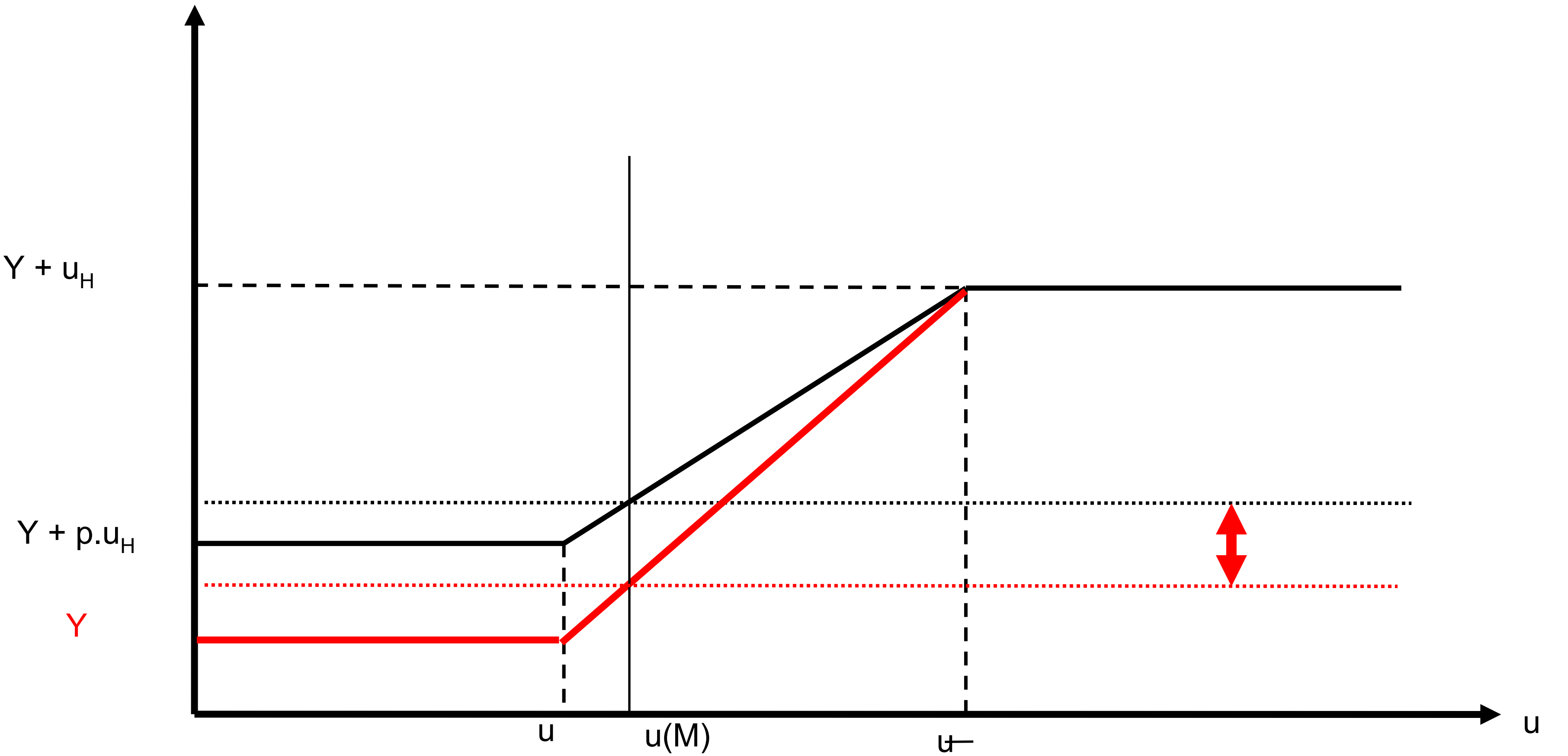


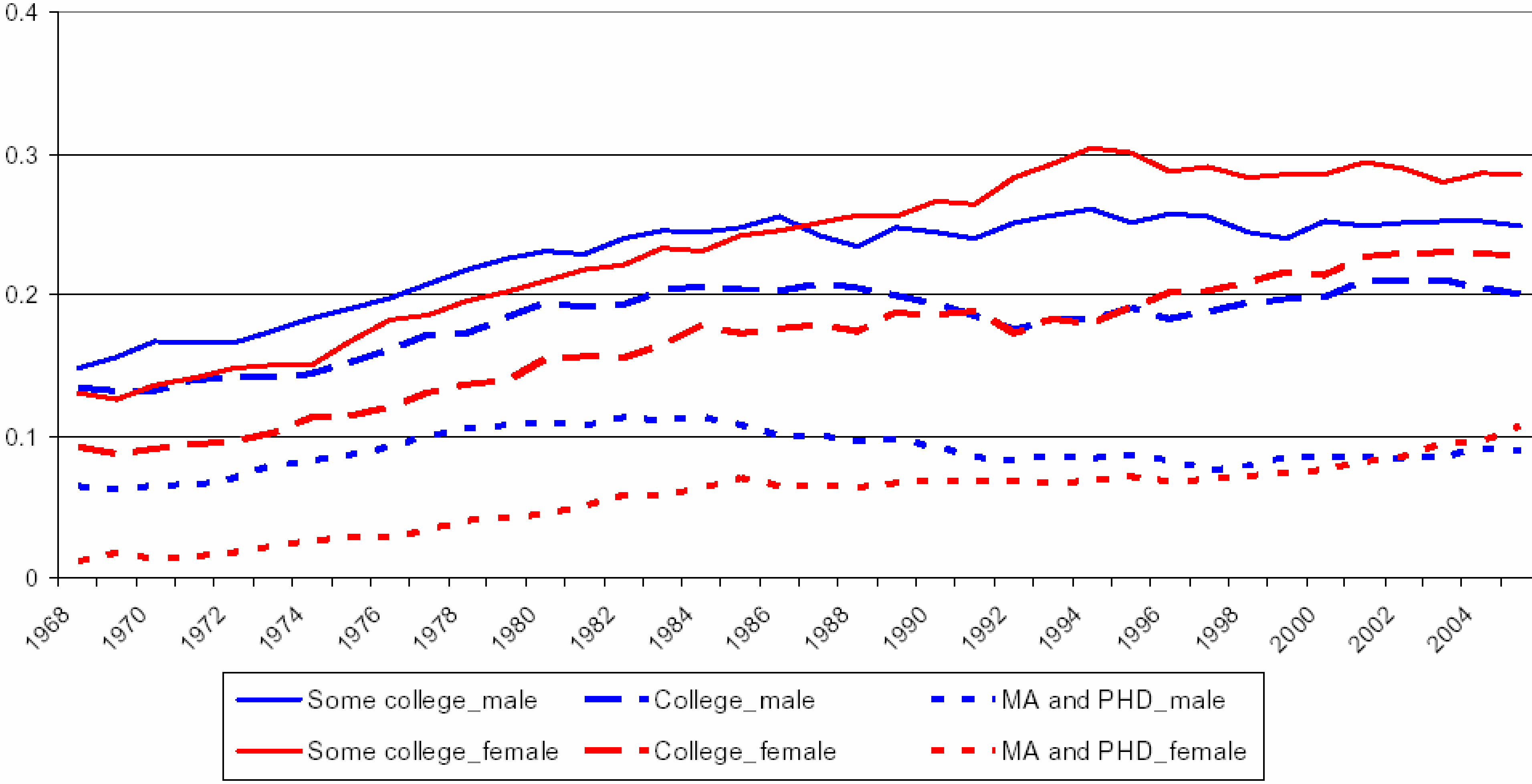
Application: legalizing abortion



Application 2: matching and investments in education (BCW AER 09)

Basic puzzle: asymmetric reactions to increasing returns to education

Figure 13: Completed Education by Sex, Age 30-40, US 1968-2005



Source: Current Population Surveys.

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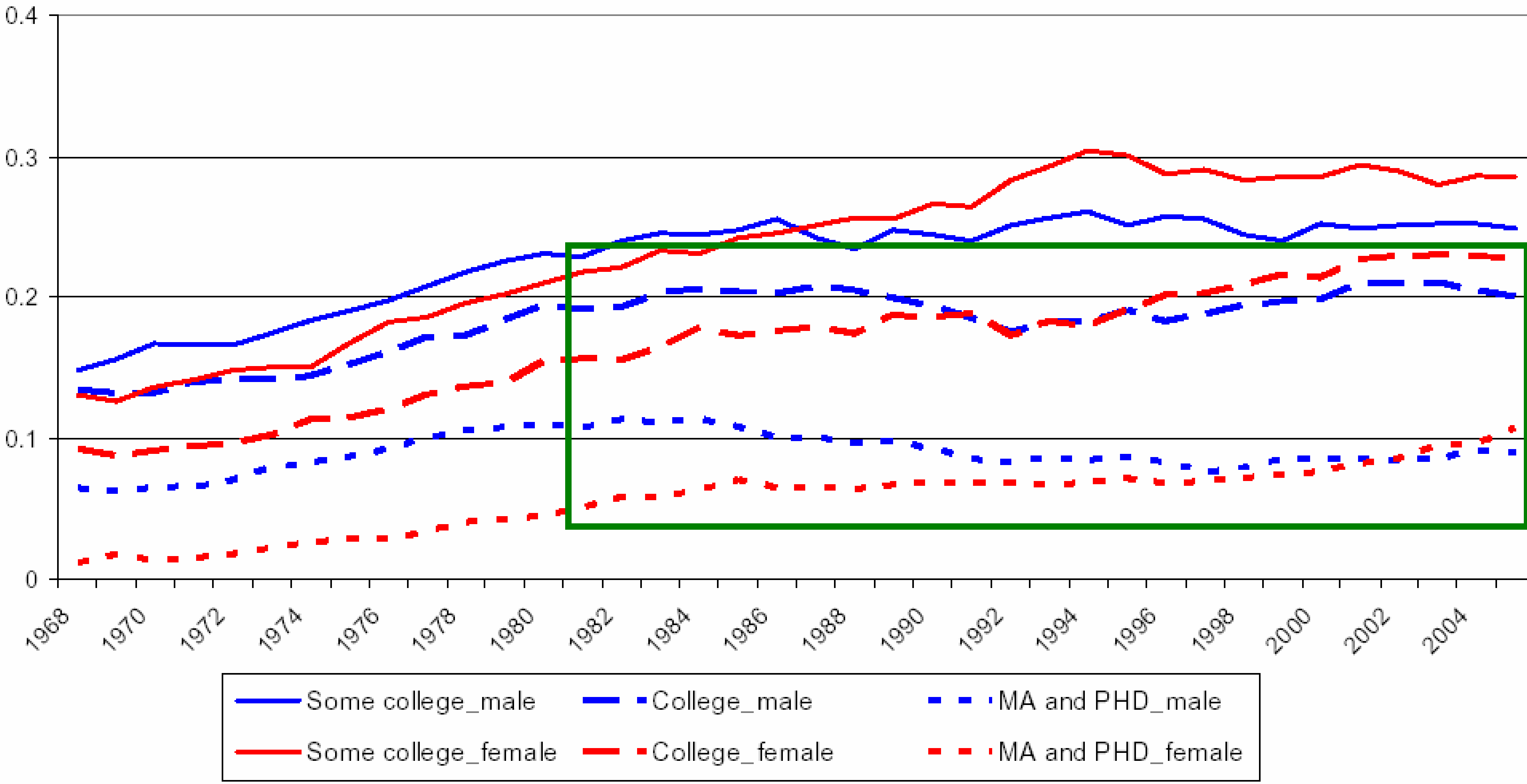

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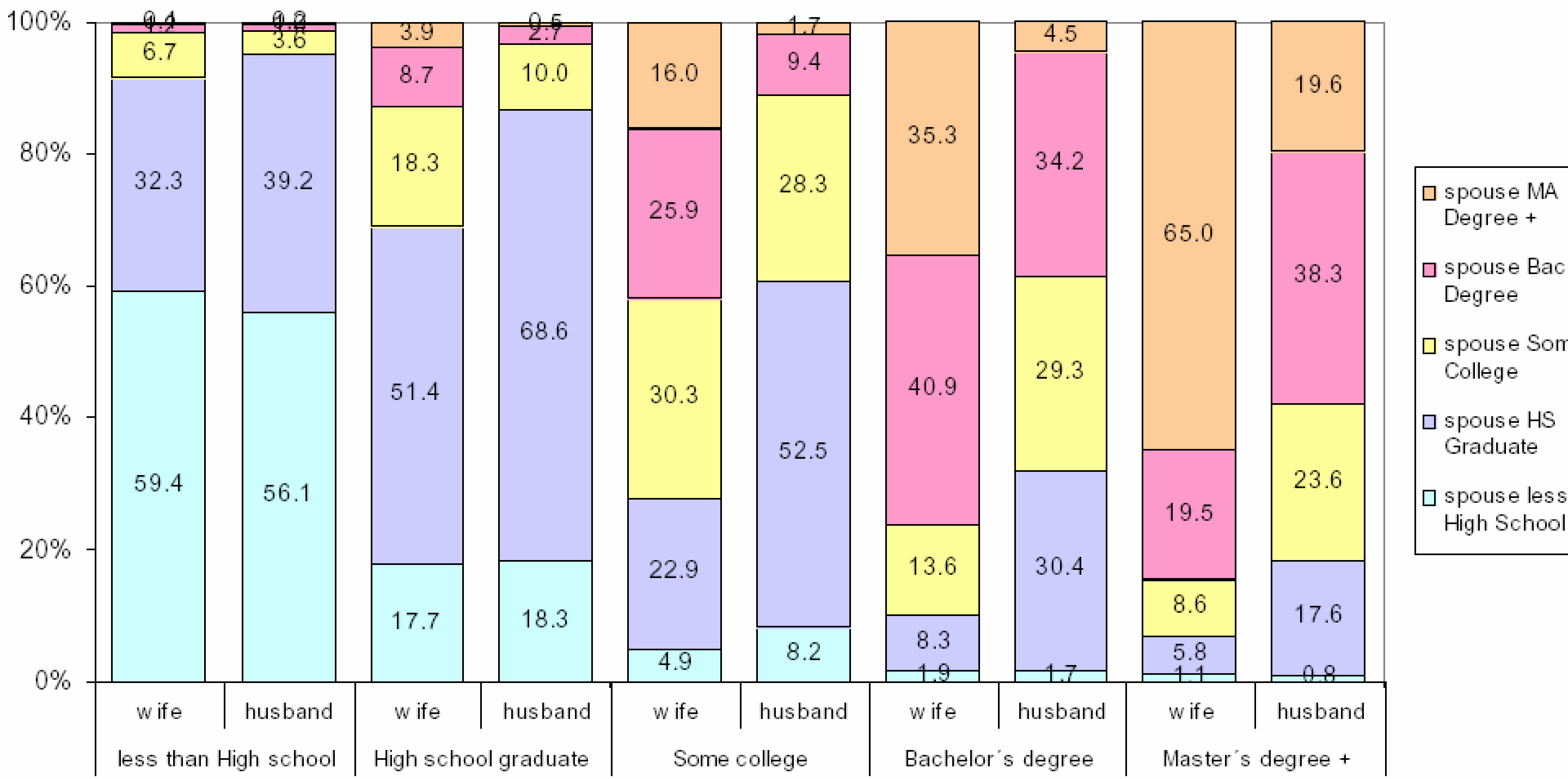
Explanation:

1. **Gender discrimination smaller for high incomes**
2. **Intrahousehold effects**
   - Returns to education have two components: market and intrahousehold
   - If larger percentage of educated women, affects matching patterns
   - Cost of not being educated are higher

Figure 15a: Spouse Education by own Education, Ages 30-40,US 1970-79



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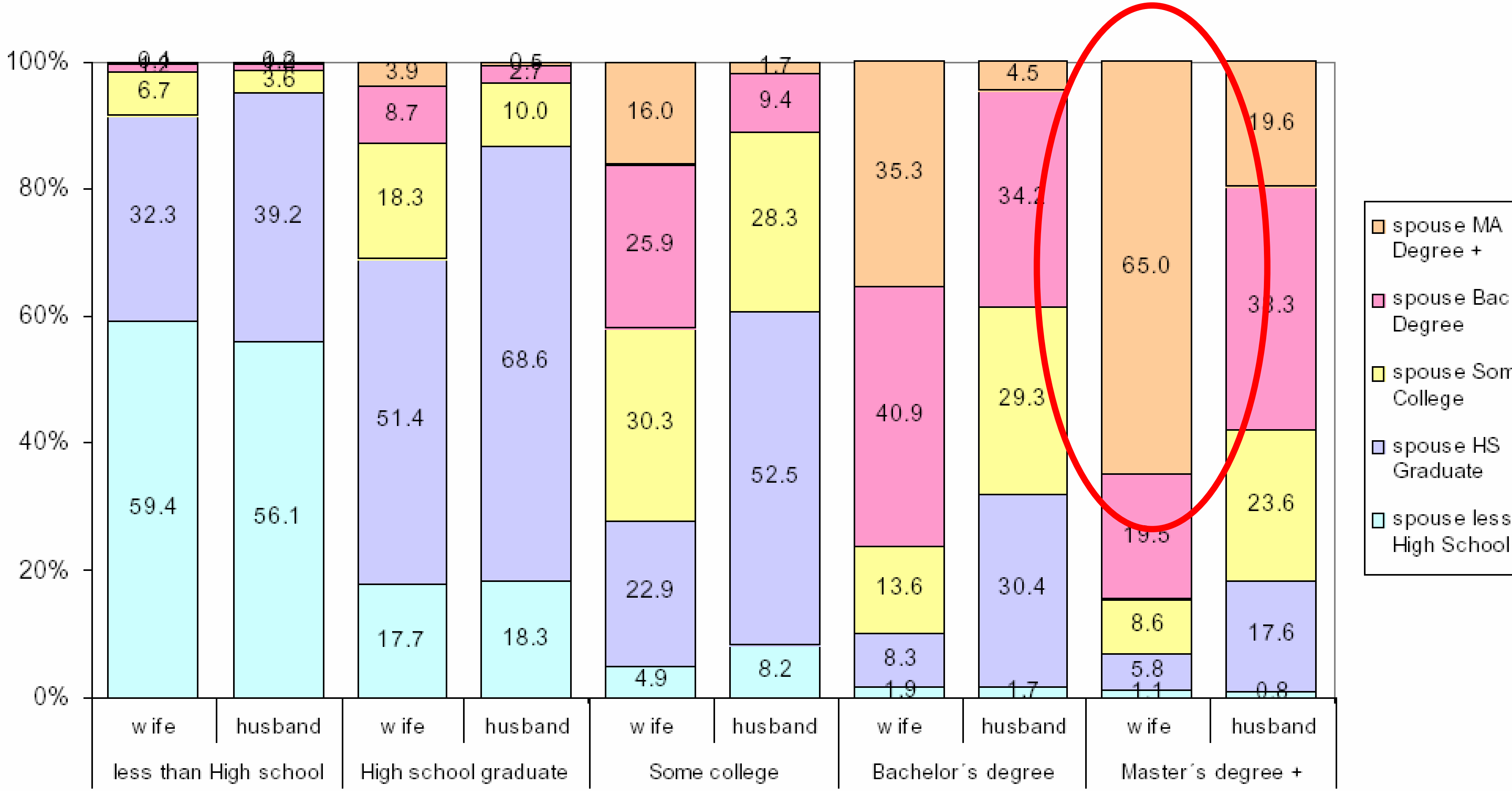

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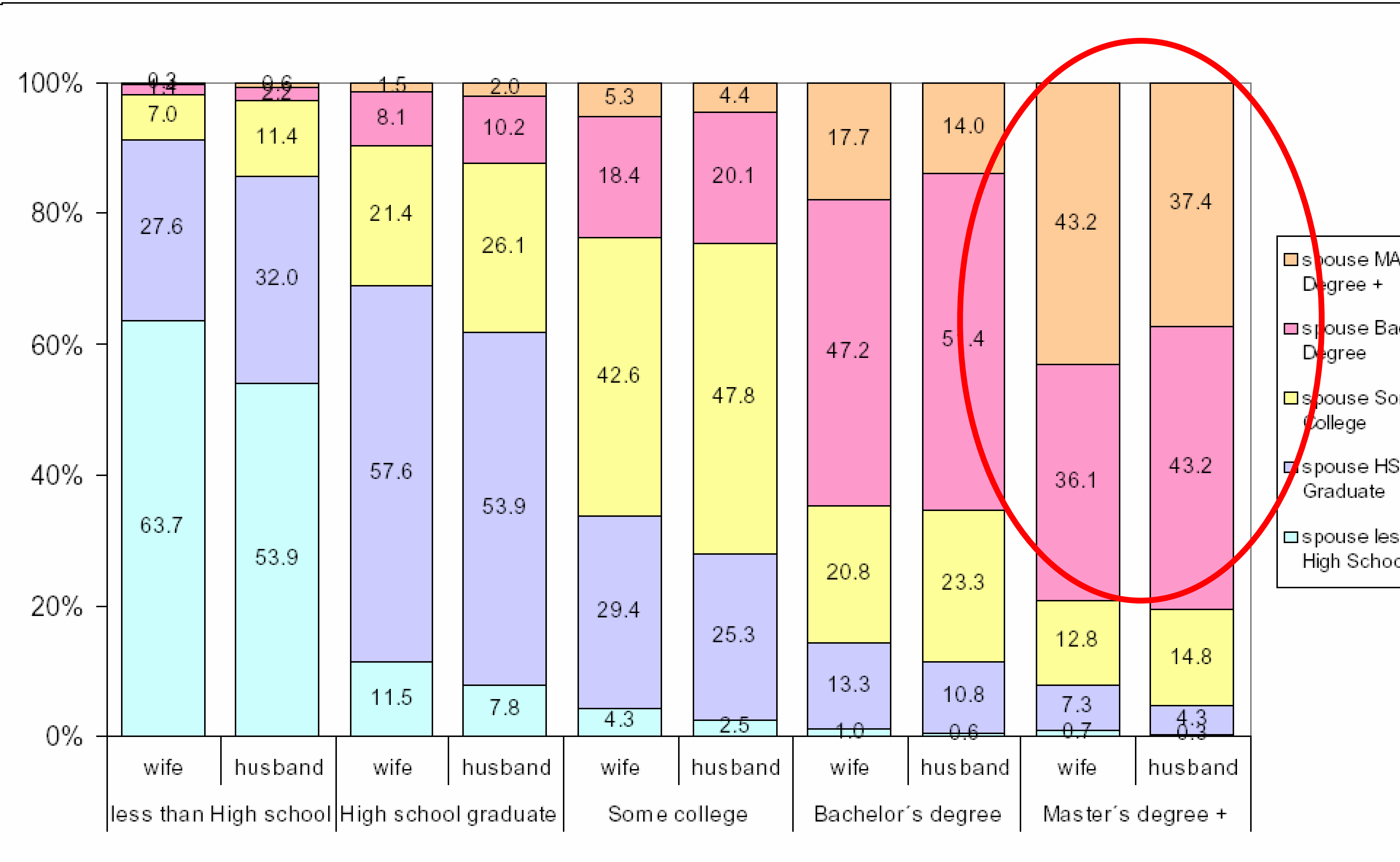
Figure 15b: Spouse Education by own Education, Ages 30-40, US 1996-2005



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The model

- Two equally large populations of men and women to be matched.
- Individuals live two periods; investment takes place in the first period of life and marriage in the second period; investment in schooling is lumpy and takes one period.
- All agents of the same level of schooling and gender receive the same wage.
- I(i) and J(j) are the schooling "class" of man i and woman j: I(i)=1 if i is uneducated and I(i)=2 if he is educated, J(j)=1 if j is uneducated and J(j)=2 if she is educated.
- Transferable utility; marital surplus if man i marries woman j:

$$s_{ij} = z_{I(i)J(j)} + \theta_i + \theta_j$$

with

$$z_{11} + z_{22} > z_{12} + z_{21}$$

The model

- Investment in schooling is associated with idiosyncratic cost (benefit), μ_i for men and μ_j for women.
- θ and μ independent from each other and independent across individuals; distributions $F(\theta)$ and $G(\mu)$.
- Shadow price of woman j is u_j, shadow price of man i is v_j; stability:

$$z_{I(i)J(j)} + \theta_i + \theta_j \leq v_i + u_j$$

therefore

$$v_i = Max\{\underset{j}{Max}\,[z_{I(i)J(j)} + \theta_i + \theta_j - u_j], 0\}$$

$$u_j = Max\{\underset{i}{Max}\,[z_{I(i)J(j)} + \theta_i + \theta_j - v_i], 0\}.$$

Findings

Compare an "old" regime a "new" regime.
In the old regime:
- lower returns to education
- more time to be spent at home
- 'social norms'

In both regimes women suffer from statistical discrimination and earn less than men; weaker against educated women. Then:

- Schooling serves as an instrument for women to escape discrimination.
- The return for education of women within marriage is higher in the new regime and the may invest more than men.
- Some women marry down and the returns of schooling of men declines.

Application 3: marriage dynamics and the impact of divorce laws (CIW)

Basic question:

Take a reform of laws governing divorce, which favors women (typically: income/wealth sharing). What would be the impact on intrahousehold allocation?

Note: UK 2000 (see Kapan)

Answer: basic distinction between existing couples and couples to be formed.

- Existing couples: unambiguously favors women
- Future couples: the law is *taken into account* at the matching stage
  - No impact on lifetime utilities
  - Only impact on timing
  - Women lose during marriage, especially at the beginning
  - Application: Wolfers,…

Conclusion

1. Matching models provide an interesting technology for
   - studying marital patterns
   - assessing the consequences in terms of intrahousehold allocation

2. Alternative approaches are possible (search,…)

3. Promising empirical perspectives : *joint* estimation of matching and household behavior